MEASUREMENT OF SHORT NOISY SIGNAL PARAMETERS WITH THE USE OF SIGN DM

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Abstract: Usually correlation analysis (CA) is realized in Pulse Code Modulation (PCM) format that needs numerous mathematical operations with the large word length that limits the fast-acting of processing as well as the use of its to the measurement of wide-band noisy signals in real time. In this paper the use of signal representation in Sign Delta Modulation (SignDM) format has been proposed in order to increase the efficiency of the real time CA of short hardly noised signals. The SignDM codes are specified by comparing the values of differences between signal samples in PCM format with a differential zone which is defined a'priori. These two bit codes reflect the signal character and are suitable for CA as well. In this paper there are presented the fast and exact algorithms of the SignDM CA, as well as the computer simulation results for resolution of CA of short noisy signals at the various differential zone and initial phases.

Keywords: correlation analysis, short time series, delta modulation.

1. INTRODUCTION

CA is widely used in acoustics, radiolocation, telecommunications, and other fields of science and technology in order to measure strongly noised signals parameters in real time. The problem of correlation measurements becomes complicated when analyzing short time series. We shall refer to them the series of $N \leq 30$ length.

It is a well known fact that the resolution of CA is connected with signal duration and by the use of PCM depends significantly on the length of time series [1-3]. The realization of CA in real time and in PCM format requires to use a large number of mathematical operations with large word length. This does not assist the use CA in PCM format for fast measurements and does not assure high resolution. Therefore it is expedient to seek the other effective algorithms of CA.

On the other hand, the high resolution does not ensure the use in CA such economical kinds of DM as linear DM and Σ-Δ-modulation [4-6]. Employing of the SignDM CA [7,8] and power spectrum determined on base of Wiener–Hinchin's transformation of SignDM correlation function enables to achieve proper resolution.

The purpose of this paper is to work out the fast, exact, and economical method to measure the parameters of short random signals on basis of the SignDM correlation - spectral analysis in real time.

2. METHOD OF CORRELATION-SPECTRAL ANALYSIS IN SIGNDM FORMAT

We propose to study the short-duration strongly noised periodical signals in real time in the following way:

1. The frequency of the periodical signals is specified on base of two successive operation - the determination of auto correlation function (ACF) in SignDM format, and next, the power spectrum determination;
2. Time or phase shifts between two short-duration strongly noised signals of the same frequency will be determined with the use of cross correlation function (CCF) in SignDM format.

The two bit SignDM code $\left\{d_i^{(x)}\right\}$, $\forall d^{(x)} \in \{-1,0,1\}$ is obtained in a following manner:

$$d_i^{(x)r} = \begin{cases} 1,\ if\ x_i - x_{i-r} > \delta^{(x)} \\ 0,\ if\ \left|x_i - x_{i-r}\right| \leq \delta^{(x)} \\ -1,\ if\ x_i - x_{i-r} < -\delta^{(x)} \end{cases}, \qquad i = \overline{1,N},\ N = ENT\left(\theta T^{-1}\right) \tag{1}$$

where $\delta^{(x)} \geq 0$ - differential zone of SignDM, assigned a'priori. Distance $r \in \left\{a \middle| a = \overline{1,s}\right\}$ between samples in PCM format determines the order of differences as well as the SignDM code. Sample rate of SignDM is equal to the Nyquist's frequency same as with PCM [7,8].

The characteristics of the SignDM coder according to (1) is shown in Figure 1. The structure of the SignDM coder on s order is shown in Figure 2. SingDM coder consists of an PCM analog - digital converter (A/D) and s same channels, each having a delay unit, subtractor and a comparator. Common for all the channels A/D generates the input signal PCM samples with sampling rate . The time of the delay in each channel relates to its order. Subtractor produces differences which are compared in the comparator with differential zone of SignDM $\delta^{(x)}$. As a result the comparator's output shows SignDM code $B_i^{(Sx)r} B_i^{(Mx)r}$.

