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Influence criterions of quality to the weight windows selection

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Abstract: Weight filtration is performed in time domain through multiplying values of appropriate samples by a weighting function and summing the yielded products along the entire window length. The number of weight windows is unlimited. In his paper author is trying to define the filtration quality and its influence to the weight window selection..

Keywords: weight filtration, criterions of quality, filtration error.

1. INTRODUCTION

Weight filter is an FIR (finite impulse response) filter. It is performed in time domain through multiplying values of appropriate samples by a *weighting function*, also called *weight window* (or *weight function*, or simply *filter*), and summing the yielded products along the entire window length. It may be defined by the following relationship:

$$y_{f\,j} = \sum_{i=-k}^{k} w_i \cdot y_{z_{i+j}} \quad \text{for} \quad j = 1,\ldots,N \tag{1}$$

where:
- $y_{z_{i+j}}$ is the filtration input signal affected by noises,
- y_{f_j} is the signal after being filtered,
- w_i is the weighting function, k is a parameter determining the window width equal $2 \cdot k + 1$,
- N is the number of samples of the measured signal to be filtered.

The basic differences between weight filter and classical FIR filter are:
- weight filter is performed in the time domain,
- it no needs the transformation to the frequency domain,
- the weight window is symmetrical.

2. WEIGHT FUNCTIONS

The basic parameters of weight filtration are: the *form* and the *width* of the weight window. The weight window is a symmetrical function with an uneven number of elements equal $2 \cdot k + 1$. In a practice the kind number of weight windows is unlimited. The only condition the weighting function has to meet is:

$$\sum_{i=-k}^{+k} w_i = 1 \tag{2}$$

3. CRITERIONS OF QUALITY

Criterions of quality can be used for the filtration's parameters selection (form and width of the window). The *filtration error* can be this criterion.

To define the filtration error the model of the measure procedure (Fig. 1) will be helpful.

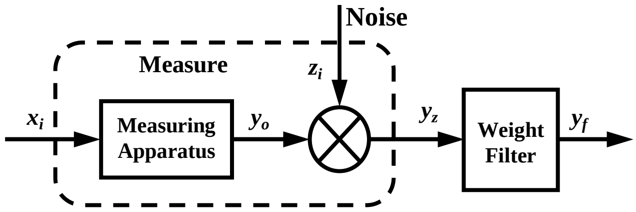

Fig.1. The model of the measure procedure

Where y_w is the discrete unaffected signal, y_z is the signal affected by the noises z, and y_f is the signal after being filtered.

The filtration error may be defined as:

f_error = what we know – what we want know

Two kinds of errors can be defined: *maximal* and *mean square*.

By virtue of the Fig.1 and filtration definition:
- absolute maximal:

$$\Delta_{\max} = \left| y_{f\,j} - y_{o\,j} \right|_{\max} \tag{3}$$

- relative maximal:

$$\delta_{\max} = \frac{\Delta_{\max}}{\left| y_{o\,j} \right|_{\max}} \tag{4}$$

and
- absolute mean square:

$$\Delta_{sq} = \frac{1}{N} \cdot \sqrt{\sum_{j=1}^{N} \left(y_{f\,j} - y_{o\,j} \right)^2} \tag{5}$$

- relative mean square:

$$\delta_{sq} = \frac{\Delta_{sq}}{\frac{1}{N}\cdot\sqrt{\sum_{j=1}^{N} {y_o}_j^{\,2}}} \tag{6}$$

Where:

y_o – the undisturbed signal on the output of the measuring apparatus,

y_f – the signal after filtration.

In a practice the real unaffected signal y_o is never known. Hence, only simulations evaluate errors defined above. However obtained results may be used in a practice.

4. RESEARCH

The Author's research is based on simulated signals affected by noises. All simulation and research were made in the Mathcad 12 program.

4.1. Standard signals

Simple periodic signals (not included higher harmonic signals) were used as the standard signals y_o during research. All of the standard signals has the normalized amplitude. The number of samples the standard signal y_o were $N = 1024$ per period.

4.2. Disturbances

There were three kinds of disturbances during author's research: the random, the periodic and the impulse.

4.2.1. The random noise

The Author has used the internal Mathcad function as the random noise generator. Generated noise had a normal distribution. The maximal disturbance amplitude were not higher than 10% of the amplitude the standard signal y_o. Samples of the random noise z_i, the standard signal y_o and disturbed signal yz_i are shown on Fig.2.

