XVIII IMEKO WORLD CONGRESS
11th WORKSHOP ON ADC MODELLING AND TESTING
September, 17 – 22, 2006, Rio de Janeiro, Brazil

DYNAMIC BEHAVIOR MODELS OF ANALOG TO DIGITAL CONVERTERS AIMED FOR POST-CORRECTION IN WIDEBAND APPLICATIONS

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Abstract: In this paper a dynamic behavior model of analog to digital converters is proposed. The model is aimed for post correction in wideband applications. The suggested post correction method is a combination of look up tables and model based correction.

The model consists of three components. The first is a component represented by a Hammerstein model; that is a static nonlinearity followed by a time invariant linear filter. The second component is a nonparametric model caused by significant deviation from the characterized integral nonlinearity and the output from the Hammerstein model. The third component contains of the remaining deviation and is considered as a random model error.

Results from simulations verify that the examined ADC can be described by an ordinary Hammerstein model and a static look-up table.

Keywords: Analog to digital converters, modeling, wideband applications.

1. INTRODUCTION

The analog to digital converter (ADC) is still a limiting part in wide bandwidth and high dynamic range systems such as digital communication and instrumentation. The main problem with today's ADCs is in the requirements on harmonic distortion and spurious free dynamic range (*SFDR*).

Characterization and testing of ADCs are interesting in many different aspects. For improvements of the ADC characteristics by signal processing methods, the error occurrence is predicted in order to compensate error source effects, so-called post-correction. ADC post-correction by table look-up methods have shown to improve performance measures such as *SFDR*, total harmonic distortion and signal-to-noise and distortion ratio. Thus, it is instrumental for the performance of systems characterized by a wide bandwidth and high dynamic range.

A survey of state-of-the-art ADC modeling and models is given in [1]. In the literature a majority of the proposed methods describe the static properties of the converter. A common solution for post-correction based on a static model is the use of look-up tables (LUT); that is the ADC output is remapped by a table look-up where the table entries are such that some performance measure is improved. It has been show that post-correction based on LUT that do not take dynamics into account is band limited, see for example [2, 3].

Error tables can be used to characterize and compensate nonlinear systems with short memory. Two types of tables considered are phase-plane and state space. In a phase-plane the error is related against amplitude and slope of the input, while for the state-space it is the error against current sample amplitude, and previous sample amplitude. In [4] a further development of the state-space method is suggested, where a generalized approach is taken with full flexibility between the dynamics (that is the number of delayed samples) and the precision of each sample. Thus, the size of the multidimensional look-up table is kept at a reasonable number. However, these methods are burdensome considering time to train the LUT as well as the requirement on memories. Accordingly, there is a need for dynamic post-correction that is easy to train and simple to implement.

In order to take the dynamic effects of the ADC into account, Volterra series for ADC modeling is commonly used [5, 6]. However, in order to fully describe the behavior of a nonlinear system by using Volterra kernels stress that the transfer function is continuous, which is generally not true for an ADC. Post-correction based on Volterra series use an inverse model.

2. THEORY

An important performance parameter of ADC is the integral non-linearity (INL), which can be described as the maximum difference between the ideal and the actual code transition levels after correction for gain and offset. In Figure 1, a typical INL as function of the transition level is viewed.

In [7], an error model was characterized, expressed as one dimensional image in the code k domain consisting of two components. The first component was the low code frequency component (LCF) represented by a polynomial approximation. The second component was a high code frequency component (HCF) caused by significant deviation from the mean value of the differential nonlinearities. The INL as function of the code transition level T_k is then described as

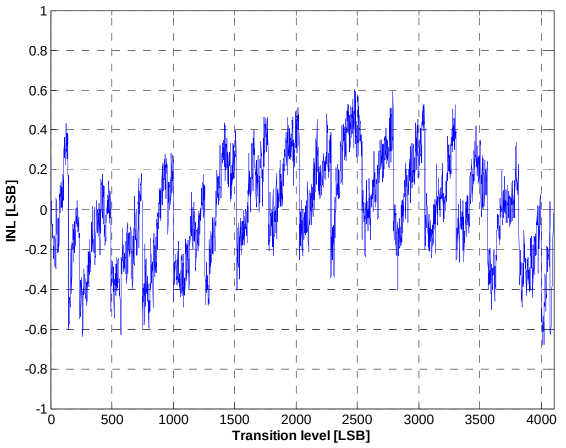

Figure 1: Exemplary measured INL from a 12 bit commercial ADC.

$$INL(T_k) = INL'_{HCF}(T_k) + INL_{LCF}(T_k) \tag{1}$$

where the first term is the contribution by the HCF and the second term the LCF. The LCF is represented by the polynomial

$$INL_{LCF}(T_k) = a_0 + a_1 T_k + a_2 T_k^2 + \dots a_L T_k^L \tag{2}$$

where L is the order of the polynomial.

