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ADC CHARACTERIZATION IN THE FREQUENCY DOMAIN BY DUAL TONE TESTING

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Abstract: The main goal of this paper is to characterize the low-frequency part of ADC transfer function by dual tone testing from intermodulation components that are independent of test signal harmonic distortion. The theoretical analyses of incoherently sampled signals in the frequency domain and of intermodulation distortion are provided and the theory is verified by experimental results.

Keywords: ADC, dual tone, frequency spectrum.

1. INTRODUCTION

As the sine-wave purity of commercially available generators is often not sufficient particularly for high-resolution ADC testing, alternative ways are being explored. An elegant measurement of ADC nonlinearities consists in the application of two independent sine waves and the observation of the intermodulation components that correspond only to the ADC distortion.

Since the single tone test is the most common one, the ADC parameters are often required to be known for this way of testing. The relation of some single and dual tone parameters was already derived in [1] for the 3rd order polynomial approximation of ADC transfer function. In this paper, more general approach is proposed. The analysis of incoherent sampling is described in section 2, the way of ADC transfer characteristic determination is proposed in section 3 and experimental results are presented in section 4.

2. PHASE DETERMINATION USING COSINE WINDOWS

The sampling of the input signal is mostly performed incoherently. It is also the case of the majority of dual tone tests because coherent sampling at intermodulation frequencies is practically impossible. To suppress the leakage effect in frequency spectrum due to incoherent sampling, signal windowing is performed before frequency spectrum computation.

In practice, cosine windows are widely applied because of their excellent characteristics. Amplitude frequency spectrum of cosine windows is generally well known. Since the phase characteristic is usually not taken into account, phase analysis of common cosine windows will be performed.

Cosine windows, $w_{\cos}$, are determined in the time domain by the formula

$$w_{\cos}(t)=\sum_{l=0}^{L-1}(-1)^l a_l \cos\frac{2\pi}{T}tl \qquad (1)$$

where L is the window order, T its duration and a_l are window coefficients. Window order determines both the main lobe width and the suppression of side lobes. Frequency spectrum of this window can be computed as [2]

$$X_{\cos}(\theta)=\left[\sin(2\pi\theta)+j(\cos(2\pi\theta)-1)\right]\frac{1}{2\pi}\sum_{l=0}^{L-1}(-1)^l a_l\frac{\theta}{\theta^2-l^2} \qquad (2)$$

where $\theta = fT$ is the normalized frequency. Phase frequency spectrum of this window is given by [3]

$$\phi_{\cos}(\theta)=\arctan\left(\frac{\cos(2\pi\theta)-1}{\sin(2\pi\theta)}\operatorname{sign}\left(\sum_{l=0}^{L-1}(-1)^l a_l\frac{\theta}{\theta^2-l^2}\right)\right). \qquad (3)$$

The sign function is within the range of the main lobe, $\pm L$, the rectangular function ± 1 that changes its value at the frequencies where θ is an integer. This is the characteristics of window coefficients a_l. Each a_l is mostly significant for $\theta \approx l$; at the frequency of $\theta = l$ it changes the sign in the fragment denominator. Thus, this sign function can be simplified by $\operatorname{sign}(\sin(\pi\theta))$. Phase spectrum is then (with regard to all phase quadrants)

$$\phi_{\cos}(\theta)=\arctan\left(\frac{-\sin^2(\pi\theta)}{\sin(\pi\theta)\cos(\pi\theta)}\operatorname{sign}(\sin(\pi\theta))\right)=-\pi\theta\,. \qquad (4)$$

Frequency spectrum, $X(k)$, of a windowed cosine wave

$$x(n)=A\cos(\omega n+\varphi) \qquad (5)$$

is according to the DFT definition

$$X(k)=\frac{A}{2N}\sum_{n=0}^{N-1}\left(e^{j(\omega n+\varphi)}+e^{-j(\omega n+\varphi)}\right)w(n)e^{-\frac{2\pi j}{N}nk} \qquad (6)$$

where the decomposition of the cosine function to complex exponentials was applied. The angular frequency, ω, can be decomposed into coherent, c, and incoherent, α, part

$$\omega=(c+\alpha)\frac{2\pi}{N} \qquad (7)$$

where c is integer and $|\alpha|\leq 0.5$. The frequency spectrum, $X_+(k)$, that is defined for positive frequencies equals

$$X_+(k)=\frac{Ae^{j\varphi}}{2N}\sum_{n=0}^{N-1}w(n)e^{\frac{2\pi j}{N}n(c-\alpha+k)} . \quad (8)$$

The amplitude frequency spectrum reaches its maximum at frequency bin c

$$X_+(c)=\frac{Ae^{j\varphi}}{2N}X_w(-\alpha). \quad (9)$$

If cosine window is used, formula (4) can be substituted into (9) for determining the phase frequency spectrum

$$\phi_+(c)=\varphi+\alpha\pi \ . \quad (10)$$

Thus, signal phase, φ, can be computed from the phase frequency spectrum, $\phi_+(c)$, and incoherency coefficient, α. This formula can be also easily derived from the convolution of the signal and window frequency spectra.

