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FAST MEASUREMENT OF POWER SYSTEM FREQUENCY IN THE FREQUENCY DOMAIN

Dušan Agrež

Faculty of Electrical Engineering, University of Ljubljana, Slovenia, e-mail: dusan.agrez@fe.uni-lj.si

Abstract: In paper, an algorithm for fast measurement and estimation of power system frequency is presented. The frequency is calculated by an interpolation of the amplitude coefficients of the discrete Fourier transform (DFT). An analysis is made to study the influence of the leakage effect when the rectangular window and the Hanning window are used. Interpolations with longer time of measurement and with larger number of points decrease the systematic errors. The proposed method achieves accuracy in measuring the frequency under a hundredth of hertz.

Keywords: power system, frequency estimation, interpolated DFT.

1. INTRODUCTION

The variable fundamental frequency is the most important parameter for the safety, stability and efficiency of the power system. Deviation of the power-line frequency from its stationary value is a measure of the imbalance between the load and generation. Frequencies lower than the nominal value indicate that the system is overloaded and frequencies higher than the nominal value indicate that the system has more generation than load. It is necessary to maintain the frequency of the power system at its nominal value ($50\,\mathrm{Hz}$ in Europe and $60\,\mathrm{Hz}$ in the United States, etc.), or as close as possible to that value. There are the average drift of the system frequency and superimposed frequency oscillations. Regular power-line frequency variations are $0.01-0.5\,\%$, but under emergency conditions, they may reach $2\,\%$ and more. Several methods for tracking and estimating the actual power-line frequency can be divided in two main groups: methods that are performed in the time domain as level crossings methods etc. [1-3], and methods of frequency estimation in the frequency domain using discrete and fast Fourier transforms (DFT and FFT) [4-6]. The measurement time in the first group varies from $25\,\mathrm{ms}$ to $30\,\mathrm{ms}$ and maximal values of errors are around $0.02\,\mathrm{Hz}$. The second group methods perform the measurements in an integer multiple of the fundamental period ($T_\mathrm{m} = 20, 40, 60, \ldots\,\mathrm{ms}$ in $50\,\mathrm{Hz}$ system) and the estimation error in measuring the frequency is about $0.01\,\mathrm{Hz}$. Real measurement procedures are often encountered with noise components (quantization and thermal noise, switch noise etc.) and harmonic distortions. In these cases some averaging and prefiltering algorithms of measurement signals (interpolations of the time samples, integration by the DFT, etc.) are performed. However, all such methods involve a compromise between the accuracy of the frequency measurement and the length of the observation period; accuracy decreases, as the period becomes smaller.

2. FREQUENCY ESTIMATION BY IDFT

In this paper, an algorithm for fast measurement and estimation of the power system frequency is presented. The algorithm uses discrete time values of the input signal sampled Δt apart. The sampled analog multi-frequency signal $g(t)$ (N is the number of sampled points; m is the index of component) can be written as follows:

$$g(k\Delta t)_N = \sum_{m=0}^{M} A_m \sin(2\pi f_m k\Delta t + \varphi_m) \qquad (1)$$

In order to estimate parameters of time dependent signals containing any periodicity (f_m, A_m, φ_m are frequency, amplitude, and phase of a single component, respectively) is most suitable to use the frequency domain. Integral frequency transformation with $e^{-j2\pi ft}$ is in principle the best approximation to periodicity in the signal. The key step of the frequency estimation is the determination of position of the measured component δ_m between the DFT coefficients $G(i)$ and $G(i+1)$ surrounding the component (Fig. 1).

The DFT of the windowed signal $w(k)\cdot g(k)$ at the spectral line i is given by [7]:

$$G(i) = -\frac{j}{2}\sum_{m=0}^{M} A_m \left[W(i-\theta_m)e^{j\varphi_m} - W(i+\theta_m)e^{-j\varphi_m}\right], \quad (2)$$

where $W(*)$ is the spectrum of the used window and θ_m is the frequency divided by the frequency resolution of the time window $\Delta f = 1/(N\Delta t)$ and can be written in two parts:

$$\theta_m = \frac{f_m}{\Delta f} = i_m + \delta_m \qquad -0.5 < \delta_m \le 0.5\,, \qquad (3)$$

where i_m is an integer value of the first approximation and the displacement term δ_m is caused by the non-coherent sampling.