We further divide the high code frequency component into two parts – $INL_{HCF}(T_k)$ and $INL_{Noise}(T_k)$, respectively. The former, $INL_{HCF}(T_k)$, depends on the design of the component (such as, a pipeline, successive approximation or other structure) and is modeled as piecewise linear. The latter component, $INL_{Noise}(T_k)$, is the part of $INL(T_k)$ that can not be described by an equation. Thus, the INL model in (1) is refined to

$$INL(T_k) = INL_{HCF}(T_k) + INL_{LCF}(T_k) + INL_{Noise}(T_k) \tag{3}$$

A static model is not sufficient to accurately describe an ADC in a wideband application. Hence, also the dynamic behavior needs to be included in the model which can be done by adding amplitude information from either previous samples $\{x(t), x(t\text{-}1), \dots\}$ amplitudes or the slope $s(t)$, for example using the first order backward difference approximation

$$s(t) = \frac{x(t) - x(t-1)}{\Delta t} \tag{4}$$

where Δt is the sampling interval and $x(t)$ is the sample amplitude at time instance t.

The dynamic behavior of the INL can alternatively be described as a frequency dependency; that is different sine wave test stimuli result in different INL data. In order to stress the dependency of (3) on the stimuli frequency ω, (3) is rewritten as

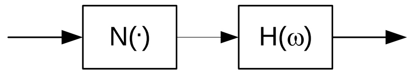

Figure 2: Block structure of the Hammerstein model. The input signal is first affected by a static nonlinear function and then by a linear time invariant dynamic system.

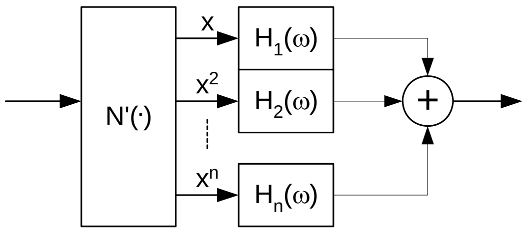

Figure 3: A block diagram representation of the Parallel Hammerstein model. The different orders are now filtered by different filters.

$$INL(T_k, \omega) = INL_{HCF}(T_k, \omega) + INL_{LCF}(T_k, \omega) + INL_{Noise}(T_k, \omega) \tag{5}$$

where ω denotes the frequency variable.

$INL_{Noise}(T_k, \omega)$ is modeled as a noise term. In particular $INL_{Noise}(T_k, \omega) = e(T_k, \omega)$ is assumed zero mean and independent identically distributed in T_k and ω.

Nonlinear dynamic models, such as $INL_{LCF}(T_k, \omega)$, can be described by different model structures. A commonly used is the Hammerstein model, which is a nonlinearity N(·), followed by a linear filter H(ω), see Figure 2. The Parallel Hammerstein model is an extension of the ordinary Hammerstein model. The difference from the standard Hammerstein model is that the contributions of different orders, i, are now filtered by different filters, $H_i(\omega)$. In this paper the Parallel Hammerstein model will be used to analyze the nonlinear dynamic parts of INL, in (5).

3. POST CORRECTION

The purpose with this model is to use it for post correction. The structure of the model may to some extend be effected by the aim to find a dynamic model that is easy to train and simple to implement.

The post correction which will be a combination of look-up tables and model based correction. The high code frequency component will be used as a LUT, since it is improbably that INL_{HCF} can corrected by any other method. In order to minimize the size of the look-up table, the INL_{HCF} is considered static. The $INL_{LCF}(T_k, \omega)$, on the other hand, is a parametric model. Hence, it can be compensated by computation.

4. METHOD

The structure of the wideband model is set in Section 2. Next step will be to find the model order of the different elements and estimate the coefficient values.

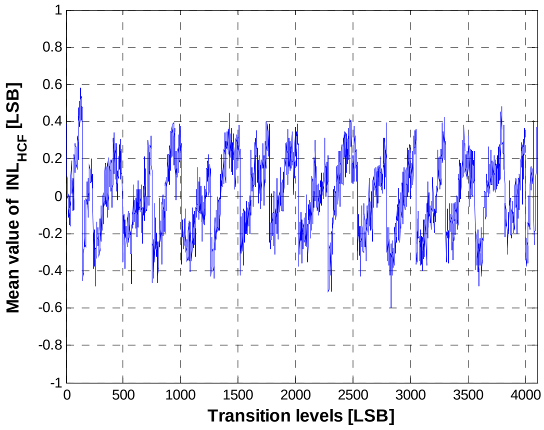

Figure 4: Estimated high code frequency component INL_{HCF} given by an average over all test frequencies.

