XVIII IMEKO WORLD CONGRESS
Metrology for a Sustainable Development
September, 17 – 22, 2006, Rio de Janeiro, Brazil

EVALUATING THE UNCERTAINTY FOR MAGNETIC SUSCEPTIBILITY USING THE DIRECTLY METHOD

Sheau-Shi Pan [1],*Hui-Ching Lu* [2], *Chi-Sheng Chang* [3]
[1] Center for Measurement Standards, Hsinchu, Taiwan, sheau.shi.pan@.itri.org.tw
[2] Center for Measurement Standards, Hsinchu, Taiwan, franceslu@itri.org.tw
[3] Center for Measurement Standards, Hsinchu, Taiwan, ChiSheng.Chang@itri.org.tw

Abstract: The volume magnetic susceptometer was established in the CMS mass laboratory in 2003. The method is referred to the experimental method and procedure developed by Davis; while the distances between weighs and the magnetic sample were measured with the laser interferometer. We calculated the effect by the offset between the magnetic source and the weights; then we evaluated the uncertainty of the χ -value in samples directly.

Keywords: magnetic susceptometer ,laser interferometer, susceptibility

1. INTRODUCTION

The magnetic suscepto-meter developed by Davis[1]at the Bureau International des Poids et Mesures(BIPM).This device was used to measure the volume magnetic susceptibility of the stainless-steel weight standards; under the assumption of linear ,homogeneous and weak susceptibility samples or standards. The suscepto-meter included the small cylindrical magnetic source sample , lifting gauge blocks and a high precision balance.

As this type of suscepto-meter was widely used; Several results of the volume magnetic susceptibility measurement on the 1 kg standards was measured. These results under the earth magnetic field [1-3] and shield the earth magnetic field [4-5].The relevant measurements on the magnetic interactions between weights and balances were found [6-7]. The method was used in assuming the magnetic source is the dipole source and the weight sample is semi-infinite slab. The nearest distances between magnetic source and the weight sample(Z_0) was evaluated by a convergent iterating way. When the gauge blocks were used to change lifting distances values Z in a discrete type that was used in evaluating the effect of magnetic effect; we can get readings of the balance which showed the magnetic effects on the stain-steel weight standard. Recently, there are more research about the magnetic properties of weights were evaluated by OIML shapes[9] and the errors due to the magnetic effects in 1 kg primary mass comparators[10].

About this work, we modified the gauge lifting blocks by combining the aluminum slide guide and a compact three axis laser interferometer to measure the Z_0 and the other values of lifting heights Z. During the experiment; the heights of the lifting samples and the readings of the balance were recorded by computer simultaneously. According these access datum; we could calculated the volume magnetic susceptibility χ and the magnetic effect to the height continuously [10]. The uncertainties of our method were caused by the offset degree of the axes between the weights and the magnetic source. We also calculated the uncertainties by geometry factor of weights and the effect of distance Z. Then the uncertainty of the suscepto-meter was evaluated directly.

2. BASIC PRINCIPLES

We assumed the field produced by the small cylinder magnetic disc was a dipole field. The magnetic field of earth was assumed uniformly in our area of experiment. The magnetic field acted on a sample with the form of a semi-infinite slab and approximated the magnetic flux as being uniformly along its axis. These fields can be considered from the magnetic dipole potential [2].

$$F = \chi F_{\max} I_a + \frac{\mu_0}{4\pi}\left(\chi H_{E_Z} + M_Z\right)\frac{m}{Z_0} I_b$$
$$\equiv F_a + F_b \tag{1}$$

where

$$\frac{3\mu_0 \chi m^2}{64\pi Z_0^{\ 4}} \equiv \chi F_{\max}, \tag{2}$$

and

$$I_a = -\frac{32\pi}{3m^2}\frac{\partial}{\partial Z_{0n}}\iiint_{V_n} H_{mag}\cdot H_{mag} dV$$
$$= -\frac{2}{3\pi}\frac{\partial}{\partial Z_{0n}}\iiint_{V_n}\frac{\rho^2+4z^2}{\left(\rho^2+z^2\right)^4}\rho d\rho d\theta dz, \tag{3}$$

$$I_b = -\frac{4\pi}{m}\frac{\partial}{\partial Z_{0n}}\iiint_{V_n}\left(H_{mag}\right)_z dV$$
$$= -\frac{\partial}{\partial Z_{0n}}\iiint_{V_n}\frac{\rho^2 - 2z^2}{\left(\rho^2 + z^2\right)^{5/2}}\rho d\rho d\theta dz, \qquad (4)$$