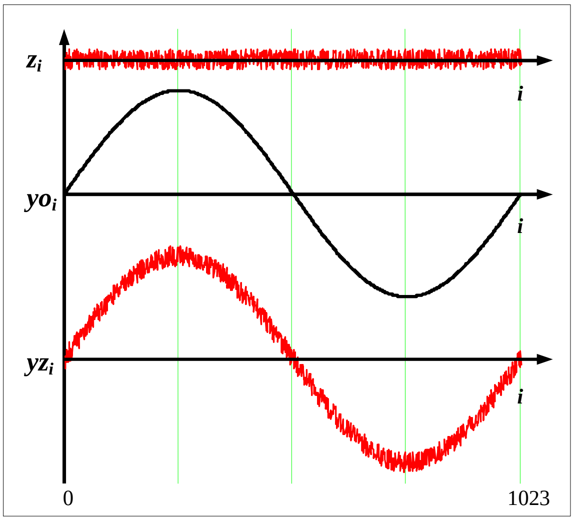

Fig.2. The input signal y_o disturbed by a random noise

The random noise from fig.2 has a normal distribution and maximal amplitude equal 10% of amplitude the standard y_o signal.

4.2.2. The periodic noise

In the case of periodical disturbances the summary signal z_i was the combination from 3 to 10 sinusoidal signals. The number and the frequencies of the component signal were random generated. Similarly random generated were amplitudes of all component signals. However its maximal amplitude couldn't be greater than 10% of the amplitude the standard y_o signal. Samples of the periodic noise z_i, the standard signal y_o and disturbed signal yz_i are shown on Fig.3.

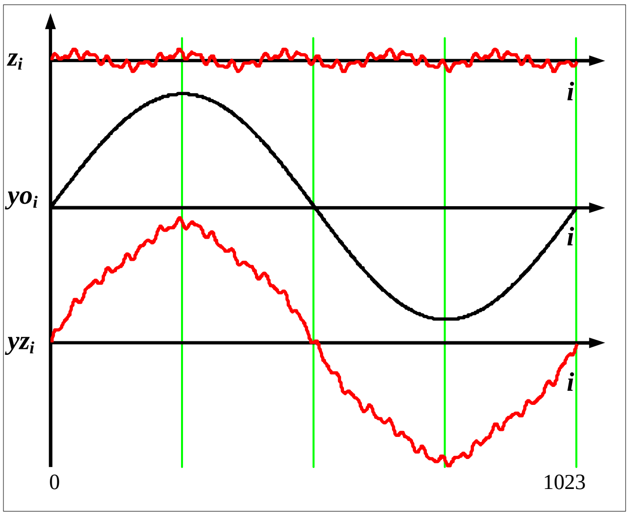

Fig.3. The sample of the input signal y_o disturbed by a periodical noise

Disturbances from fig.3 are described by equation:

$$z_i = 0.055\cdot\sin(15\cdot\omega_0 T) + 0.03\cdot\sin(90\cdot\omega_0 T) + \\ + 0.02\cdot\sin(150\cdot\omega_0 T) \tag{7}$$

Where ω_0 was a pulsation of the standard signal y_o.

4.2.3. The impulse noise

Impulses noises were generated by the δ Kronecker function included as the internal Mathcad function. Amplitude of this noises, as previous cases, was not greater than 10% of the amplitude the standard y_o signal. Impulse noises had a periodical distribution.

4. 3. The weight windows

In his research the Author has made a thorough analysis tens of weight windows like: square, triangle, sinusoidal, Hamming, Hanning, Blackman, Kaiser and Gauss etc.

In this paper the Author has described obtained results for some samples of chosen windows such as:

– square window:

$$w_{sqr_i} = \frac{1}{2\cdot k+1} \tag{8}$$

– Hamming window:

$$w_{Hm_i} = 0.08 + 0.46\cdot\left(1-\cos\left(\frac{2\cdot\pi\cdot i}{2\cdot k}\right)\right) \tag{9}$$

– Gauss window:

$$w_{Gs\,i} = \frac{1}{\sqrt{2\cdot\pi\cdot\sigma}}\cdot\exp\left[-\frac{1}{2\cdot\sigma^2}\cdot(i-\mu)^2\right] \qquad (10)$$

where: i = 0..2·k+1 and $\mu = i/2$, $\sigma = \mu/1.7$.

Forms of this three weight windows are shown on Fig.4.

