XVIII IMEKO WORLD CONGRESS
Metrology for a Sustainable Development
September, 17 – 22, 2006, Rio de Janeiro, Brazil

ESTIMATION OF THE CLOCK SIGNAL JITTER USING THE TIME-INTERVAL MEASUREMENT SYSTEM

Marek Zielinski [1], *Marcin Kowalski* [2], *Dariusz Chaberski* [3], *Sławomir Grzelak* [4]

[1] Institute of Physics, Nicolaus Copernicus University, Torun, Poland, marziel@phys.uni.torun.pl
[2] Institute of Physics, Nicolaus Copernicus University, Torun, Poland, markow@phys.uni.torun.pl
[3] Institute of Physics, Nicolaus Copernicus University, Torun, Poland, daras@phys.uni.torun.pl
[4] Institute of Physics, Nicolaus Copernicus University, Torun, Poland, slawg@phys.uni.torun.pl

Abstract: Accumulated clock signal jitter is a significant source of errors in many measurement systems. In this paper a new method of estimation of clock signal jitter will be discussed. This method enables to calculate an accumulated jitter and time-interval error [TIE] as a function of the clock cycles. The time-interval measurement system [TIMS], accommodated for jitter measurements and their application for estimations of jitter parameters will be also presented and discussed.

Keywords: jitter, phase fluctuations, time-interval measurements.

1. INTRODUCTION

Jitter is defined as phase fluctuations in a clock or data signal which are result of noise or other disturbances. Jitter is caused by thermal noise, instabilities in the oscillator electronics and external interferences disturbing oscillator. Random jitter, which is described by a Gaussian probability distribution is characterized by its standard deviation value. Deterministic jitter described by a non-Gaussian probability density function is characterized by its peak-to-peak value.

The period jitter is usually measured as the difference between the longest period and the shortest period T_k. A high-quality oscillator has an accumulated period jitter less than 50 picoseconds. Period jitter, shown in Fig.1, is the maximum change in a clock's output transition from its ideal position.

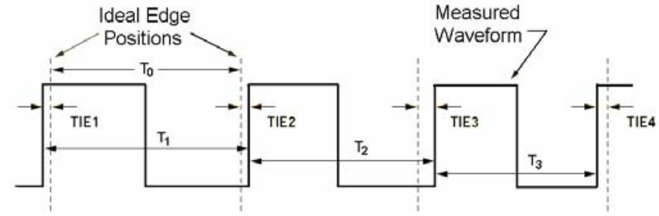

Fig. 1. Clock Signal Jitter

Cycle-to-cycle jitter J_{CC} is a difference between consecutive clock periods. Time interval error is a difference between real and ideal positions of clock output transitions.

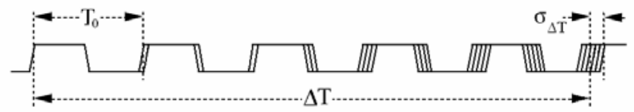

Fig. 2. Clock Long-term Accumulated Jitter

Long-term jitter, shown in Fig. 2, measures the maximum change in a clock's output transition from its ideal over a large number of cycles [1 – 3]. To measure long-term jitter, a technique called differential phase measurement can be used. In this method, the long-term jitter is the time difference of the first rising edge and the time delayed edge and can be expressed by the rms value of time fluctuations. Long-term jitter can be also characterized by the square root of Allan variance. In many measurement systems, where the time base is applied, TIE caused by the accumulated jitter is very important, because the uncertainty of the obtained results strongly depends from TIE.

2. TIME-INTERVAL MEASUREMENT MODULE

The TIE caused by the long-term accumulated jitter of the clock can be measured using the method of time-interval interpolation.