Where χ is the effective volume magnetic susceptibility of the standard, the parameter μ_0 is the vacuum permeability, identically equal to $4\pi\times10^{-7} N\cdot A^{-2}$. χH_{E_Z} is an induced magnetization that has the same effect as a permanent magnetization M_Z ;. H_Z is the z-component of the total magnetic field $\vec{H}$ from the magnetic sample. The first term in Eq. (1) is given by $F_a = {(F_1 + F_2)}/{2}$, and the second term by $F_b = {(F_1 - F_2)}/{2}$. The initial force measurement F_1 is made with the north pole of the magnet pointing down and a second measurement F_2 is made (at the same Z_0) with the north pole pointing up. With Z measured by laser interferometer directly; equations are modified as follows:

$$F_1 = \chi F_{\max} I_a + \frac{\mu_0}{4\pi}\left(\chi H_{E_Z} + M_Z\right)\frac{m}{Z_0} I_b$$
$$= \Delta m_1 \times g \qquad (5)$$

$$F_2 = \chi F_{\max} I_a + \frac{\mu_0}{4\pi}\left(\chi H_{E_Z} + M_Z\right)\frac{(-m)}{Z_0} I_b$$
$$= \Delta m_2 \times g \qquad (6)$$

$$F_1 + F_2 = 2\chi F_{\max} I_a = (\Delta m_1 + \Delta m_2)\times g \qquad (7)$$

$$F_1 - F_2 = \frac{\mu_0}{2\pi}\left(\chi H_{E_Z} + M_Z\right)\frac{m}{Z_0} I_b$$
$$= (\Delta m_1 - \Delta m_2)\times g \qquad (8)$$

$$\chi(x, r, h, Z, \mathrm{Im}) = (\Delta m_1 + \Delta m_2)\times g / (2F_{\max} I_a) \quad (9)$$

$$u_x = \frac{\partial\chi}{\partial x}\Delta x \qquad (10)$$

$$u_r = \frac{\partial\chi}{\partial r}\Delta r \qquad (11)$$

$$u_h = \frac{\partial\chi}{\partial h}\Delta h \qquad (12)$$

$$u_Z = \frac{\partial\chi}{\partial Z}\Delta Z \qquad (13)$$

$$u_{\mathrm{Im}} = \frac{\partial\chi}{\partial \Delta m}\Delta(\mathrm{Im}) \qquad (14)$$

Where,

Δx : Offset of the central of the magnetic sample
Δr : Uncertainty of radius measured in weight
Δh : Uncertainty of height measured in weight
ΔZ : Uncertainty of distance Z measurement
$\Delta\,\mathrm{Im}$: Uncertainty from UMT5- balance reading

u_x : Uncertainty of χ by Δx
u_r : Uncertainty of χ by Δr
u_h : Uncertainty of χ by Δh
u_Z : Uncertainty of χ by ΔZ
u_{Im} : Uncertainty of χ by $\Delta\,\mathrm{Im}$

The combined uncertainty u_χ was given by the equation (10) - (14):

$$u_\chi = \sqrt{u_x^2 + u_r^2 + u_h^2 + u_Z^2 + u_{\mathrm{Im}}^2} \qquad (15)$$

The degree of freedom ν_{eff} is got ten by

$$\nu_{eff} = \frac{u_\chi^4}{\frac{u_x^4}{\nu_x} + \frac{u_r^4}{\nu_r} + \frac{u_h^4}{\nu_h} + \frac{u_Z^4}{\nu_Z} + \frac{u_{\mathrm{Im}}^4}{\nu_{\mathrm{Im}}}} \qquad (16)$$

Combined factor (k) can be given by a statistical table[12].

Figure 1 shows the magnetic sample with moment m and Z_0 is the distance between the center of the magnet and the standard weight. Z_0 was determined by following steps:

1. Let the moving guide touch the movable platform; the positions were recorded by laser interferometer that serves as the starting point.
2. Movable platform moved and the moving guide sliding down slowly. The position datum of the moving guide and the readings of balance were recorded by the laser interferometer and the UMT5 -balance simultaneously.
3. As the moving guide touched the magnetic sample; the readings of balance changed suddenly and values fell out of reading range. The corresponding readings of the laser interferometer and the balance were checked. Z_0 was determined accordingly.
4. When the moving guide was lifted from the top of magnetic sample; the reading of the balance fell into reading range again.
5. Let the moving guide slide to a position higher than movable platform. The movable platform was placed back and was then loaded with the weight sample. The top of the slide guide touched the top of the weight sample. Davis' structure was changed to Z direction to record the readings of the interferometer and the balance.