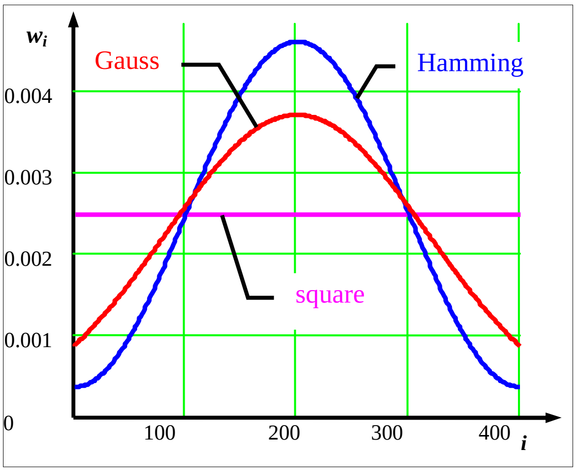

Fig.4. Samples of weight windows: square, Hamming and Gauss

The width of all windows from Fig.4 was 201 samples.

4.4. Results of the research

The usability the described criterions of quality was checked by Author's special written program based on the Mathcad 12 software environment.

The program has evaluated values of the relative filtration errors: maximal and mean square, on the window width-dependent.

The number of all obtained results is to big to make full presentation in this paper. Only results obtained for the three windows (8,9,10) described above will be shown for this reason as the sample.

Functions of the relative maximal error (4) for random noises (Fig.2) are shown on Fig.5.

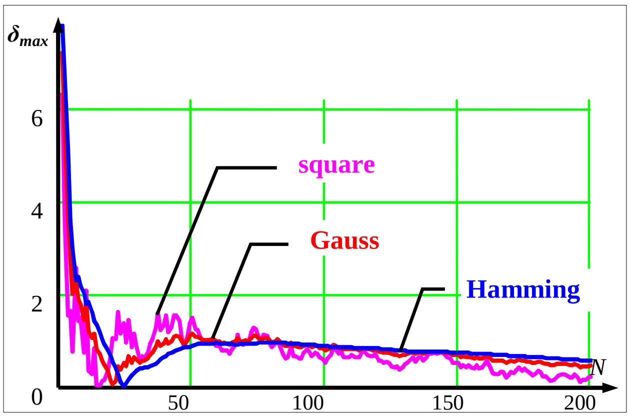

Fig.5. Relative maximal filtration error (square, Hamming and Gauss windows) for random noise

All of the error's function contains some relative extremes (minimum). The number of extremes is the kind of the window-dependent. The places of the minimum are also the kind of noises-dependent. The square window produced the largest number of minimum. All of three errors function are showing declining tendency with growing of the window width.

The mean square relative errors (6) for random noises, on the window width-dependent, are shown on Fig.6. The value of this error are declining with the windows width. Only the error for square windows has some local extremes, but they are smaller than in the relative maximal error case.

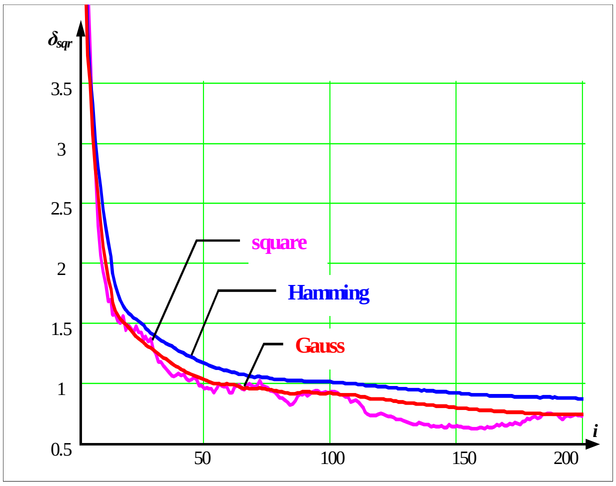

Fig.6. Relative mean square filtration error for: square, Hamming and Gauss windows

Samples of Gauss window filtrated signals y_f (for different window width) are shown on Fig.7.

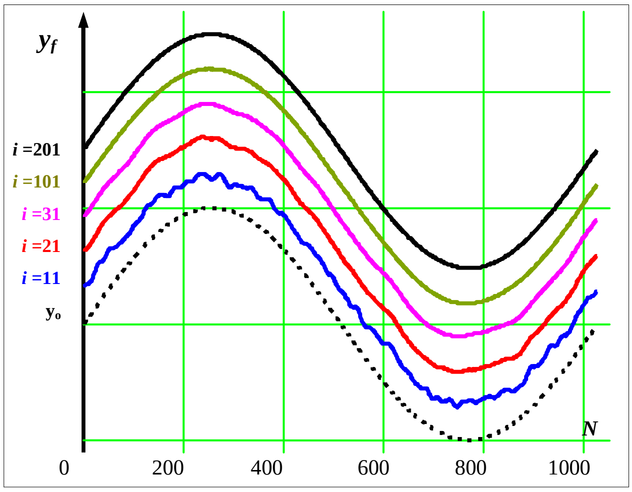

Fig.7. Weight filtration of the random noise with Gauss window (for different windows width)

Where i is the window width.