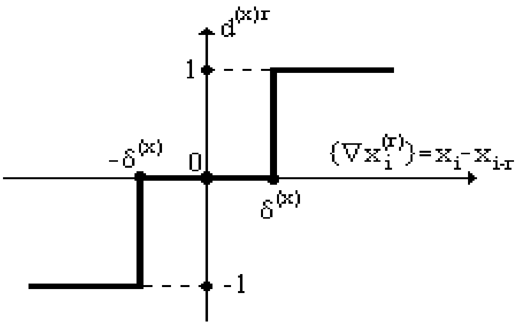

Fig. 1. Characteristic of the SignDM coder

Representing SignDM code in binary form in accordance with the principles

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where - sign's bit, - module's bit, , we obtain the algorithm for the multiplication of SignDM codes

. (2)

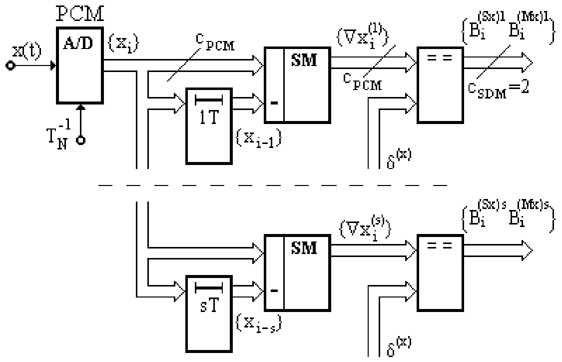

Fig. 2. Structure of the SignDM coder

The *r*-order ACF and CCFs in SignDM format have the forms [7]:

$$\kappa_{xx}^{(r)}(m)=\frac{1}{N-m}\sum_{k=0}^{N-m-1}d_k^{(x)r}d_{k+m}^{(x)r}; \qquad (3)$$

$$\kappa_{xy}^{(r)}(m)=\frac{1}{N-m}\sum_{k=0}^{N-m-1}d_k^{(x)r}d_{k+m}^{(y)r}, \qquad (4)$$

$$\kappa_{xy}^{(r)}(-m)=\frac{1}{N-m}\sum_{k=0}^{N-m-1}d_{k+m}^{(x)r}d_k^{(y)r} \qquad (5)$$

Here, $\kappa_{xx}^{(r)}(m)$, $\kappa_{xy}^{(r)}(m)$, $\kappa_{xy}^{(r)}(-m)$ - correlation function estimators for time shifts $m=\overline{0,P}$, $-m=\overline{-P,-1}$, PT – correlation interval, $NT=\theta$ - the length of the realization of random signal, T^{-1} - sampling rate.

As a result of Wiener–Hinchin's transformation of ACF

$$\hat{G}_{xx}^{(SDM)}(k)=\sum_{m=0}^{P}\kappa_{xx}^{(r)}(m)\exp(-j\frac{2\pi}{P+1}mk), \qquad (6)$$

is the spectral power density estimator $\hat{G}_{xx}^{(SDM)}(k)$, $k=\overline{1,P}$, which leads to the recognition of a determined periodical signal from the noises.

As a result of Wiener-Hinchin's transformation of CCF the estimator of the cross spectral power density is obtained

. (7)

We were verifying the proposed method on the basis of DFT of ACF in PCM format and DFT (6) of ACF in SignDM format with the use of computer simulation.

We used the pink noise *ξ(t)* according to S.O.Rice which was formed with *L* sine terms with the use of random phases and the same amplitudes *A* as follows:

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where *φ(t)* – random phase. To this noise we added sine signal with amplitude A_x: . Then signal-noise ratio is as follows:

$$SNR_x=10\lg(A_x^2/LA^2),\ \text{dB}$$

We used too the normalized correlation functions in PCM and SignDM formats appropriately:

$$\rho_{PCM}(m)=\hat{K}_{xx}(m)/\hat{K}_{xx}(0),$$

$$\rho_{SDM}(m)=K_{xx}(m)/K_{xx}(0)$$

We define the ratio SNR$_G$ with the following relationship

$$SNR_G=10\lg\frac{G_x}{G_{s.l.\max}}$$

where G_x, $G_{s.l.max}$ – appropriately main lobe of estimator of spectrum of signal *x(t)* and maximal side lobe of one.

The results of computer simulation based on the proposed method are presented in Fig. 3 and 4. In Fig. 3a and 4a are shown mixtures of signal and noise *x(t)+ξ(t)* when SNR$_x$=-9dB and –14dB appropriately. Here the length of time series is *N*=30 and number of noise terms is *L*=15. Apart from it, in the Fig.3 and 4 there are shown: the normalized ACF estimators $\rho_{PCM}(m)$ in PCM format and $\rho_{SDM}(m)$ in SignDM format as well as estimators of PCM and SignDM power spectra. These averaged results were obtained on base of 100 different realizations of mixture of the sine signal with noise. Solid lines corresponds the signal initial phase $\varphi_0=\pi/2$ rad whilst the dashed lines corresponds 0 rad of one.

It is necessary to note that the resolution of CA of short time series in SignDM format depends essentially on initial phases of the signal and noise components.

