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INNOVATIVE DEVELOPMENTS AND INTEGRATION OF SINE-APPROXIMATION AND TIME INTERVAL ANALYSIS METHODS FOR PRIMARY VIBRATION CALIBRATION BY HETERODYNE INTERFEROMETRY

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Abstract: In these paper, innovative developments in sine approximation and time interval analysis methods (SAM and TIA) for primary vibration calibration by heterodyne interferometry are described in detail. The frequency demodulation of novel TIA is based on estimation of time intervals between neighbouring peaks and valleys instead of traditional positive or negative zero-crossings of Laser Doppler Interferometer (LDI) signal, while the simplified SAM uses digital reference signal to generate quadrature signals and perform phase demodulation and differentiation to restore time history of velocity. Because of simplified algorithm and low requirements on hardware, these developments lead to a successful integration of SAM and TIA, both efficient and cost effective, for accurate result and reliable comparison in a single measurement, and to a novel implementation of national medium frequency vibration standard (10 Hz to 10 kHz) in National Institute of Metrology (NIM), China. The validity and accuracy of the novel methods is proved by simulation and will be further verified by a bilateral comparison underway.

Keywords: primary vibration calibration, Sine approximation method, time interval analysis method, heterodyne interferometry.

1. INTRODUCTION

Heterodyne laser interferometry technique proves its value in vibration calibration, especially when vibration displacement is in nanometer range at high frequency with moderate acceleration level applied [1-3]. Two methods, SAM and TIA, have been established, well investigated and implemented in various versions for calibration of complex sensitivity of accelerometers based on phase demodulation and frequency demodulation respectively [4-8].

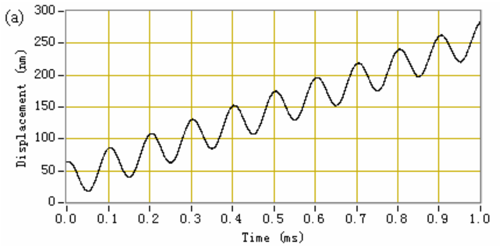
The current heterodyne implementation of SAM and TIA are independent in hardware and software with respect to signal acquisition and processing. High-speed data acquisition cards (50 MS/s) and high-resolution time interval analysers (50 ps) have proved to be possible hardware choices for good measurement results [4]. Unfortunately, it is almost unavoidable in either case to store and process a large size of data samples. Recent study has shown that lower sampling rate can speed up the calculation procedure of TIA to some degree, but result in intolerably inaccurate measurement results [9].

However, it is not necessarily to be so with newly developed algorithm for TIA, for which frequency demodulation is based on estimation of time intervals between neighbouring peaks and valleys of LDI signal instead of successive positive or negative zero-crossings. It makes possible to apply a relatively slow sampling rate (5 MS/s) to digitise LDI signal that can be used for both SAM and TIA. On the other hand, the simplified SAM of heterodyne version uses digital reference signal to generate quadrature signals and perform phase demodulation and differentiation to restore time history of velocity. All these lead to the possibility of single channel sampling of LDI signal with normal speed data acquisition card instead of dual channel high-speed one or expensive high-resolution time interval analyser, and to a novel implementation and integration of SAM and TIA, both efficient and cost effective, for accurate results and reliable comparison in a single measurement.

The innovative integration and successful implementation of simplified SAM and novel TIA for the national medium frequency vibration standard provides NIM, China with better primary vibration calibration capabilities over a wide frequency range from 10 Hz to 10 kHz. Based on sufficient theoretical investigation and experimental verification, these two methods are valid and efficient enough to be adopted, among other more conventional primary vibration calibration methods, for a bilateral comparison in vibration measurement underway.

2. THEORETICAL BACKGROUND AND PRACTICAL CONSIDERATIONS

The author's previous investigation of primary vibration calibration by homodyne interferometry has revealed the stimulation of slow relative motions shown as drift between the device under test and the laser interferometer at high frequency when moderate acceleration level is applied [10]. Two main motion disturbances are caused by hum of power amplifier and resonance of vibration isolator. These motion disturbances physically exist during the operation of every vibration calibration system even with the best power amplifier and vibration isolation system available employed. The phenomenon is more obvious at high vibration frequency, which is shown in Fig. 1(a). The slow drift can be easily recognized from displacement time history recovered by phase demodulation, with a sampling rate of 5 MS/s for the digitisation of LDI signal whose carrier frequency is 1.25 MHz for a sampling period of 1 ms at 10 kHz. The influence from slow motion disturbances can be effectively suppressed by numerical differentiation of displacement time history. The resulting velocity time history is demonstrated in Fig. 1(b) with high frequency noisy content amplified to some extent.