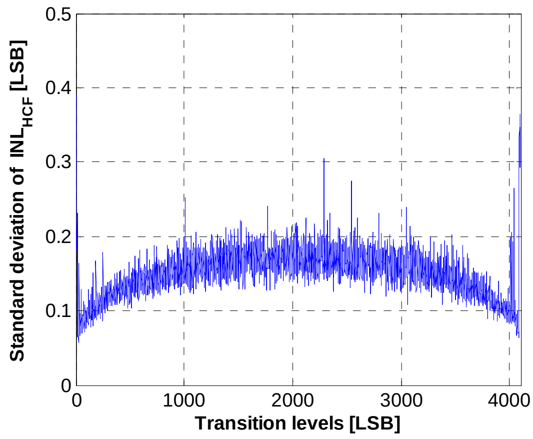

Figure 5: Standard deviation of the INL_{HCF}.

4.1 Characterizing ADC

Different methods for characterizing ADCs are described in [8]. Sine wave fit and histogram tests are two of these methods.

Fitting sine waves is performed by applying a sine wave with specified parameters on the input of the converter, take a record of data, and fit a sine wave function to the record to minimize the sum of squared difference between the function and the data.

The test of ADCs with statistical analysis is based on the building of a histogram, which gives the number of occurrences of each code at the output of the converter. This histogram is then compared with the probability density function (pdf) of the stimulus signal. Sine waves are commonly used as stimulus signals, due to the easiness with which they can be generated and with their spectral purity.

However, sine waves have (per definition) narrow bandwidth. It has been shown [3] that a broad band model can be obtained from several sine waves histograms on different frequencies. The suggested stimuli are sine waves with frequencies spread over the whole bandwidth. Consequently, there will be a set of INL characterizations, one for each frequency. By using sine waves both histogram and sine wave fit can be used as characterization method.

4.2 Estimating high code frequency component INL_{HCF}

Initially it is investigated whether the claim that the structure dependent part $INL_{HCF}(T_k)$ being static is a reasonable assumption for ADC modeling, or not. For that purpose, several INL characterizations are performed for different input frequencies. For each characterization a estimate of the static polynomial $INL_{LCF}(T_k)$ in (2), is executed. Thereafter the characterized INL is reduced by the obtained estimate. That is eliminating the effect of polynomial nonlinearities given by (2). This procedure results in a sequence that is described by the sum of $INL_{HCF}(T_k)$ and $INL_{Noise}(T_k)$, one sequence for each of the different test cases. Under the assumption that the structure dependent part is independent of test frequency, the mean value of the set is the estimate of $INL_{HCF}(T_k)$ and may be used for LUT purposes. Further, the standard deviation of the set gives some information regarding $INL_{Noise}(T_k)$. If the standard deviation is small, the $INL_{HCF}(T_k)$ can be considered as more or less static and that the LUT can omit dynamic effects. Result from our device under test indicates that $INL_{HCF}(T_k)$ is static.

4.3 Estimating the low code frequency component

In order to estimate $INL_{LCF}(T_k, \omega)$ each set of INL characterizations are reduced by the $INL_{HCF}(T_k)$ estimate obtained by averaging in Section 4.2. This procedure gives us a set of data that is described by the sum of $INL_{LCF}(T_k, \omega)$ and $INL_{Noise}(T_k)$ – one sequence for each test frequency. For each data set (or each test frequency) the parameters of a L – order polynomial coefficients $a_2 \ldots a_L$ are estimated by linear regression The parameters a_0 and a_1, which are the offset and gain respectively are by construction equal to zero since they have already been compensated for in the INL calculations. The resulting parameters are plotted versus test signal frequency; that is their frequency response which can be evaluated by standard methods.

5. RESULTS

In this section some results based on simulations will be presented. The test object is a 12 bit, 210 MHz ADC (AD9430, by Analog Devices). The simulation model is provided by the ADC manufacturer. The ADC is characterized over the frequency range 10 – 90 MHz, which gives 80 MHz bandwidth, slightly less than 80 % of the Nyquist frequency. All test frequencies are chosen so that coherent sampling is achieved. In the provided simulation model the $INL_{LCF}(T_k, \omega)$ model is assigned to be a third order polynomial, that is L=3 in (2).