As Fig. 3 and 4 show, in most cases CA in 2-bit SignDM format has much higher resolution than CA in the multibit PCM format, in particularly for short time series up to *N*=30. Additionally, Wiener-Hinchin's transformation increases the resolution of CA used for the recognition, in real time, of short-duration strongly noised periodical signals.

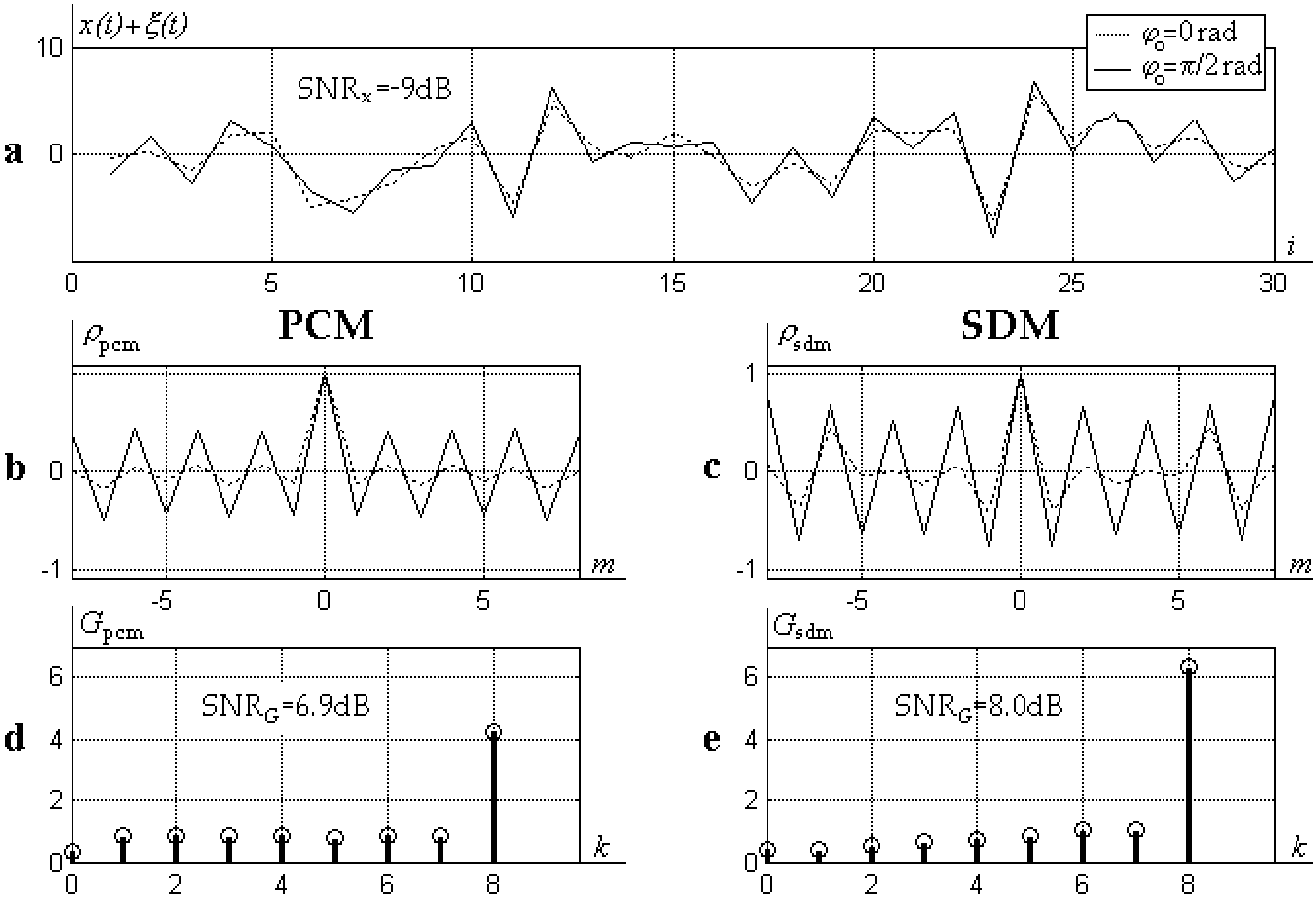

Fig. 3. a) mixture *x(t)+ξ(t)*, *N*=30, *L*=15, SNR_x= -9 dB; b) normalized PCM correlation function, *P*=16; c) normalized SignDM correlation function, *P*=16; d) estimator of PCM power spectrum; e) estimator of SignDM power spectrum

3. CONCLUSION

Algorithms of correlation analysis in SignDM format, showed ih this work, are fast and economical because operate for 2-bit words. It is necessary to note that resolution of CA of short time series in SignDM format depends essentially on initial phases of the signal and noise components. In most cases CA in SignDM format has higher resolution than CA in the multi bit PCM format, in particularly for short time series up to *N*=30. Additionally, Wiener-Hinchin's transformation increases the resolution ability of CA used for the measurement of short noisy signal parameters in real time even though N=30 and SNR_x is less then –9dB.