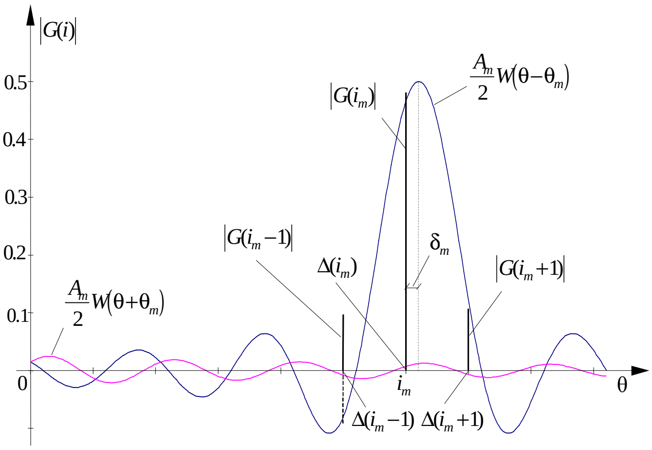


Fig. 1. Leakage influence of the negative part of the sinus spectrum with rectangular window at the positive part of the spectrum ($i_m = 6$)

The amplitude coefficients surrounding one component in the signal (Fig. 1) are composed of the short-range leakage contribution of the window spectrum weighted by the amplitude of the frequency component and the long-range leakage contribution of the negative part considering only one-component signal.

$$\left|G(i_m)\right| = \left| -\frac{j}{2}A_m W(\delta_m)e^{j\varphi_m} + \frac{j}{2}A_m W(2i_m+\delta_m)e^{-j\varphi_m} \right| =$$

$$= \frac{A_m}{2}\left|W(\delta_m)\right| \pm \left|\Delta(i_m)\right| \quad (4)$$

At a first approximation, we can idealize circumstances for displacement δ_m estimation of a single component and in the equation (4) neglect the second part and long-leakage contributions of other signal components. That means in practice we have been measuring long enough or the frequency components are sufficiently interspaced.

2.1 Stationary case

The frequency of the measured component can be estimated by means of an interpolation of the DFT coefficients. For two-point interpolation the local maximum in the amplitude part of DFT with the largest coefficients $|G(i)|$ and $|G(i+1)|$, surrounding the position of component m, have to be found. Considering the equation (4), we can write as follows:

$$|G(i_m)| = \frac{A_m}{2}\left|W(\delta_m)\right| \;,\; |G(i_m \pm 1)| = \frac{A_m}{2}\left|W(1-\delta_m)\right| \quad (5)$$

The unknown amplitude A_m can be easily eliminated by ratio of coefficients.

$${}_2\alpha_m = \frac{\left|G(i_m \pm 1)\right|}{\left|G(i_m)\right|} \approx \frac{\left|W(1-\delta_m)\right|}{\left|W(\delta_m)\right|} \quad (6)$$

If the function $W(\theta)$ of window used is analytically known, the $\delta_m = f(\alpha_m)$ can be expressed (In paper, well known rectangular and Hanning windows are used). For the rectangle window, the following equations are valid, where the amplitude part of Dirichlet kernel is used [8]:

$$\left|W_{\mathrm{R}}(\delta_m)\right| = \frac{\sin(\pi\delta_m)}{\pi\delta_m / N};$$

$$\left|W_{\mathrm{R}}(1-\delta_m)\right| = \frac{\sin(\pi(1-\delta_m))}{\pi(1-\delta_m)/N} \quad (7a)$$

$${}_2\alpha_{m\mathrm{R}} \approx \frac{\delta_m}{1-\delta_m} \quad \Rightarrow \quad {}_2\delta_{m\mathrm{R}} \approx \frac{\alpha_{m2\mathrm{R}}}{1+\alpha_{m2\mathrm{R}}} \quad (7b)$$

or expressed by the DFT coefficients:

$${}_2\delta_{m\mathrm{R}} = s_\delta \frac{\left|G(i_m+1)\right|}{\left|G(i_m)\right| + \left|G(i_m+1)\right|} \quad (7c)$$