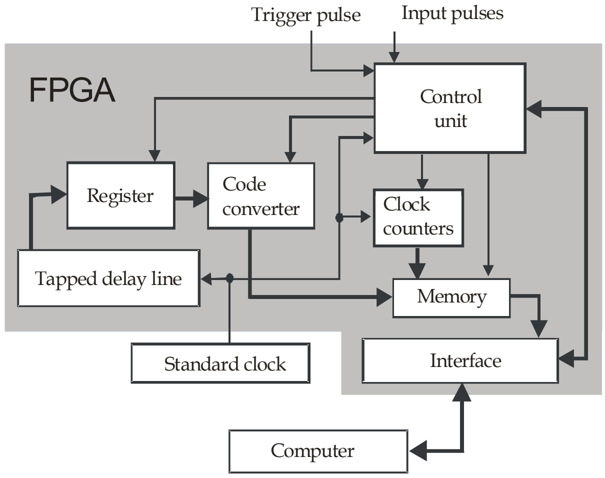

Fig. 3. Block Diagram of TIMM

The high-resolution time-interval measurement module [TIMM] implemented in the single FPGA structure consists of: a 32-segment tapped delay line, 32-bit data register (which can be set in one of 64 different states), two clock counters, control unit and memory block (2048 x 16) [4, 5].

Redundancy of the clock counters allows to eliminate readout errors caused by timing. The block diagram of the TIMM is shown in Fig. 3.

In this system the time-intervals between trigger pulse and each of the clock rising edges are precisely measured and collected in the memory. As the trigger pulse can be taken any one of the clock rising edges. Such measurement system should have a high-stability standard clock and high-resolution tapped delay line.

The TIMM implemented in single FPGA device Virtex XCV 300 has resolution equal to 100 ps. Similar TIMM implemented in the XC2VP4 device has resolution 60 ps and nonlinearity errors less than 50 ps as it is shown in Fig. 4.

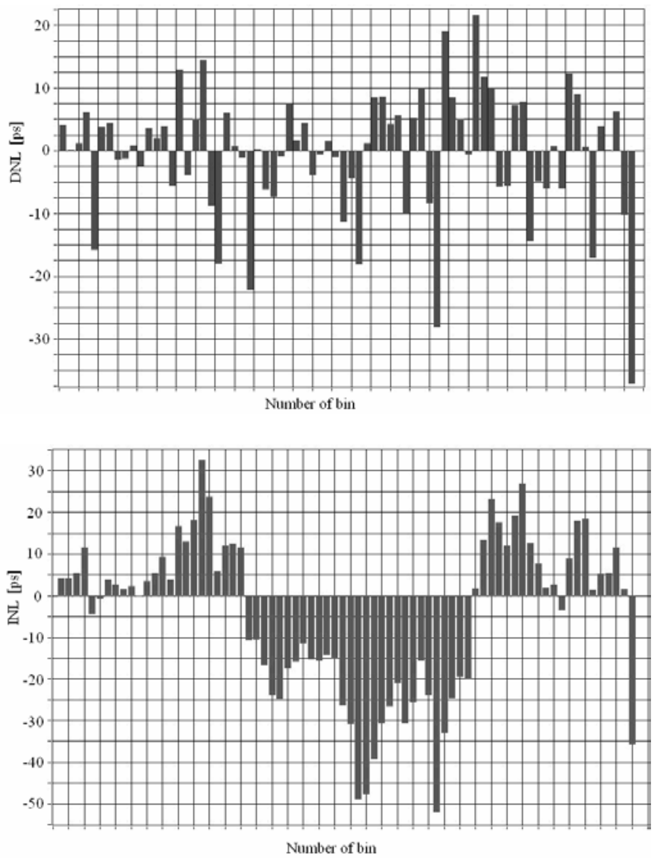

Fig. 4. Differential (DNL) and integral (INL) nonlinearity errors of the TIMM

3. PRINCIPLE OF MEASUREMENT

Signal from the oscillator under test is fed to the input of TIMM [6]. The measured time-intervals between low-to-high transitions of oscillator, occurring after trigger pulse, collected as a time-stamps during many measuring cycles are considered as random variables Δt_N, which are described by the Gaussian probability density function. The standard deviation of variables Δt_N is of course different from the standard deviation of single period fluctuations ε.