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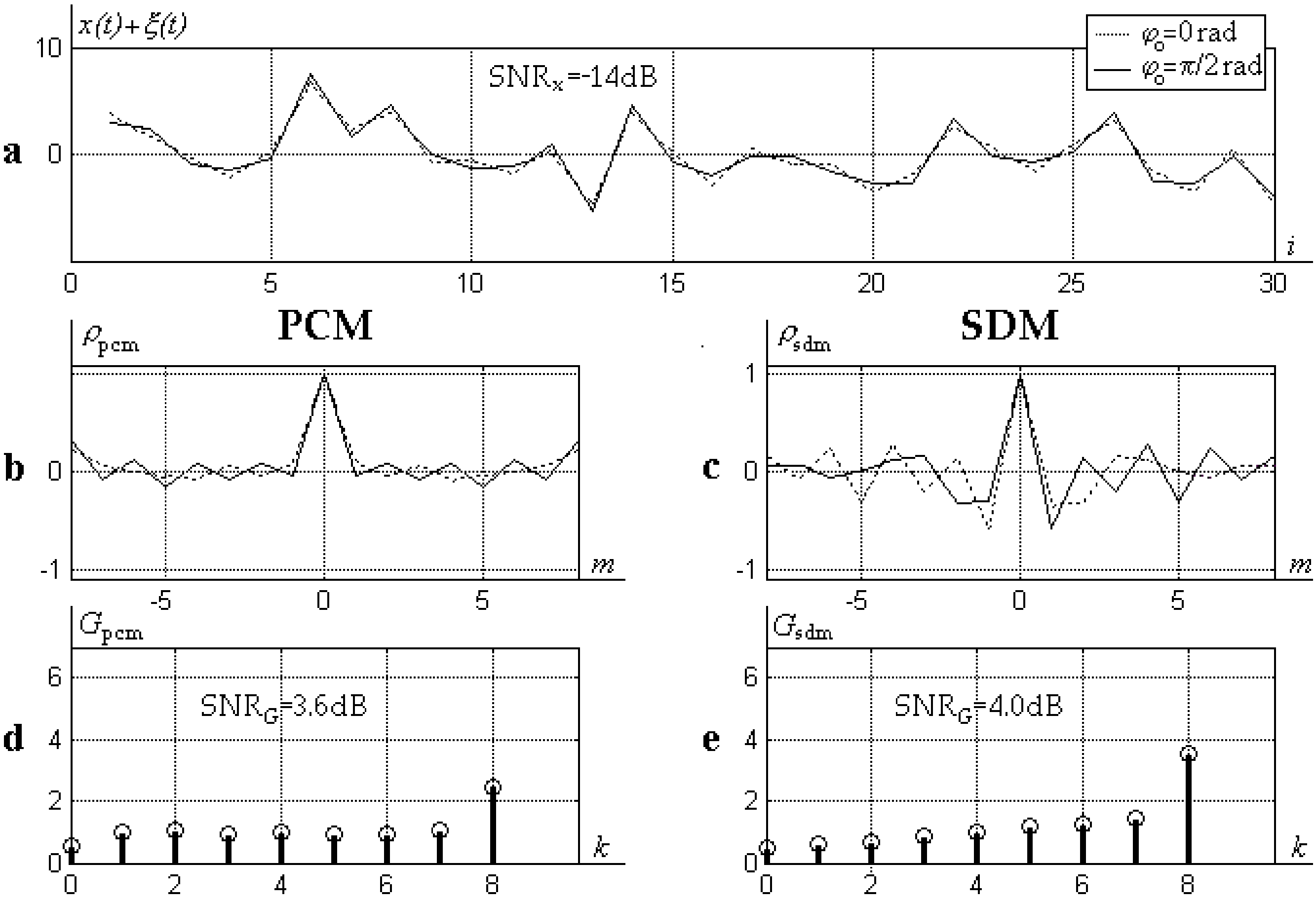

Fig. 4. a) mixture $x(t)+\xi(t)$, N=30, L=15, SNR$_x$= -14 dB; b) normalized PCM correlation function, P=16; c) normalized SignDM correlation function, P=16; d) estimator of PCM power spectrum; e) estimator of SignDM power spectrum