The estimate of the high code frequency $INL_{HCF}(T_k)$, according to Section 4.2, is shown in Figure 4. In Figure 5, the standard deviation of the same data is shown. One may note that the standard deviation is small in relation to the magnitude of $INL_{HCF}(T_k)$, Even though the standard deviation is not negligible; it is small enough to justify that $INL_{HCF}(T_k)$ is considered as static.

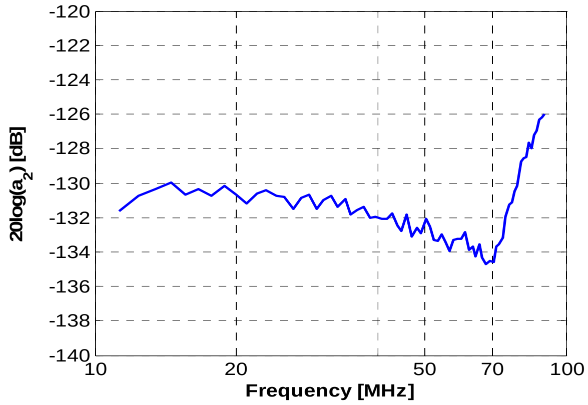

Figure 6: The frequency response from parameter a_2.

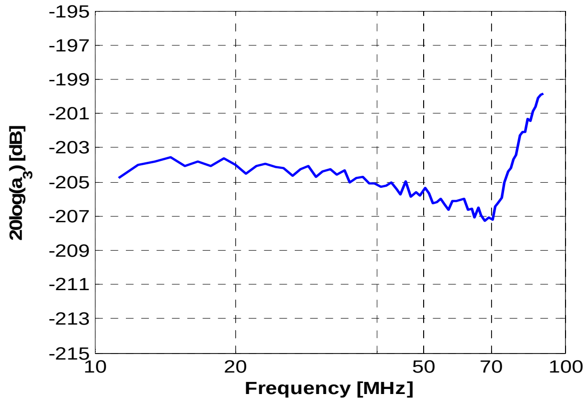

Figure 7: The frequency response from parameter a_3.

The frequency response for the two parameters for nonlinearities, (that is a_2 and a_3, respectively) are shown in Figure 6 and Figure 7, respectively. From the figures it is clearly observable that the parameters obtain a frequency dependency. Moreover, it is interesting to note that the frequency response is comparable with the frequency response of a second order system with complex-valued roots, which is illustrated in Figure 8.

From Figures 6-7, one may note that the magnitude of the coefficients increase at frequencies above 70 MHz. This is in accordance with the data sheet, where the *SFDR* starts to decrease at that frequency.

A further conclusion is that the frequency response for a_2 and a_3 obey similar behaviour. As a result of this observation the parallel Hammerstein structure suggested in section 2 and described in Figure 3 can be modified to an ordinary Hammerstein structure shown in Figure 2; motivated by the fact that both coefficients have equal time invariant filter.

CONCLUSIONS

In this paper a dynamic behavior model of an ADC is proposed. The model is aimed for post correction in wideband applications. The suggested post correction method is a combination of look up tables and model based correction.

The model consists of three components. The first is a component represented by a Hammerstein model. This component will be used for model based correction. Since this component includes dynamic behavior it will broaden the frequency range for the post correction method. The second component is a nonparametric model caused by significant deviation from the characterized integral nonlinearity and the output from the Hammerstein model. This component results in a LUT. The third component contains the remaining deviation and can be considered as a random model error.

Results from simulations shows that the examined ADC can be described by an ordinary Hammerstein model with a third order nonlinearity and a second order filter. The model error can be considered small compared to the static look-up table.

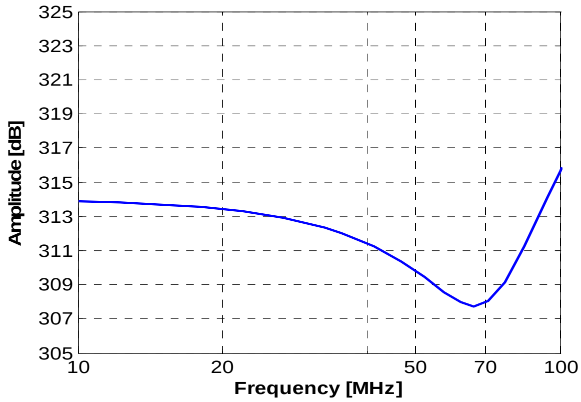

Figure 8: frequency response from a second order filter with two complex roots.

ACKNOWLEDGMENTS

This work was supported by the University of Gävle, the Royal Institute of Technology, the Knowledge Foundation (KKS), Ericsson AB, the Graduate School of Telecommunications (GST), Note AB, Syntronic AB, Racomna AB, and Rohde & Schwarz GmbH.

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