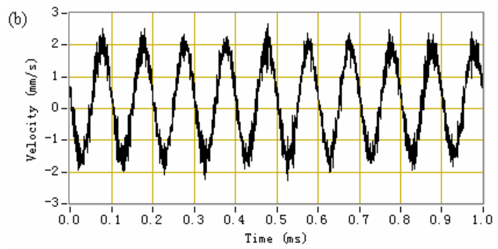


Fig. 1. Displacement (a) and velocity (b) time histories of vibration motion at 10 kHz superimposed mainly by motion disturbances (a) and noise (b).

In fact, the frequencies and velocity amplitudes of various relative motions are of the most interesting aspect for frequency demodulation, which are calculated by frequency domain analysis method and shown in Table 1. In this case, only two main motion disturbances are taken into account: drift caused by hum of power amplifier (about 50 Hz) and vibration isolator (about 4 Hz). Even lower frequency disturbance caused by resonant effect of vibration isolator does exit, but has negligible influence on acceleration measurement results at vibration frequency lower than 10 kHz.

Table 1. Frequencies and velocity amplitudes of vibration motion and motion disturbances at high vibration frequencies.

Vibration frequency (kHz)	Vibration velocity (mm/s)	Frequency of disturbance1 (Hz)	Velocity of disturbance1 (mm/s)	Frequency of disturbance2 (Hz)	Velocity of disturbance1 (mm/s)
4	3.00	49.79	0.03	3.86	0.01
5	2.49	50.43	0.02	4.17	0.01
6.3	2.05	50.03	0.02	4.11	0.01
7	1.90	50.02	0.02	4.16	0.01
8	1.71	50.00	0.02	4.14	0.01
9	1.63	49.96	0.02	4.06	0.01
10	1.43	49.97	0.03	3.97	0.01

It should be noted that the acceleration amplitude under measurement is quite stable because the long-term drift has no significant influence on the comparably very high acceleration level measured, normally about 100 m/s^2. Therefore, high measurement accuracy for voltage amplitude and initial phase of accelerometer output signal can be achieved since its sensitive element corresponds to the change of the acceleration. The specific procedure for calculation of this voltage signal can be referred to [1].

By taking the real situation influence quantities into account, three important conclusions can be drawn which lay a solid basis for the feasibility and measurement accuracy of innovative implementation of SAM and TIA: (1) Though time interval analyser can measure time intervals of a LDI signal with a resolution of 50 ps, this measurement accuracy ensured by hardware can not assure the target low uncertainty of the whole calibration system because the relative motions obscure the ideal time intervals modulated only by vibration motion under measurement between successive zero-crossings; (2) To reduce the unfavourable influence from relative motions, integer cycles of motion disturbances should be covered for signal processing and further calculation based on their periodical characteristics; (3) For suppression of drift effect, TIA based on frequency demodulation of LDI signal is preferred for acceleration measurement. As for SAM, numerical derivative is recommended to be used for displacement time history recovered from phase demodulation before least square approximation is applied for the calculation of velocity amplitude and initial phase.

3. SIMPLIFIED SAM

The SAM of heterodyne version has been described a lot in previous publications [2,4,7-9]. Almost in each case, a reference signal generated by mixing of two signals from internal quartz oscillator of Bragg cell and external synthesis generator is required for the generation of quadrature signals, a basis of phase demodulation. It is proposed in [2] that the reference signal can be generated by digital signal processing procedure on the precondition that the exact value of carrier frequency can be acquired accurately enough. This is not easy because the drive frequency of

Bragg cell may drift from time to time, which makes it difficult to track the actual instant frequency value even with extra instrument for frequency measurement. On the other hand, it is unnecessary to do so since the information of interest is the Doppler modulation content carried in a narrow bandwidth rather than the instant carrier frequency value itself. Further more, the unfavourable effect on frequency or phase modulation caused by frequency instability of the carrier signal (normally in the order of 10^{-6}) is minor compared with that caused by relative motions.

After the down mixing of original LDI signal, the modulated measurement signal can be expressed as:

$$u_m(t) = \hat{u}_m \cos[\omega_C t + \varphi_M(t) + \varphi_0] \tag{1}$$

with a modulation term

$$\varphi_M(t) = \hat{\varphi}_M \cos(\omega t + \varphi_s) \tag{2}$$

whose amplitude $\hat{\varphi}_M$ is proportional to the displacement, $\hat{\varphi}_M = \dfrac{4\pi\hat{s}}{\lambda}$. φ_s stands for displacement initial phase angle and ω for vibration angular frequency. The initial phase angle of the modulated measurement signal is denoted by φ_0 and the transformed carrier angular frequency by ω_C.