The estimate of incoherency coefficient, $\hat{\alpha}$, can be computed from the amplitude frequency spectrum as

$$\hat{\alpha}=\hat{k}_1-c \quad (11)$$

where $\hat{k}_1$ is the estimate of the exact input frequency (in frequency bin units) that can be determined e.g. by the formula [4]

$$\hat{k}_1=\frac{\sum_{k=c-L}^{c+L}k\cdot X^2(k)}{\sum_{k=c-L}^{c+L}X^2(k)} . \quad (12)$$

3. ADC TRANSFER FUNCTION DETERMINATION

ADC transfer function or the *INL* is the representation of the digital output code of an ADC as a function of the input signal value. This curve is commonly measured by means of histogram based method applying a signal with known pdf on ADC input. Great disadvantage of this method is a large number of samples that is required for an accurate estimate.

An interesting method was published in [5]. It proposes the computation of the ADC transfer function from the amplitude frequency spectrum using Chebyshev polynomials. Although this approach essentially reduces the number of required samples, it cannot distinguish test signal and ADC harmonic distortion.

The last mentioned limitation will be the subject of this section. The polynomial approximation will be analyzed for intermodulation components in dual tone mode that are known to be independent of test signal harmonic distortion.

Consider an output data, $y(t)$, gained by acquiring dual tone signal, $x(t)$,

$$y(t)=\sum_{i=0}^{p}b_i x(t)^i , \quad (13)$$

$$x(t)=A(\cos(\omega_1 t+\varphi_1)+\cos(\omega_2 t+\varphi_2)) \quad (14)$$

where b_i are polynomial coefficients and p is the polynomial order. With regard to the simplicity of further analysis, only fifth order polynomial (p=5) is considered in this paper. However, this value is mostly sufficient for the estimation of the low-frequency ADC nonlinearity because the strongest (i.e. the most important) higher harmonic components that appear at real ADCs are usually the second, third and the fifth one.

By substituting (13) into (14) and decomposing the result into separate cosine functions with power of one, three types of components appear: DC, harmonic and intermodulation. It can be shown that possible test signal harmonic distortion causes the rise of DC and harmonic components and only the intermodulation components remain unaffected.

Intermodulation components have a general form of

$$C_{r,s}\cos(\pm r(\omega_1 t+\varphi_1)\pm s(\omega_2 t+\varphi_2)) \quad (15)$$

where r, s are positive integers and (r+s) determines the intermodulation order. Intermodulation coefficients, $C_{r,s}$, are computed in Table 1.

Table 1. Amplitudes of intermodulation coefficients ($C_{r,s}=C_{s,r}$).

Intermodulation order	$C_{r,s}$	Amplitude
2	$C_{1,1}$	$b_2A^2+3b_4A^4$
3	$C_{1,2}$	$\frac{3}{4}b_3A^3+\frac{25}{8}b_5A^5$
4	$C_{1,3}$	$\frac{1}{2}b_4A^4$
4	$C_{2,2}$	$\frac{3}{4}b_4A^4$
5	$C_{1,4}$	$\frac{5}{16}b_5A^5$
5	$C_{2,3}$	$\frac{5}{8}b_5A^5$

If the amplitudes of intermodulation components are known, absolute values of coefficients, b_i, of the ADC transfer polynomial can be estimated by solving the set of equations that can be represented as

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & \frac{3}{4} & 0 & \frac{25}{8} \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{3}{4} & 0 \\ 0 & 0 & 0 & \frac{5}{16} \\ 0 & 0 & 0 & \frac{5}{8} \end{bmatrix} \begin{bmatrix} b_2A^2 \\ b_3A^3 \\ b_4A^4 \\ b_5A^5 \end{bmatrix} = \begin{bmatrix} C_{1,1} \\ C_{1,2} \\ C_{1,3} \\ C_{2,2} \\ C_{1,4} \\ C_{2,3} \end{bmatrix} \quad (16)$$

where the coefficients, $C_{r,s}$, can be computed from the power frequency spectrum by averaging all intermodulation components with the same index r,s and s,r. The averaging cannot correct bias of the amplitude estimate caused by noise but it decreases amplitude variance.

