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MEASUREMENT OF PHOTON PRESSURE

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Abstract: Center for Measurement Standards has established a system to probe micro force down to nano Newton. Based on a very stable torsion strip pendulum with restoring torque constant 5.25×10^{-6} N m/Rad. Light beam of a high power Nd:YAG laser was reflected by the mirror attached on pendulum frame. Result in angular deflection of the pendulum. The deflection was measured by an angular interferometer with resolution 1.95×10^{-7} Rad. Modulation signal, generated by varying power of laser, shift the pendulum from its standing position. The angular deflection equivalent torque is the same as photon generating torque to within 22%.

Keywords: photon pressure, torsion pendulum.

1. INTRODUCTION

Photon pressure is mechanical behavior of radiation. The amount is about several nano Newton when one watt laser reflected on the surface. Direct measurement of photon mechanical behavior is critical.

Micro cantilever or MEMS device could have stiffness lower than 1 N / m. With proper displacement measurement one should be able to detect photon pressure with certain accuracy. However, it was unable to coat low absorption films on structures at this scale without interfering stiffness. The laser will surely damage the surface with nominal metal film coating.

Torsion pendulum is considered as the most sensitive mechanical device on the earth. It was wildly used in experiments of fundamental science. For example, determine Newtonian constant or test of equivalent principle [1, 2]. Conventional torsion pendulums are highly dependent on torsional stiffness of fiber implemented in experiments. The surface to volume ratio is relative large. Environmental change for example temperature variation, moisture and pressure would cause the pendulum severe drift [1].

During the experiment setup for measuring Newtonian constant, Quinn et al has discover a load dependent torque of thin strip. This additional term plays an important role in designing torsion pendulum [3]. By careful putting on pendulum mass, the restoring torque could dominate by load dependent term. This is of great importance since this term is free from environment induced drifting [4].

NIST microforce realization project has design an electrostatic balance to bridge force to electric standards. The system build in NIST has sensitivity down to nano Newton [5]. With certain modification electrostatic force balance should be able to probe photon pressure with great accuracy.

Center for Measurement Standards has launched a microforce project since 2003. The goal of this project is to directly measure sub-micro Newton force with capability of tracing to mass standards [4, 6]. The force was generated by gravitation attraction and the angular deflection of torsion pendulum was quantitatively measured.

Using similar system setup with improved sensitivity, we could direct measure the mechanical behavior of light quanta. This was known as photon pressure or photon momentum.

2. EXPERIMENTAL SETUP AND RESULTS

Photon, name of light qunta, behaves the quantum nature of radiation. The energy carried by a photon is described as

$$E_{photon} = h\nu \tag{1}$$

Where h is the Plank constant

$$h = 6.63 \times 10^{-34} \quad \text{J s} \tag{2}$$

and ν is the frequency of radiation.

Now, considering the optical power of a laser beam, it is the energy flow in unit time. In the language of photon, the optical power is equivalent to the counts of photon per second.

$$P = \frac{Nh\nu}{t} \tag{3}$$

According to Einstein theory of special relativity, the momentum of photon p is written as

$$p = \frac{h\nu}{c} \tag{4}$$

Where c is speed of light in above equation.

If a photon normal incident on a mirror then totally reflected, photon changes its direction oppositely. Which result in momentum difference Δp

$$\Delta p = 2p \tag{5}$$

The force exert on mirror, F_{photon}, is the total momentum difference per unit time. From equation (3),(4) and (5) we conclude the applied force ,F ,on a mirror with power *P* is written as

$$F=\frac{\Delta p}{t}=\frac{2p}{t}=\frac{2P}{c} \quad (6)$$

As shown in eq.(6), a laser of 1 W output power will generate force approximately 6.67×10^{-9} Newton on the reflected plane.

2.1. Torsion strip pendulum

The major part of this experiment is based on a torsion strip pendulum. The torsion strip pendulum is an improved system of conventional torsion pendulum. The restoring torque constant of conventional fiber torsion pendulum τ_{wire} is written as

$$\tau_{wire}=G\frac{\pi r^4}{2L} \quad (7)$$

Where *G* is shear modulus, r is cross section radius of fiber and *l* the length of fiber. It is obvious the restoring torque constant is quartic proportion to fiber diameter and it is load independent. If we further replace the fiber with a long strip. The restoring torque constant under the condition $t << b << L$, could be written as

$$\tau_0=G\frac{bt^3}{3L} \quad (8)$$

Where t is thickness, b is width and L is length of the strip in above equation. Due to bifilar structure of strip. There is an additional restoring torque constant τ_1.

$$\tau_1=Mgb^2\frac{\left[1+\left(\frac{t}{b}\right)^2\right]}{12L} \quad (9)$$

Where *Mg* is the load applied on the strip. Obviously τ_1 is load dependent. The Overall restoring torque constant τ is

$$\tau=\tau_0+\tau_1 \quad (10)$$

With heavy loading on the strip, the total restoring torque constant is dominated by τ_1. It makes the torsion pendulum very stable and nearly zero drift. Compare to previous experiment, heavy load torsion strip pendulum is free from environment disturbance. This is of great importance since torsion pendulum is a sensitive devices base on small restoring constant. It took long time for single data taking. The drift became a severe problem though.