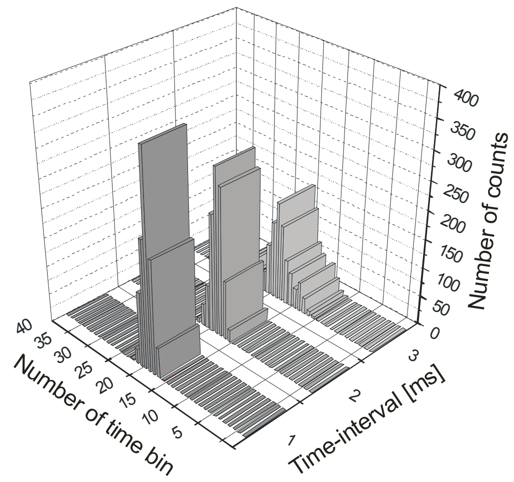

Fig. 5. Probability density function of time interval fluctuations obtained for three different time intervals

$$\Delta t_N = NT_0 + \sum_{n=1}^{N} \varepsilon_n \tag{1}$$

The probability density function of the fluctuations ε can be written as:

$$p(\varepsilon) = \frac{1}{\sqrt{2\pi}\sigma_\varepsilon} e^{-\frac{(\varepsilon - T_0)^2}{2\sigma_\varepsilon^2}} \tag{2}$$

where σ_ε is a standard deviation of ε.

For each oscillator transition beginning from the trigger pulse, standard deviation depends on correlation between fluctuations ε_i. When there is no correlation between fluctuations ε_i then variance of the time-interval Δt_N, is a sum of variances

$$\sigma_{\Delta t_N}^2 = \sigma_1^2 + \sigma_2^2 + \ldots + \sigma_N^2 . \tag{3}$$

Assuming that each oscillator period can be described by similar fluctuations, variance of the time-interval Δt_N can be written as:

$$\sigma_{\Delta t_N}^2 = N\sigma_\varepsilon^2 \tag{4}$$

If correlation occurs, the variance of the time-interval errors can be greater:

$$\sigma_{\Delta t_N}^2 \le N^2 \sigma_\varepsilon^2 \tag{5}$$

Generally the variance of the time-interval errors can be described by:

$$\sigma_{\Delta t_N}^2 = N^a \sigma_\varepsilon^2 , \tag{6}$$

where : $1 \le a \le 2$.

The variance of the time-interval measured by the clock oscillator depends significantly from the number of oscillator cycles.

Fluctuations of the time-intervals, consist of N clock periods disturbed by the sum of fluctuations ε_i (where i = 1, 2, 3,…N) also have Gaussian distribution but different standard deviation, as is shown in Fig.5.

Considering equations (2) and (6), the maximum of probability density function for N oscillator cycles is given by:

$$p(t=\Delta t_N)=\frac{1}{\sqrt{2\pi}\,\sigma_{\Delta t_N}}=\frac{1}{\sqrt{2\pi N^a}\,\sigma_\varepsilon}=cN^{-a/2}\ , \qquad (7)$$

where: $$c=\frac{1}{\sqrt{2\pi}\,\sigma_\varepsilon}\ . \qquad (8)$$

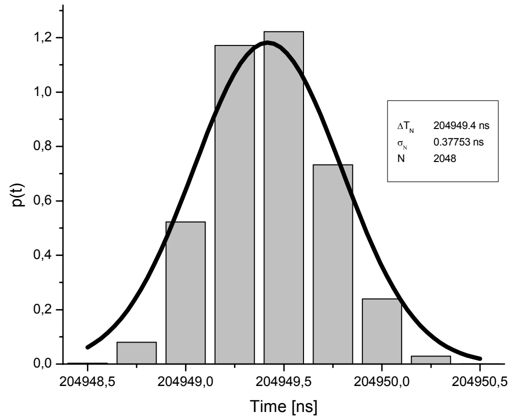

Fig. 5. The result of standard deviation estimation for the time-interval consists of N clock cycles

Two parameters, *c* and *a*, that describe the clock jitter can be estimated from the envelope of the Gaussian distributions obtained from earlier collected time-stamps.

$$\sigma_{\Delta t_N}=\frac{N^{a/2}}{c\sqrt{2\pi}} \qquad (9)$$

Parameters *c* and *a* enable to estimate a standard deviation $\sigma_{\Delta tN}$, as it is shown below in Fig. 6.

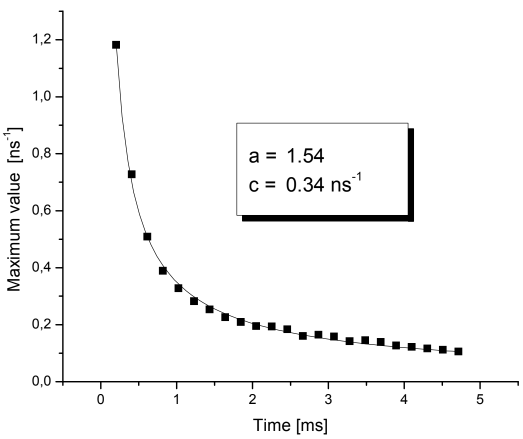

Fig. 6. Envelope of the Gaussian distributions for the different time-intervals that consist of N clock cycles

Generally parameter *a* describes correlation between fluctuations of oscillator period and *c* describes their stability.