The sign of displacement $s_\delta = \mathrm{sign}(\delta_m)$ can be estimated with the absolute value of the difference of the phase DFT coefficients $s_\delta = \mathrm{sign}\left(\left|\arg[G(i_m)] - \arg[G(i_m+1)]\right| - \pi/2\right)$. This approach is much less sensitive to systematic errors and noise than a well-known approach with the difference of the coefficients surrounding the largest one $|G(i_m)|$: $s = \mathrm{sign}\left(|G(i_m+1)| - |G(i_m-1)|\right)$.

For the Hanning window, it can be written as follows:

$$\left|W_{\mathrm{H}}(\delta_m)\right| = N\frac{\sin(\pi\delta_m)}{\pi\delta_m\left(1-\delta_m^2\right)};$$

$$\left|W_{\mathrm{H}}(1-\delta_m)\right| = N\frac{\sin(\pi(1-\delta_m))}{\pi\delta_m(1-\delta_m)(2-\delta_m)} \quad (8a)$$

$${}_2\alpha_{m\mathrm{H}} \approx \frac{1+\delta_m}{2-\delta_m} \quad \Rightarrow \quad {}_2\delta_{m\mathrm{H}} \approx \frac{2 \cdot {}_2\alpha_{m\mathrm{H}} - 1}{{}_2\alpha_{m\mathrm{H}} + 1} \quad (8b)$$

$${}_2\delta_{m\mathrm{H}} = \frac{2\left|G(i_m+1)\right| - \left|G(i_m)\right|}{\left|G(i_m)\right| + \left|G(i_m+1)\right|} \quad (8c)$$

In both cases in denominator $\sin(\pi x/N) \cong \pi x/N$ is taken into consideration because $|\delta_m| \le 0.5$ and N is usually large $N >> 1$ ($N = 256, 512, 1024, \dots$). The error of this approximation decreases even more in the quotient. By the ratio α_m not only the amplitude is eliminated but in presented cases the unwanted sinus function in numerator as well.

For the three-point interpolation the long-range leakage influences can be equalized in the quotient α_m in approximation $|\Delta(i_m - 1)| \approx |\Delta(i_m)| \approx |\Delta(i_m + 1)|$ and with subtraction eliminated by pairs [9].

$$ {}_3\alpha_m = \frac{|G(i_m)| + |G(i_m - 1)|}{|G(i_m)| + |G(i_m + 1)|} = $$

$$ = \frac{|W(\delta_m)| \pm |\Delta(i_m)| + |W(1+\delta_m)| \mp |\Delta(i_m - 1)|}{|W(\delta_m)| \pm |\Delta(i_m)| + |W(1-\delta_m)| \mp |\Delta(i_m + 1)|} $$

$$ {}_3\alpha_m \approx \frac{|W(\delta_m)| + |W(1+\delta_m)|}{|W(\delta_m)| + |W(1-\delta_m)|} \tag{9} $$

When the Hanning window, for which the spectrum is analytically known ($|W_{\mathrm{H}}(\theta)|_{N>>1} = |\sin(\pi\theta)| / |\pi\theta(1-\theta^2)|$), is used, all three coefficients of the maximum have the same sign (the main lobe is extended to four frequency resolution intervals $4\Delta f$) and it can be expressed as:

$$ {}_3\alpha_{m\mathrm{H}} \approx \frac{\dfrac{1}{\delta_m(1-\delta_m^2)} + \dfrac{1}{\delta_m(1+\delta_m)(2+\delta_m)}}{\dfrac{1}{\delta_m(1-\delta_m^2)} + \dfrac{1}{\delta_m(1-\delta_m)(2-\delta_m)}} = \frac{2-\delta_m}{2+\delta_m} $$