The digital reference signal with angular frequency ω_r can be generated precisely by a procedure as:

$$u_r(t) = \hat{u}_r \cos \omega_r t \tag{3}$$

After digital mixing and forward-and-reverse low pass filtering, the expected quadrature signals are generated in the form of:

$$u_1(t) = \frac{1}{2}\hat{u}_m\hat{u}_r \cos[(\omega_C - \omega_r)t + \varphi_M(t) + \varphi_0] \tag{4}$$

$$u_2(t) = \frac{1}{2}\hat{u}_m\hat{u}_r \sin[(\omega_C - \omega_r)t + \varphi_M(t) + \varphi_0] \tag{5}$$

where the modulation phase is

$$\varphi_{Mod} = (\omega_C - \omega_r)t + \varphi_M(t) + \varphi_0 \tag{6}$$

It should be noted that $\hat{u}_m$ is the amplitude of electric modulated measurement signal and $\hat{u}_r$ is the amplitude of digital reference signal, and that it is not necessary to meet the relationship $\hat{u}_m = \hat{u}_r$.

The traditional goal is to obtain quadrature signals in base band similar to homodyne interferometer quadrature output signals but with better quality by keeping $\omega_C \approx \omega_r$ as precisely as possible. Consequently, an electric path for generation of a reference signal or a device for measuring instant drive frequency of Bragg cell is needed. After the phase demodulation of the same LDI signal data as for Fig. 1(a) but within a longer time period of 20 ms, displacement time history is demonstrated in Fig. 2(a), with 50 Hz relative motion clearly seen.

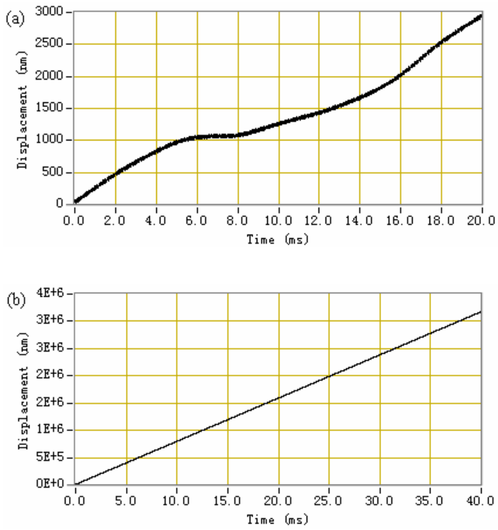


Fig. 2. Displacement time histories of vibration motion at 10 kHz based on phase demodulation of quadrature signals with (a) $\omega_C \approx \omega_r$ and (b) $\omega_C \neq \omega_r$.

When there is a constant difference between transformed carrier and reference frequencies, a speedily varying phase equal to $(\omega_C - \omega_r)t$ is added to original modulation phase component representing the measurand. If this frequency difference is high enough to include the actual bandwidth of the LDI signal modulated mainly by vibration motion, the consequent phase demodulation will reveal a displacement series almost linearly increasing with time as shown in Fig. 2(b). In this case, 250 kHz frequency difference is used for the generation of quadrature signals and serves as a new carrier frequency modulated by vibration motion and other motion disturbances. The linearly increasing phase component appears as a velocity constant in Fig. 3, which presents the resulting time history after the digital differentiation of the displacement series shown in Fig. 2(b).

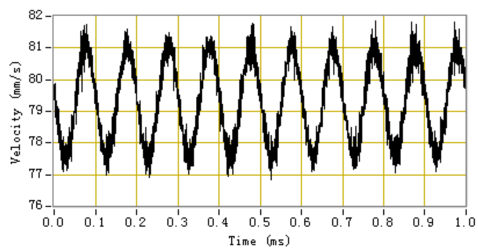


Fig. 3. Velocity time history after digital differentiation of displacement series in Fig. 2(b).

The processing of this velocity series by conventional sine-approximation leads to the amplitude and initial phase of the measured velocity, and finally to those of acceleration under measurement.

4. INNOVATIVE TIA

The TIA of heterodyne version has been implemented with time interval analyser or high speed ADC card for data acquisition. Though their common goal is to acquire the time intervals between successive zero-crossings, the former hardware choice can provide this information directly and the latter makes it possible to locate the instant time series of successive zero-crossings by third order polynomial fitting [4]. However, the previous investigation on TIA algorithm for the latter case has indicated its two disadvantages as both time consuming and inaccurate [9].