Usually, measurement cycle of many systems is limited up to several milliseconds. To determine the uncertainty of the time-interval measurement system it is necessary to know a time-fluctuation characteristic of the instrument. It should be pointed out, that only two parameters *a* and *c* enable for standard deviation estimation of the time-interval which is determined by the limited number of the clock cycles.

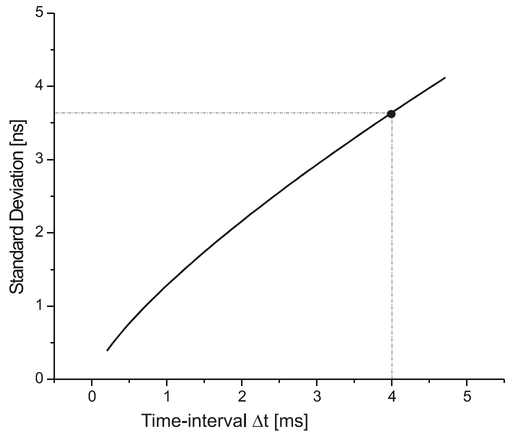

Fig. 7. Estimation of the standard deviation for measured time-interval

4. CALLIBRATION

During the time-stamp measurements two different and independent oscillators operate simultaneously. The first oscillator is the system clock oscillator and the second is an oscillator under test. Effective variances observed as variances of the collected time-stamps are the sum of the clock variance and oscillator under test variance calculated for designed time-interval. From this reason it is very important to know the variance or characteristic of standard deviation (parameters *a* and *c*) for the clock oscillator embedded in the measurement system.

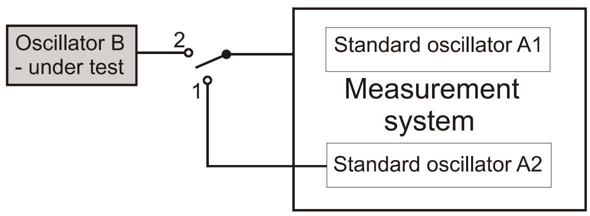

Fig. 7. Estimation of the standard deviation for measured time-interval

The measurement system should have a second standard oscillator, which enables calibration procedure. During the calibration procedure the signal from the second standard oscillator A2 is fed to the input of TIMM. In this way it is possible to calculate the pair of parameters (c_A and a_A) for standard oscillator which will be used as standard clock during tests. In case of two independent standard oscillators the effective variance of the designed time-interval will be a sum of two identical variances. The probability density function can be written as:

$$p_A(t=\Delta t_N)=\frac{1}{\sqrt{4\pi}\,\sigma_{A,\Delta t_N}}=\frac{c_A}{\sqrt{2N^{a_A}}}\ . \qquad (10)$$

The characteristic of standard deviation will be given by the equation (9).

During the tests when the signal from the oscillator under test is fed to the input of measurement system the effective variance will be the sum of two different variances. In this case the probability density function can be written as:

$$p_{A+B}(t = \Delta t_N) = \frac{1}{\sqrt{2\pi\left(\sigma^2_{A,\Delta t_N} + \sigma^2_{B,\Delta t_N}\right)}} = \frac{c_{A+B}}{\sqrt{N^{a_{A+B}}}}. \quad (11)$$

The characteristic of standard deviation of the oscillator under test will be given by:

$$\sigma_{B,\Delta t_N} = \sqrt{\frac{N^{a_{A+B}}}{2\pi c_{A+B}} - \sigma^2{}_{A,\Delta t_N}}\ . \quad (12)$$

As it is shown, the standard deviation of the jitter fluctuations can be predicted after simple calculations which is very important because it means that such operations can be processed also in real-time.

5. SIMMULATIONS AND TESTS

The results of simulation and measurements are shown in Fig. 8, and Fig. 9.