The displacement term can be expressed with quotient ${}_3\alpha_{m\mathrm{H}}$ or with amplitude coefficients from (9):

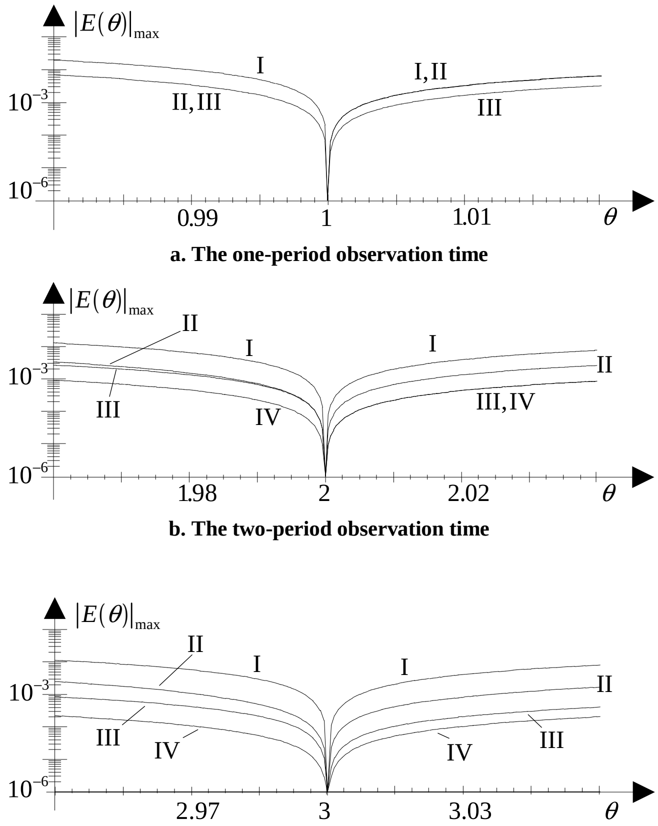
$$ {}_3\delta_{m\mathrm{H}} \approx 2\frac{1-\alpha_{m3\mathrm{H}}}{1+\alpha_{m3\mathrm{H}}} = \frac{2\cdot(|G(i_m+1)| - |G(i_m-1)|)}{|G(i_m-1)| + 2|G(i_m)| + |G(i_m+1)|} \tag{10} $$

For the rectangular window (the main lobe is extended only to two frequency resolution intervals) the equation can be rearranged in the following form:

$$ {}_3\delta_{m\mathrm{R}} \approx s_\delta \frac{(|G(i_m+1)| + |G(i_m-1)|)}{2|G(i_m)| + s_\delta(|G(i_m+1)| - |G(i_m-1)|)} \tag{11} $$

We checked the error of the frequency estimation $E(\theta) = (i+\delta) - \theta$ (θ is the true value of the relative frequency) for one sinus component in the signal with double scan varying both frequency and phase ($A_m = 1$; $0.98 \le \theta \le 1.02\ (\cdot 2, \cdot 3)$, $\Delta\theta = 0.0001$ and $-\pi/2 \le \varphi \le \pi/2$, $\Delta\varphi = \pi/180$; $N = 1024$). The absolute maximum values of errors (from 181 iterations) at the given relative frequency were compared for two-point (7c) and three-point interpolation (11) with the rectangular window and with the Hanning window for two-point (8c) and three-point interpolation (10) (Fig. 2).