The values of $b_i A^i$ can be computed from (16) using the least mean square method. When signal (fundamental) amplitude, A, is estimated from the amplitude frequency spectrum, absolute value of coefficients, b_i, of the ADC transfer polynomial can be determined.

Signs of the components have to be computed with a help of the phase frequency characteristics. Initial absolute phases of both signals φ_1 and φ_2 have to be estimated first applying (10). The phase given by their linear combination ($\pm r\varphi_1 \pm s\varphi_2$) according to (15) can be either the same or the opposite of the phase frequency spectrum at the intermodulation frequency ($\pm r\omega_1 \pm s\omega_2$). Phase difference of π signifies that the appropriate intermodulation coefficient has minus sign.

If some coefficients of intermodulation components, $C_{r,s}$, consist of two or more terms $b_i A^i$ ($C_{1,1}$, $C_{1,2}$ in Table 1), the solution of (16) can give negative values of some lower-order $b_i A^i$. Negative value signifies opposite sign of the appropriate lower-order $b_i A^i$ and this sign has to be taken into account when determining polynomial coefficients b_i.

For the purpose of coefficient comparison, it is advantageous when the value of each coefficient has roughly equal influence on signal harmonic distortion. This is not fulfilled in case of polynomial coefficients, b_i, of the ADC transfer function; thus, they can be recomputed to the normalized coefficients, B_i, according to the formula

$$B_i = \mathrm{sign}(b_i)\sqrt[i]{|b_i|}\ . \tag{17}$$

The approximation polynomial of the ADC transfer function using the normalized coefficients is then

$$y(t) = \sum_{i=0}^{p} \mathrm{sign}(B_i)\left(|B_i| x(t)\right)^i\ . \tag{18}$$

ADC transfer function is determined by the polynomial (13) with coefficients b_i up to p-th order. If the ADC nonlinearity is needed to be observed in the amplitude frequency spectrum for a common pure sine wave signal, a simple conversion of polynomial coefficients can be applied [6]

$$A_h = \sum_{i=0}^{d} \frac{(2i+h)!}{2^{2i+h-1} h!(i+h)!} b_{2i+h} A_1^{2i+h} \tag{18}$$

where A_h is the amplitude of h-th harmonic component and $d = (p-h)/2$ for $(p-h)$ even, while $d = (p-h-1)/2$ for $(p-h)$ odd.

When the most significant harmonic components A_h are known, ADC spectral parameters such as the *THD*, *SFDR* can be determined. However, the estimation of ADC transfer curve is more universal because it determines not only some ADC integral parameters but also the low-frequency *INL*. The curve can be used for increasing the conversion accuracy by correcting ADC output data. Note that the transfer function of a real ADC can be estimated only for the sine wave particularly at a specific frequency (or two close frequencies) and ADC parameters can vary at different input conditions.

4. EXPERIMENTAL RESULTS

The proposed method was verified by a simulation first. Simulation parameters were chosen to be close to the practical ones. The simulated ADC had the accuracy of 24 bits, the error of transfer function was approximated by five order polynomial $\Delta f(x) = -(18x)^2 - (13x)^3 + (10x)^4 + (9x)^5$ and ADC noise was simulated by white Gaussian noise with standard deviation of 200 LSB. The test signal contained two sine waves at close frequencies without any distortion or noise. Since test signal harmonic distortion has no effect on the estimated parameters, it was not applied because of better illustration of ADC behavior. Data record of 64 kSa was analyzed.

Amplitude frequency spectrum measured by the simulated ADC is shown in Fig. 1. While harmonic components (signed by big numbers in Fig. 1) are significant only up to the third one, all intermodulation components (signed by small numbers in Fig. 1) corresponding to the fifth order polynomial are clearly visible. So, even in the case of distortionless test signal, the reconstruction of the ADC transfer function from harmonic components is not possible.