When the experiment proceeds; the readings corresponding to the laser interferometer and the balance will be recorded by control program in PC.

According to these measurements; we can use formula(1)-(9) to calculate the force factors.

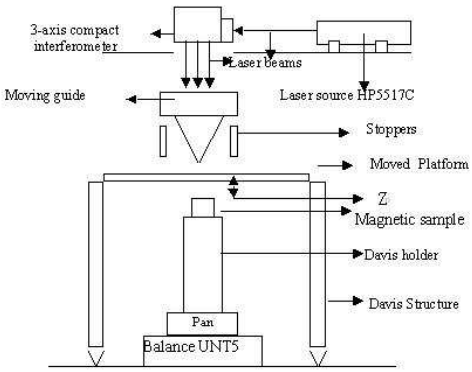

Fig. 1. Susceptometer in CMS

Because we want to evaluate the effect of offset position in sample weighed. The related position of weight and magnetic sample was shown in fig. 2. A cylindrical type neodymium-iron-boron magnet was used as the source of the magnetic field. The dimensions of the simple cylinder are nominally 6 mm in diameter and 5 mm in height. It was confined by a aluminum ring on the Davis holder. The confined ring is about 0.1 mm larger than the diameter of the small magnetic sample and the offset of the central of the magnetic sample was indicated by ΔX . Z_0 is the distance between the bottom of the weight and top of the magnetic sample. The uncertainties of ΔX and Z_0 were evaluated.

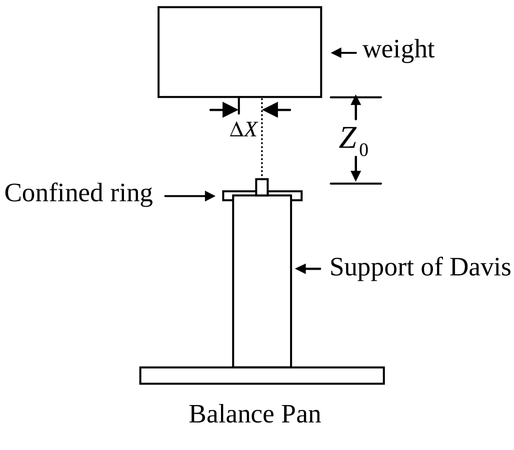

Fig. 2. Offset of the central line of the magnetic sample

3. RESULT

The values of susceptibility χ in two stainless-steel weights (ST1,ST2), CHYO, Titatium (TI) and Alacrite (ALA) were evaluated. They are shown in the fig.3 to fig.7.These figures expressed the values of χ and the uncertainties under the offset distance ΔX (2.5 mm). Because the reading of the balance is not sensitive by offset's effect. The huge offset's distance ΔX is moved. When the distance Z is larger than 30 mm; the magnetic force is weak and the uncertainty becomes larger in CHYO, TI and ALA.