The innovative TIA has no special requirements on data acquisition facility because its algorithm focuses on the time intervals between neighbouring peaks and valleys instead of zero-crossings. Even with a small number of samples per period of the LDI signal that is close to the Nyquist-limit, the time locations of all its peaks and valleys can be obtained fairly accurately with this new algorithm while polynomial fitting will certainly fail to find exact time locations of zero-crossings. The key lies in the well-known fact that LDI signal is stable and affected little by various electric disturbances compared with homodyne interferometry signal.

The originally sampled LDI signal may consists of only a few samples per period as shown in Fig. 4(a) where 5MS/s sampling rate is applied for the carrier frequency of 1.25 MHz. Four samples per period as an average is definitely not sufficient for direct detection of either zero-crossings or peaks and valleys. Therefore, software-based resampling that computes new values based on existing signal samples is a must for new TIA algorithm. A cubic spline interpolation algorithm to compute the resampled values is applied to originally sampled signal, yielding smooth transitions between the samples, shown in Fig. 4(b). The spline interpolation requires the interpolant to pass through all the sampled data for the computation of the interpolation and uses piecewise-continuous cubic polynomials for interpolation of the data set avoiding problems associated with higher degree, single polynomial interpolation.

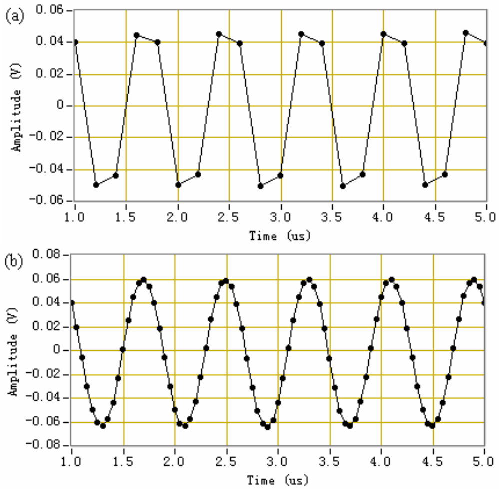


Fig. 4. Plots of originally sampled LDI signal (a) and resampled signal with spline interpolation (b) .

For the peaks and valleys detection, a progressive quadratic fit on a window consisting of a certain number of data points is applied iteratively with the slide of the window [11]. For the fit, it finds the second derivative, and checks to see if the slope changes, to determine if there is a peak or a valley. The quadratic fit algorithm returns the peak locations as floating point numbers, not as integer index values. This feature is an advantage of the algorithm because it effectively interpolates between the data points while finding peaks and valleys. The function can therefore measure a peak or valley that has greater amplitude than any data points near the peak or valley. This interpolation provides a good indication of the true value of the peak or valley in the original signal.

Based on the time locations t_i of all the peaks and valleys of the modulated measurement LDI signal, a series of time intervals between neighbouring peaks and valleys can be calculated by $\Delta t_i = t_{i+1} - t_i, i = 0,1,..., N-1$. A non-equidistant time series of instant frequencies can be obtained by the relationship:

$$f(t^*_i) = \frac{1}{\Delta t(t^*_i)} \tag{7}$$

where $t^*_i = \frac{t_i + t_{i+1}}{2}, i = 0,1,..., N-1$.

The actual velocity series is given by

$$v(t^*_i) = \frac{\lambda}{4} f(t^*_i) \tag{8}$$

It needs to be mentioned that a higher demodulation resolution for velocity calculation can be acquired based on time intervals between neighbouring peaks and valleys, which is twice as that of conventional TIA algorithm. The amplitude and initial phase of velocity can be obtained then by the calculation of sine-approximation, which finally leads to those of acceleration.

5. SIMULATIONS

5.1. Simulation model

The theoretical model for transformed LDI signal is expressed in equation 1. However, the real measurement condition is more complicated affected by influence from noise and disturbance from relative motions, etc. The LDI signal then can be written as:

$$u_m(t) = (\hat{u}_v + 1)\hat{u}_m \cos[\omega_C t + \varphi_0 + \varphi_p + \sum_{i=0}^{n-1} \hat{\varphi}_{Mi} \cos(\omega_i t + \varphi_{si})] \tag{9}$$

where $\hat{u}_v$ is voltage amplitude deviation affected by random noise and denoted as percent of ideal signal voltage amplitude $\hat{u}_m$. φ_p denotes the phase disturbance in degree. The last item consists of various modulation terms of vibration and motion disturbances with different angular frequencies and initial phase angles because in reality, low frequency motion disturbances caused by hum and resonance also appear as modulation components in LDI signal.