The next picture (Fig.8) contains values of relative maximal errors for periodical disturbances (Fig.3) described by (7) for all three kinds of windows (8,9,10).

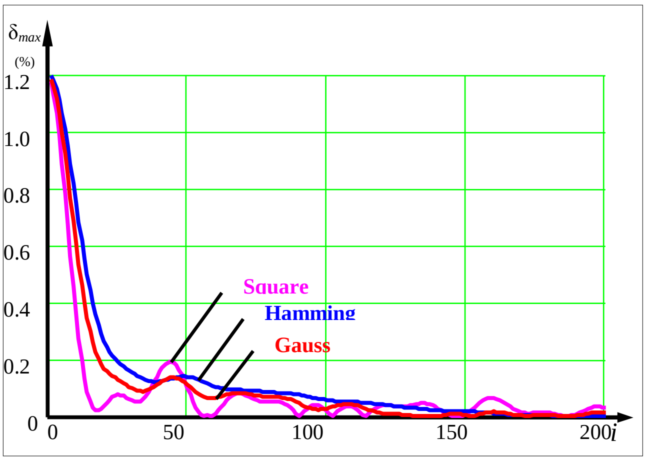

Fig.8. Relative maximal filtration error (square, Hamming and Gauss windows) for periodical disturbances

The mean square relative errors (6) for periodical disturbances are shown on Fig.9.

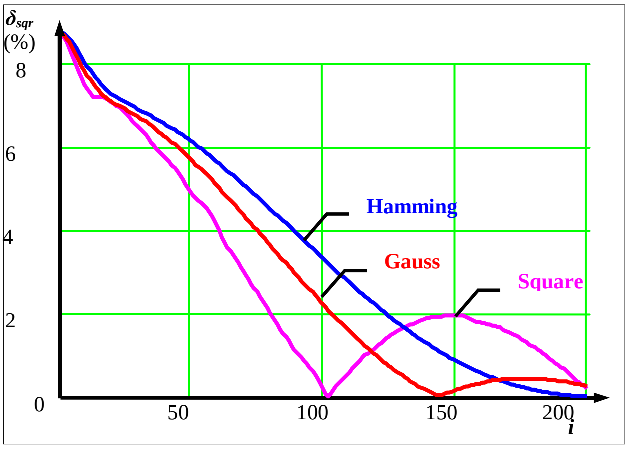

Fig.9. Relative mean square filtration error for: square, Hamming and Gauss windows

Samples of Hamming window filtrated signals y_f (for different window width) are shown on Fig.10.

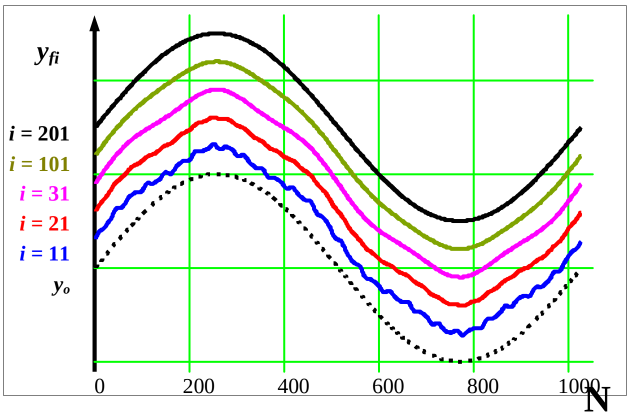

Fig.10. Weight filtration of the random noise with Hamming window (for different windows width)

Where *i* is a window width.

5. CONCLUSION

The author has described his results of research for influence criterions of quality for weight window selection.

Summary ending conclusions:

- the real values of errors are never known because the real signal y_o is never known too,
- values of both described by the Author errors can be obtained only during simulation of noises as well input signals y_o,
- obtained results of simulation can be helpful in a practice for a kind and a width of the window selection,
- both values of relative errors (maximal and mean square) were declining with the windows width,
- the number of local minimum were on the windows form and the kind of noises dependent,
- the number of local minimum were greater for the relative maximum error,
- for this reason the mean square error was better for the filter parameters selection,
- the value of filtration error ((4) or (6)) can be useful for form of the weight windows optimization.
- both criterions of relative filtration errors don't give the explicit answer which one of filters was better. The final choice of the filter is on the kind of the signal, noises and the human's knowledge depended.

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