As a rule the power-line frequency is measured in the time multiple of the fundamental period for eliminating the harmonic interference. At the one-period observation time, we can use a three-point interpolation with the rectangular window or a two-point interpolation with the Hanning window (Fig. 2a.: II or III). At the two-period time is better to use the Hanning window (Fig. 2b.: III or IV), and at the three-period observation time the best results are with Hanning window and the three-point interpolation (Fig. 2c.: IV). This interpolation automatically adds the sign of displacement and the error band is around $|E(\theta)|_{\max} \approx 10^{-4}$.



a. The one-period observation time

b. The two-period observation time

c. The three-period observation time

Fig. 2. Maximal absolute errors of the frequency estimation with interpolations of DFT for the rectangular window (I - two-point and II - three-point) and the Hanning window (III - two-point and IV - three-point)

2.2 Non-stationary conditions

The selection of an algorithm ((7c), (8c), (10) or (11)) for the frequency estimation in the non-stationary circumstances has following requirements: reducing the time of measurement, that the estimated average frequency could be as close as possible to the instantaneous frequency; the relative frequency should be close to the integer values ($i = 1, 2$ or 3) to reduce the leakage influences of the other harmonic components; reducing the noise; robustness etc. The optimum for upper requirements can be the estimation with the two-point interpolation and the Hanning window (8c). The Hanning window has good capability of filtering the harmonic interferences [10]. In fact, the rectangular window function would exhibit a heavier bias in the frequency estimation due to the scallop loss while other windows (like higher order cosine windows) exhibit a larger main lobe and a worse tone resolvability.

The two-point interpolation using Hanning window has the following properties: the sign of displacement is expressed implicitly; it uses only two coefficients and the noise is smaller than with the three-point interpolation; if the second and the third harmonic component are not in the signal than it can be used at $\theta \approx 1$; if it is used at $\theta \approx 2$, only the second harmonic component should not be in the signal because the spectrum main lobe is too wide ($\propto 4\Delta f$).

The coherence can be improved also with the averaging of the estimation results at two widths of the measurement interval: N_{a} is the number of points where the number of oscillation of more than two is reached for the first time $f_{\mathrm{a}} = (2+\delta_{\mathrm{a}})/\Delta \mathrm{f}$, $\delta_{\mathrm{a}} \geq 0$; with $N_{\mathrm{a}}-1$ width, the displacement takes the opposite sign $\delta_{\mathrm{b}} < 0$.

$$\bar{f} = \frac{|\delta_{\mathrm{b}}|}{|\delta_{\mathrm{a}}|+|\delta_{\mathrm{b}}|} f_{\mathrm{a}} + \frac{|\delta_{\mathrm{a}}|}{|\delta_{\mathrm{a}}|+|\delta_{\mathrm{b}}|} f_{\mathrm{b}} \qquad (12)$$

3. SIMULATIONS AND EXPERIMENTAL RESULTS

The method has been tested by the real signals, which have been measured in the three-phase power system when the short-circuit appear in the line L3. Three voltages (Fig. 3: $\hat{u}_0$ is the maximal value of voltage in the stationary state at the beginning) and current in line L3 (Fig. 4: $\hat{i}_0$ is the maximal value of current in the stationary state at the beginning) were sampled in the time interval of seven periods before the protection against short-circuit finally disconnect line L3. Sampling frequency was $f_{\mathrm{S}} = 1\,\mathrm{kHz}$ and we have only 20 points in the period of measured signals. Signals were quantized by 12-bits A/D converter.

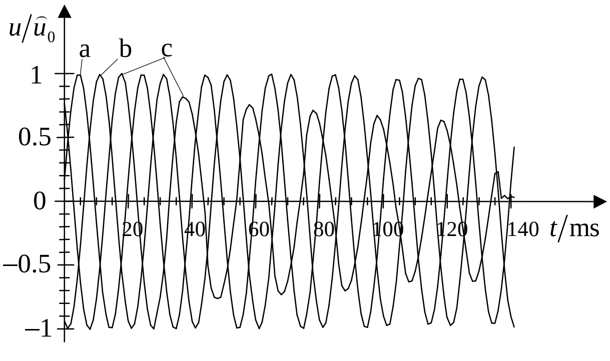


Fig. 3. Voltages in the three-phase power system when the short-circuit appear in line L3: a – voltage in line L1, b - voltage in line L2, c - voltage in line L3