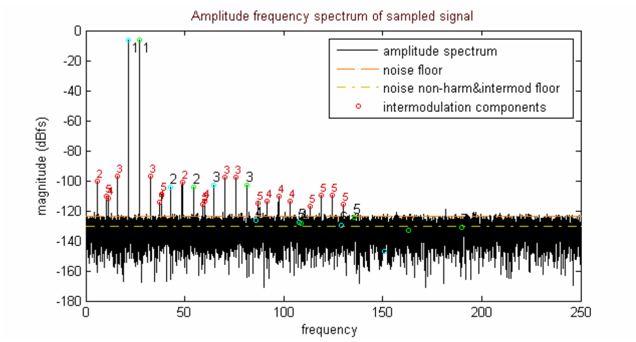

Fig. 1. Amplitude frequency spectrum measured by the simulated ADC

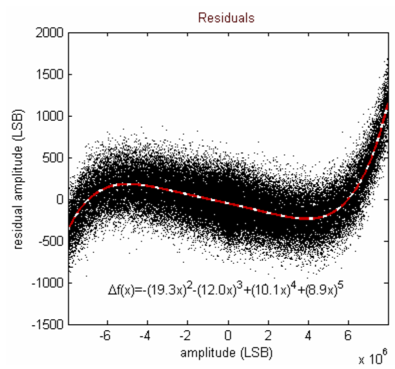

Fig. 2. Residuals of dual tone sine wave fitting and the computed error of ADC transfer function

By means of an algorithm of dual tone sine wave fitting, two fundamental harmonic components were removed from the output signal and the residual signal was plotted in dependence on the output signal amplitude (see Fig. 2); in the case of a pure test signal, this figure follows the ADC transfer function. The error polynomial of the ADC transfer function was computed applying the proposed algorithm and the resulting polynomial curve was also plotted in Fig. 2 (DC and linear component were computed from residuals).

The estimated ADC transfer function is close to the simulated one and estimation errors of the polynomial coefficients have only low impact on errors of the computed ADC parameters. Estimation errors are caused only by noise in the frequency spectrum; thus, the more samples are processed, the lower is the noise floor and the higher is the signal to noise ratio and consequently the accuracy.

Practical measurements were performed on 24-bit Σ-Δ ADC AD7793 at sampling frequency of 500 Hz and two SR DS360 generators. Amplitude frequency spectrum measured by the ADC for dual tone input signal is shown in Fig. 3.

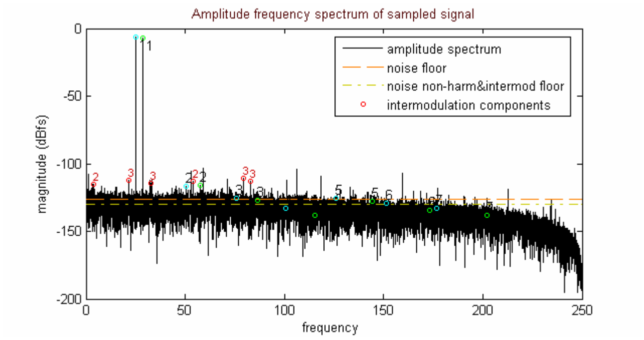

Fig. 3. Amplitude frequency spectrum of the real signal

The approximation polynomial of the ADC transfer function was computed by the proposed algorithm. The third order approximation polynomial was chosen with regard to the strongest intermodulation components. To be able to compare the result, the approximation polynomial of the ADC transfer function was also computed from single tone measurement when supplying the ADC by a pure sine wave. The results are summarized in Table 2.

Table 2. Experimental result of single and dual tone tests.

tones	B_2 coeff. (LSB)	B_3 coeff. (LSB)	*INL* (LSB)	*THD*/*IMD* (dB)
1	–20.4	–12.7	101	–101.1
2	–18.4	–13.1	99	–100.6

In publication [1], it was derived for low-order polynomial approximation that the single tone *THD* roughly equals the dual tone *IMD*. Thus, in the last column of Table 2, the *THD* is computed for single tone test and the *IMD* for the dual tone one.

Although intermodulation coefficients are almost buried in noise (see Fig. 1), the coefficients estimated from the single and dual tone test are in a good agreement. Note that the relative deviations of the computed parameters (*INL*, *THD*/*IMD*) are much less than the relative deviations of the estimated coefficients (B_2, B_3).

5. CONCLUSION

In this paper, phase analysis of incoherently sampled signals and the derivation of low-frequency ADC transfer function independent of test signal harmonic distortion were presented. The derivation was demonstrated on 5^{th} order polynomial approximation of ADC transfer function but the highest order is theoretically not limited. The only problem might be the error accumulation caused by the fact that lower order coefficients are computed using higher order coefficients. Nevertheless, experimental measurements showed that the estimation error of polynomial coefficients has relatively low impact on practical parameters.

The main advantage of the knowledge of the ADC transfer function is that it can be used not only for the determination of several integral ADC parameters but also for the correction of the ADC nonlinearity and consequently for increasing the ADC performance.

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