Fig. 1. Torsion strip pendulum

Figure 1 show the torsion strip pendulum developed in Center for Measurement Standards for measuring photon pressure. Pendulum is made of aluminum alloy. Two corner cubes placed in the center of the pendulum frame as angular reflector. High reflective plane mirrors were attached on opposite side of the pendulum. The distance from the center of plane mirrors to center of pendulum frame is 70 mm. Pendulum mass was placed by the rim of the frame. They were made of brass with diameter 30 mm and height 52 mm. The strip is made of Cu-Be foil of thickness 27 μm and width 1.2 mm. Total loads is set to 80% of yield. The free running motion of torsion strip pendulum in present experiment is about 5 milli degree and period nearly 190 seconds in nominal environment. With vacuum system evacuated as described below, the free running motion is lower than 3 milli degree.

In order to determine the restoring torque constant. The moment of inertia of pendulum should be determined first. It is straightforward to calculate the moment of inertia of pendulum mass. However, the pendulum frame is combine of several parts with different materials and complex shape. It is too complicate to evaluate moment of inertia of pendulum frame analytically. We measure the moment of inertia of pendulum frame by change the position of pendulum mass and compare the difference of angular frequency respectively. From eq.(8),(9),(10), change position of pendulum mass will not variate restoring torque constant τ.

$$\tau=Const.=I\times\omega^2 \quad (11)$$

Figure 2 is the curve fitting plot of moment of inertia and reciprocal quadratic angular frequency. Giving information of pendulum frame and pendulum mass, the moment of inertia of pendulum frame is measured to 4.8×10^{-3} kg m^2.

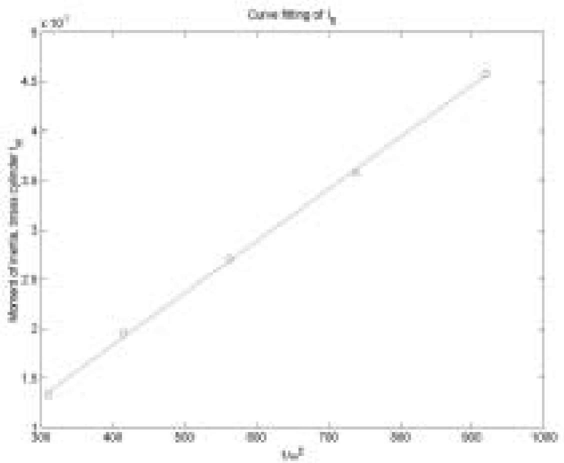

Fig. 2. Determination of moment of inertia of pendulum

Put together both the moment of inertia of pendulum frame and pendulum mass times the quadratic free running angular frequency, the restoring torque constant in present experiment is $\tau = 5.25\times10^{-6}$ rad/ N m.

Whole pendulum system was housed in a vacuum chamber to further isolating our system from environment air and acoustic disturbance. As shown in Fig. 3, there were three view ports. Center one is path way of angular measurement laser beams. The other two on opposite side are dedicated for passing high power laser to mirrors on the pendulum frame. The chamber was first evacuate to 5 mtorr, then shut the valve and leave the system “cool down” for a couple days before the experiments start.

Fig. 3. Experimental setup

2.2. Angular interferometer

The angular deflection of torsion strip pendulum is precisely measured by angular interferometer and heterodyne laser measurement system. Two parallel beam separate by 25.7 mm. The path difference of separate beam reflected from two cube corner reflectors is measured and converts to angular deflection.

Heterodyne laser measurement give 5 nm resolution on displacement measurement. This is equivalent to angular resolution of 1.95×10^{-7} rad. The resolution is 250 times smaller than free running motion of torsion strip pendulum in present experiments.

2.3. Lasers

There were two lasers implement in experiment setup. One is dedicated for angular measurement. The angular measurement laser is about 200 μW He-Ne laser with wavelength 633 nm. The laser used for generating photon pressure is a diode pumped Nd:YAG laser, with wavelength 1064 nm and output power is higher than 1.6 W. According to eq.(6), The laser could generate approximately 10 nN force on the mirror.

This laser system also equipped with a cavity enhanced frequency doubling unit which could convert the IR laser to green with 1 W output power. However, this is reserve for future works.