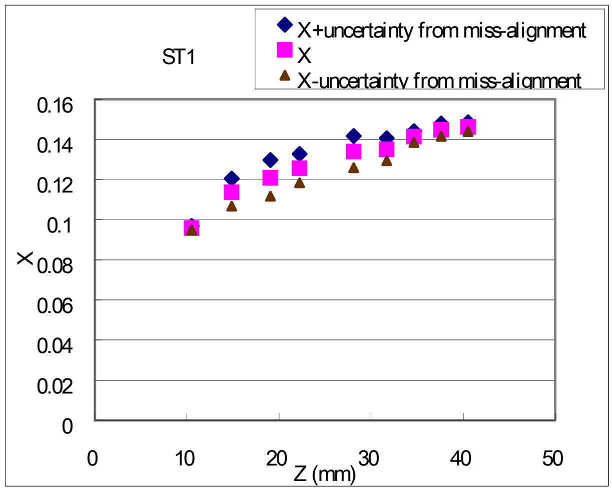

Fig. 3. χ values of ST1

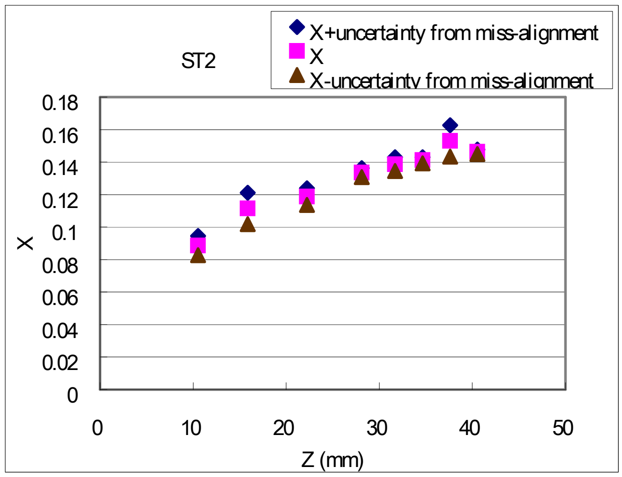

Fig. 4. χ values of ST2

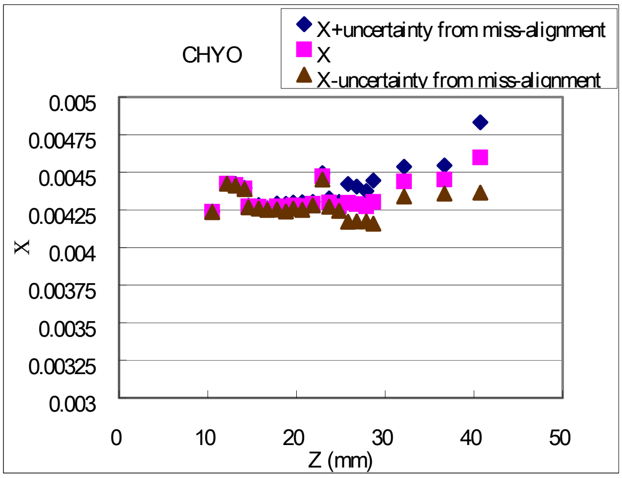

Fig. 5. χ **values of CHYO**

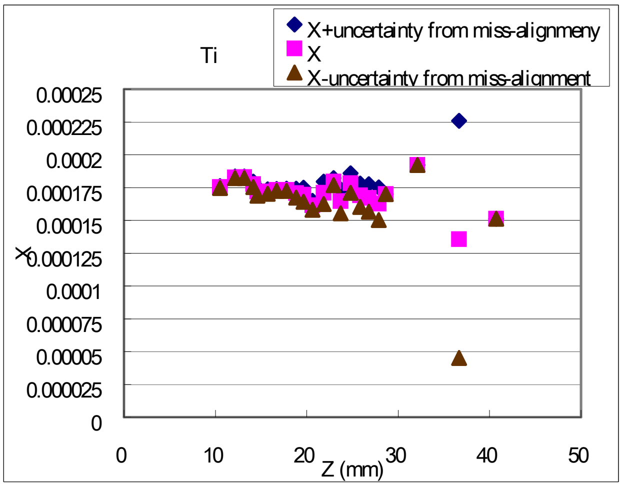

Fig. 6. χ **values of TI**

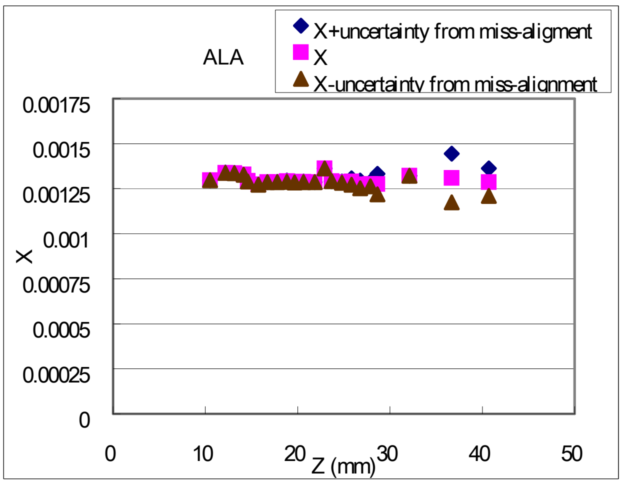

Fig. 7. χ **values of ALA**

The dimensions of these samples are listed in table 1.