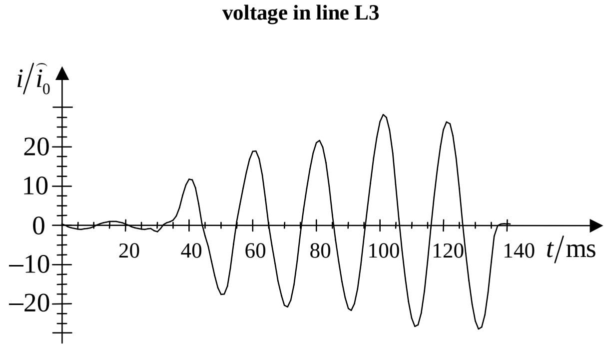


Fig. 4. Current in line L3

In Fig. 5, the results of the algorithms tracking the frequencies of voltages in lines L1 and L2 are shown. Frequencies of the uncorrupted phases are changing opposite from stationary state at the beginning ($f/f_0 = 0.9998$, $f_0 = 50\,\mathrm{Hz}$). We can see that the three-point interpolation with the Hanning window using the two-period measurement time (curves b - L1 and d - L2 in Fig. 5) smoothes the frequency changes in comparison with the two-point interpolation (curves a - L1 and c - L2 in Fig. 5) due to more points using in regression. Frequency can be also estimated on the voltage and current in the faulty phase L3 (Fig. 6), but changes are much larger.

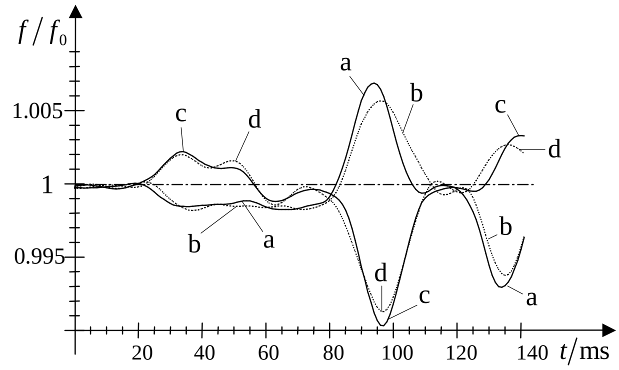


Fig. 5. Estimations of the frequency changes of the voltage in line L1 (a – two-point interpolation and b – three-point interpolation) and voltage in L2 (c – two-point interpolation and d – three-point interpolation)

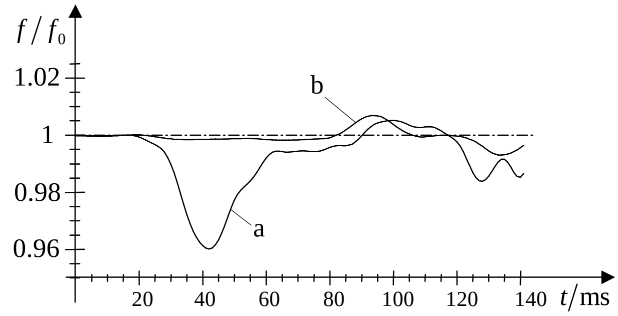


Fig. 6. Estimations of the frequency changes of the voltage in line L3 (a) and current in line L3 (b)

4. CONCLUSIONS

A simple algorithm for fast measurement and estimation of the power system frequency is presented. The frequency is calculated from the amplitude DFT coefficients. The interpolations with larger number of points and with longer time of measurement decrease the systematic errors, with smaller number of points there is an inverse appearance. At the one-period observation time, the best results are with the rectangular window (the two- point and the three-point interpolations). At the two-period time is better to use the Hanning window and the two-point interpolation and at the three-period time, the best results are with the Hanning window and the three-point interpolation. This interpolation automatically adds the sign of displacement and the error level is below $|E(\theta)|_{\max} \approx 10^{-4}$. The compromise for the efficient measurement of the power system frequency is at the two-period measurement time using the Hanning window and the two-point interpolation.

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