2.4. Experiment results

The laser power was modulated by varying diode current. Angular deflection of torsion strip pendulum reveals the variation of laser power modulation as shown in Fig. 4. As shown in the figure, it is not a good modulation scheme. Since laser has a threshold, it came out with tens of milli watt when lasing. The schemes in this experiment can not generate smooth power modulation.

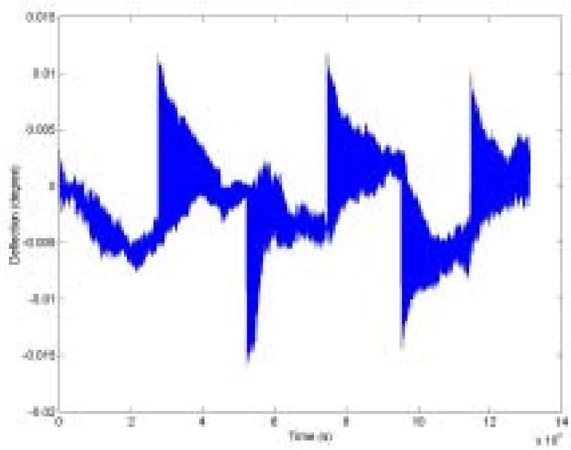

Fig. 4. Modulated signal of photon pressure

Figure 4 is the plot of angular deflection of torsion strip pendulum due to photon pressure modulation. The modulation signals took 20000 seconds from zero to maximum power and hold for 10000 seconds. Angular deflection sampling at 1 Hz, 131072 samples was took in current experiment.

In order to eliminate the “kicking” due to laser threshold, we further modify the modulation signals and elongating the “hold” time at both maximum power and zero. The experiment result as shown in figure 5. Sampling condition is the same as previous experiment. Totally 207701 samples was took. Clearly, the modulated photon pressure with longer holding gives higher contrast signals.

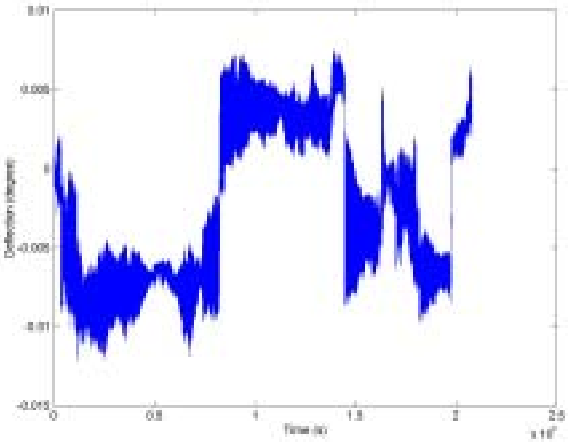

Fig. 5. Long holding modulated signal of photon pressure

3. DISCUSSION

The angular deflection in present experiments are dominate by high power Nd:YAG laser. The power of angular measurement laser is 3 orders of magnitude smaller. In the mean time, the torque is nearly cancelled by parallel beams across axis of rotation.

Torque generated from laser is the results of photon force multiply displacement from mirror to center of pendulum frame. The torque of torsion strip pendulum is measured by angular deflection, θ, multiply the restoring torque constant τ. Torque from photon and torsion pendulum is equal while the system is stable. Therefore

$$T = \tau \times \vartheta = F_{photon} \times l \qquad (12)$$

Quantitative determination of photon generating force according is relative difficult in present measurement data. Measured angular deflection due to photon force is between 7 ~ 10 milli degree. The measured angular deflection of torsion strip pendulum is equivalent to force applied on the mirror with magnitude between 9~13 nN. According to equation (6), a laser 1.6 W output power would generate force nearly 10.7 nN. Compare to power measured by power meter, the measurement result in present experiments are no more than 22% exceed and no less than 16% lower than the laser should have.

It should be notice that the gravitational attraction turns out to be “noise source” in present experiment. Since the gravitation force of a 60 kg adult approach the measurement system in distance of 0.5 m will contribute nearly 5 nN on the pendulum mass. Unfortunately, this noisy force is considerable large and unable to shield. Remote control and data manipulation could reduce gravitational disturbance due to working person.

4. OUTLOOK

In present experiments, the relation between modulated laser power and the angular deflection of torsion strip pendulum could be easily identified. However, due to environment disturbance, mostly from gravitation noise, it is uneasy to give a precise result of photon generating force. Poor modulation scheme further limits the data cycling period.

In the on going experiment, we have implemented a polarization modulation method. By rotating a half wave plate before polarizer, the laser power is sinusoidal modulated. Preliminary experiment results shows relative good contrast signals.

A gravitation force feedback system is now under construction to damp the free running motion of torsion strip pendulum. Further experiments with gravitation force generating system could quantitatively convert photon pressure to gravitation force. Finally, link the radiation power to Newtonian constant.

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