To achieve reasonable simulation results, all the influence quantities mentioned above have to be well known from real tests of a certain system. In our simulation tests, the data of relative motions in table 1 are used. The random noise level of the signal is 3% in relation to its voltage amplitude, and the phase disturbance 0.2°. Other conditions concerning the simulated signals generated are: acceleration level equal to value obtained from real experiments (from about 75 to 90 m/s2), vibration frequency from 4 to 10 kHz, sampling rate 10 MS/s, sampling period 20 ms, and effective resolution 8 bits.

Since the initial phase angles of carrier frequency, vibration motion and motion disturbances have significant influences on measurement accuracy, they are randomly generated within the angle range from 0° to 180° for each simulation test. On account of this influence, one hundred simulation tests are performed under all the same test conditions with the only exception of four different random-generated initial displacement phase angles. Based on simulated LDI digital signal with limited precision described above, the arithmetic mean and experimental standard deviation of the calculated deviations between nominal acceleration amplitude and initial phase values and those calculated by SAM and TIA can be estimated.

2.2. Simulation results

In Table 2, some of the results from the numerical simulation tests over frequency range from 4 kHz to 10 kHz are presented to illustrate the effectiveness of SAM and TIA for good measurement results of acceleration amplitude and initial phase. This high frequency range is troublesome and of particular interest because long term drift with large displacement amplitude is stimulated and also modulated the LDI signal.

For amplitude and initial phase measurement, the mean and standard deviation values for both methods are below 0.1% and 0.1° with confidence level of 95% though they increase gradually with higher frequencies.

Table 2. Comparison results of simulation tests for SAM and TIA from 4 to 10 kHz.

Vibration frequency (kHz)	SAM				TIA			
	Amplitude		Initial phase		Amplitude		Initial phase	
	Mean (%)	Standard deviation (%)	Mean (°)	Standard deviation (°)	Mean (%)	Standard deviation (%)	Mean (°)	Standard deviation (°)
4	0.00	0.00	0.00	0.00	-0.01	0.01	0.00	0.01
5	0.00	0.00	0.00	0.00	-0.01	0.01	0.00	0.01
6.3	-0.01	0.00	0.00	0.00	-0.01	0.02	0.00	0.02
7	-0.01	0.00	0.00	0.00	0.01	-0.02	0.00	0.01
8	-0.01	0.01	0.00	0.00	-0.01	0.04	0.00	0.03
9	-0.01	0.02	0.00	0.01	0.00	0.04	0.00	0.03
10	-0.01	0.03	0.00	0.02	-0.01	0.03	0.00	0.02

Obviously, the results from the simulation tests confirm the expected good performance and agreement of the two methods, due to: (1) integer cycle of relative motions is used as sampling period and their influence is minimized by sine-approximation; (2) non-linear effects of the photo detector as well as of all signal pre-processing stages do not affect the integrity of the Doppler modulation content as only high frequency AC signals are transmitted; (3) random influences such as air current disturbance, interferometer residual effect and frequency instability of oscillator in Bragg cell, have not been taken into account.

6. CONCLUSION

A successful implementation of novel SAM and TIA (heterodyne version) has been described. Instead of using high resolution time interval analyser or high speed data acquisition card to estimate time intervals between successive zero crossings of LDI signal, the novel TIA is based on the estimation of time intervals between neighbouring peaks and valleys of the signal, which is digitised by a sampling rate of 5MS/s. On the other hand, the simplified SAM makes use of the same LDI signal as for TIA and digital reference signal provided by signal processing procedure to generate required quadrature signals for phase demodulation. This innovative development allows, among other things, a possible integration of SAM and TIA based on the same data acquisition hardware of one ADC card within one vibration calibration system.

The resampling procedure and peak/valley detection algorithm for TIA is efficient and powerful, even when the sampling rate of ADC card is close to the Nyquist-limit, which would be definitely impossible if conventional TIA based on detection of time locations of successive zero crossings were applied. To reduce the significant influence on frequency/phase modulation from relative motions at high frequency, the simplified SAM performs phase demodulation and differentiation to restore time history of velocity before the amplitude and initial phase of acceleration under measurement are calculated. The novel algorithm for TIA and SAM results in not only lower and less requirements on hardware for data acquisition and reference signal generation but also a reliable comparison between frequency and phase demodulation methods in a single measurement, which is further investigated by simulation experiments taking into account various influences observed under real situation.

Based on the research results of SAM and TIA mentioned above, the national medium frequency vibration standard of NIM has been further developed and employed, among other national vibration standards, in a bilateral comparison from 10 Hz to 10 kHz under way.

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