We can calculate the force- factors and the uncertainties by table 1 and equations (1) to (16). These results of χ at the distance (Z_0) and the degrees of freedom (ν_{eff}) are shown in table 2. The cover factor (K) is listed too. After these works; we can get the un-certainties of magnetic susceptibility (χ) in table 3. The values of χ in ST1 and ST2 are larger than the other samples; the effects of offset are also larger than others. These two stainless-steel samples are better in χ measurement at a longer Z distance; but they are worse in standard used.

Table 1. Dimensions of weights

ITEMS	Radius (mm)	Height (mm)	Freedom
ST1	27.404 ± 0.007	27.001 ± 0.016	9
ST2	27.401 ± 0.008	26.987 ± 0.014	9
CHYO	26.791 ± 0.004	55.494 ± 0.005	9
TI	29.811 ± 0.003	24.955 ± 0.007	9
ALA	30.025 ± 0.024	24.955 ± 0.014	9

Table 2. uncertainty table of χ

ITEM	ST1	ST2	CHYO	TI	ALA	ν
u_x	0.00284	0.00623	6.2E-5	1.95E-5	3.63E-5	21
u_r	4.11E-7	3.8E-7	1E-9	1E-9	6E-9	9
u_h	5.9E-4	4.84E-4	8.51E-6	6.14E-7	2.52E-7	9
u_Z	1.1E-5	1.1E-5	4.9E-7	2.1E-8	1.5E-7	11
u_{Im}	0.0165	0.0214	9.34E-5	1.16E-5	2.4E-5	21
u_χ	0.0168	0.0223	0.000113	2.27E-5	4.36E-5	
ν_{eff}	20.9826	20.8727	19.2209	13.5943	14.1496	
k	2.08	2.08	2.093	2.145	2.145	

Table 3. values of χ

ITEMS	ST1	ST2	CHYO	TI	ALA
$\chi \pm k\sigma$	0.129 ±0.037	0.129 ±0.049	0.00432 ±9.4E-5	0.00017 ±4.9E-5	0.0013 ±9.4E-5

4. CONCLUSION

The values of susceptibility (χ) were measured directly and the analysis of uncertainties were evaluated by ISO-GUIDE[12]. As the measurements of distance can be measured precisely. We will develop the related methods in measurement of magnetic moment.

REFERENCES

[1] R.S.,Davis, New method to measure magnetic susceptibity" ,Meas. Sci. Technol. **4** ,pp.141-147,1993.

[2] R.S., Davis, Determining the magnetic properties of 1 kg mass standards, J. Res. Natl. Inst. Stand. Technol.,100 (1995) pp.209-225

[3] H.C. Lu and C. S. Chang, Evaluating the Magnetic Property of One Kilogram Mass Standard in Center For Measurement, APMF'2003 Proceeding pp.57-60,2003.

[4] J.W. Chung, J.-Y. Do,B.-S. Chon and R.S. Davis, ffect of earth magnetic field on measurement of volume magnetic susceptibility of mass, Metrologia,**37**,pp.65-70,2000.

[5] J.W. Chung, K.S. Ryu and R.S. Davis, Uncertainty analysis of the BIPM susceptometer, Metrologia, 38 , pp.535-541,2001.

[6] Michael Glaser, magnetic interactions between weights and weighing instruments, Meas. Sci. Technol. **12**, pp.709-715,2001.

[7] R Davis and M Glaser, agnetic properties of weights,their measurements and magnetic interactions between weights and balances, Metrologia **40** 339-355,2003.

[8] M. Oblak, Magnetic properties of OIML shaped weights, IMEKO TC3 19th International Conference for Force, Mass and Torque Measurements; Theory and Application in Laboratories and Industries,pp82,2004.

[9] R.S. Davis and M.-J Coarasa, Errors due to magnetic effects in 1kg primary mass comparators, IMEKO TC3 19th International Conference for Force, Mass and Torque Measurements; Theory and Application in Laboratories and Industries,pp88,2004.

[10] Sheau-shi Pan, H. C. Lu and Chi-Sheng Chang , Instruments for measuring the magnetic properties of one kilogram mass standard in center for measurements standards(CMS),p89,2004.

[11] Sheau-shi Pan, Chiu-Hsien Chen and Chi-Sheng Chang ,Evaluation of the uncertainty due to Abbe □ error for primary Rockwell hardness standard system, XVII IMEKO World Congress Meterology in 3rd Millennium June 22-27,Dubrovnik, Croatis pp.1000-1004,2003.

[12] Guide to the Expression of Uncertainty in Measurement, ISO, 1995.