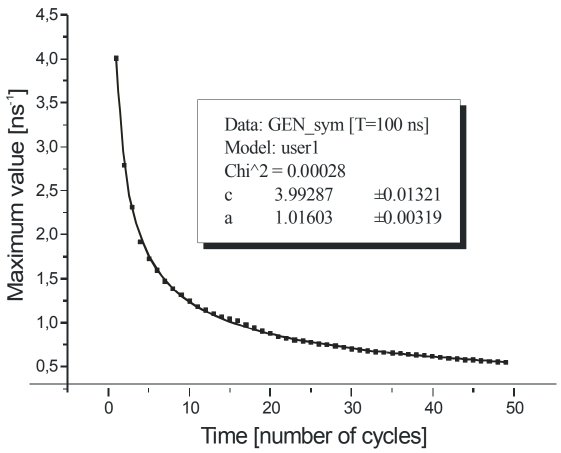

Fig. 8. The result of simulation for the oscillator with non correlated fluctuations ε

Of course, the variance of TIE estimator also depends on time-interval range (the number of clock cycles) and a phase fluctuations of the standard oscillator, which generates a time base in TIMM.

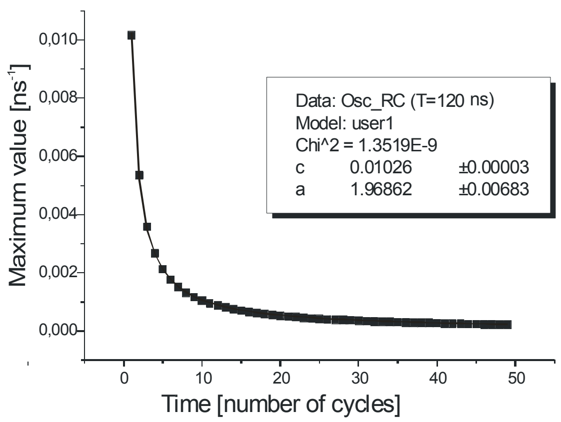

Fig. 9. The result of measurements for the RC oscillator with correlated fluctuations ε

The measurement procedure consists of:

- calibration of standard oscillator,
- data collection,
- parameters (a, c) estimation.

Using equation (12) it is possible to predict the value of time-interval error (accumulated jitter) after N oscillator cycles.

4. CONCLUSION

The new method of jitter characterization enables to predict TIE, which is a very significant parameter for measurement systems, data transmission systems and many other systems, where using standard oscillators for time base is necessary [1, 6]. Only two parameters describe oscillator fluctuations up to several milliseconds. Fortunately the typical range of measuring cycle for many systems is similar. Using the fast FPGA devices for the system implementation it is possible collect time-stamps with resolutions 60 ps and test high stability oscillators. Such simple method enables the uncertainty calculations for the measuring systems using the time base.

The next investigations will be focused on the limits of this method and applications of this method.

ACKNOWLEDGMENTS

This work was supported by the Polish Ministry of Education and Science Grant No 3 T10C 015 30

REFERENCES

[1] J. Kalisz, „Review of methods for time interval measurements with picosecond resolution", Metrologia, No. 41, pp. 17 – 32, 2004.

[2] A. Hajimiri, S. Limotyrakis, and T. H. Lee, "Jitter and phase noise in ring oscillators", IEEE Jour. of Solid-State Circ. VOL. 34, NO. 6, pp 790 – 804, 1999.

[3] J. Kalisz, "Determination of short-term error caused by the reference clock in precision time-interval measurement and generation IEEE Trans. Instrum. Meas. 37, pp. 315–316, 1988.

[4] M. Zieliński, R. S. Dygdała and P. Płóciennik, "High-resolution digital counting systems for applications in atomic physics", 5-th International Symposium on Measurement, Technology and Intelligent Instruments, pp 359 – 364, Giza – Cairo, Egypt, 2001.

[5] R. Frankowski, S. Grzelak, M. Kowalski, D. Chaberski and M. Zieliński, " Time-interval measurements, using tapped delay lines implemented in a programmable logical structures" , 12th IMEKO TC4 International Symposium Zagreb, Croatia , 2002.

[6] M. Zieliński, R. S. Dygdała, K. Karasek and A. Zawadzka, "Real-time multichannel scaler measurement of oscillator instabilities", Rev. of Sci. Instruments, Vol. 71, No. 6, pp. (2577 – 2581), 2000.