XVIII IMEKO WORLD CONGRESS
Metrology for a Sustainable Development
September, 17 – 22, 2006, Rio de Janeiro, Brazil

FREQUENCY VALUATION IN CURVE FITTING ALGORITHM

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Abstract: The possibility to use the Curve Fitting Algorithm (CFA), for the Power Quality (PQ) parameters valuation foreseen by the EN 50160 Rule, is faced in terms of frequency determination. The comparison between the loop algorithm performance in the frequency determination and the thrown one, using a Taylor Series, truncated in terms of foreseen uncertainty, is presented.

Keywords: Frequency Determination, Curve-Fitting-Algorithm , Power Quality.

1. INTRODUCTION

Although on the subject of Power Quality there are innumerable contributions by the scientific community, the definition of the power quality parameters hasn't been still completed [1,2,3,4]. The ideal sine wave, that should be present in a electrical network, is normally modified by many corrupting factors [5,11], for instance:

- Disturbances;
- Unbalance;
- Distortions;
- Voltage fluctuations;
- Voltage flicker;

The effects of these deviations can be sometimes very dangerous for the electrical systems connected to the electrical net, so it became important to study them. To quantify these effects, the study of the Power Quality parameters is necessary.

The technique to verify Power Quality voltage parameters we consider in this paper is the Curve-Fitting-Algorithm [5,6,7,8,9,10,11] that uses least square error estimation to find the magnitude and phase of the signal frequencies with respect to a time observation window.

Curve fitting measures the sum of squared of residual values between the wave form under observation and the fitted curve. By an iterative algorithm, the CFA minimized this residual obtaining the least squared error and so selecting the best fit curve for the examining waveform and consequently its amplitude and phase.

Least squares curve fitting has both computational and theoretical advantages once correctly determined frequency of the signal in the observation window.

The frequency valuation in Power Quality is the basis for all parameters evaluation foreseen by the EN 50160 Rule.

The Curve Fitting Algorithm is a particularly promising one, allowing the complete set calculation of parameters defined by the Rule, in PQ.

The value of the frequency is the basic input for the calculation in the CFA, that can be initially supposed be the nominal one forecast by the Rule.

The real value must be valued only by a series of successive adjustment that require a loop normally too length to allow the real time operations.

2. CURVE-FITTING-ALGORITHM

The well known best-fit procedure based on Langrange Multipliers, operates on the:

$$O = \int_0^T [y(t) - \alpha \sin(\omega t) - \beta \cos(\omega t)]^2 \, dt \quad (1)$$

where the *y(t)* is the signal in the time window 0↔T, α and β are the sinusoidal and cosinusoidal components that represent the better choice of *y(t)* at the nominal angular frequency ω.

From the (1) it is possible to determine the relations:

$$\begin{aligned} \frac{\partial O}{\partial \alpha} &= \int_0^T [y(t) - \alpha \sin(\omega t) - \beta \cos(\omega t)][-\sin(\omega t)] dt = 0 \\ \frac{\partial O}{\partial \beta} &= \int_0^T [y(t) - \alpha \sin(\omega t) - \beta \cos(\omega t)][-\cos(\omega t)] dt = 0 \end{aligned} \quad (2)$$

The (2) generate the algebraic system

$$\begin{aligned} \int_0^T y(t)\sin(\omega t) dt &= \alpha \int_0^T \sin^2(\omega t) dt + \beta \int_0^T \sin(\omega t)\cos(\omega t) dt \\ \int_0^T y(t)\cos(\omega t) dt &= \alpha \int_0^T \sin(\omega t)\cos(\omega t) dt + \beta \int_0^T \sin^2(\omega t) dt \end{aligned} \quad (3)$$

Values of α is so expressed as:

$$\alpha = \frac{\left[\frac{\sin(\omega T)\cos(\omega T)}{2\omega} + \frac{T}{2}\right] \cdot \int_0^T y(t)\sin(\omega t) dt}{\left[\frac{\sin(\omega T)\cos(\omega T)}{2\omega} + \frac{T}{2}\right]^2 - \left[\frac{\sin^2(\omega T)}{2\omega}\right]^2} - \quad (4)$$

$$-\frac{\frac{\sin^2(\omega T)}{2\omega}\cdot\int_0^T y(t)\cos(\omega t)dt}{\left[\frac{\sin(\omega T)\cos(\omega T)}{2\omega}+\frac{T}{2}\right]^2-\left[\frac{\sin^2(\omega T)}{2\omega}\right]^2} \quad (4a)$$

in similar way the values of β is so expressed as:

$$\beta=\frac{\left[\frac{\sin(\omega T)\cos(\omega T)}{2\omega}+\frac{T}{2}\right]\cdot\int_0^T y(t)\cos(\omega t)dt}{\left[\frac{\sin(\omega T)\cos(\omega T)}{2\omega}+\frac{T}{2}\right]^2-\left[\frac{\sin^2(\omega T)}{2\omega}\right]^2}- \quad (4b)$$
$$-\frac{\frac{\sin^2(\omega T)}{2\omega}\cdot\int_0^T y(t)\sin(\omega t)dt}{\left[\frac{\sin(\omega T)\cos(\omega T)}{2\omega}+\frac{T}{2}\right]^2-\left[\frac{\sin^2(\omega T)}{2\omega}\right]^2}$$

The amplitude A and the phase φ of the sinusoid, in the time window, with respect to the α and β parameters are expressed by the relation :

$$\alpha\sin(\omega t)+\beta\cos(\omega t)=A\sin(\omega t+\varphi) \quad (5)$$

expressly linked each other by the

$$A=\sqrt{\alpha^2+\beta^2} \quad (6)$$

and by the

$$\varphi=\arctan\frac{\beta}{\alpha} \quad (7)$$

In the real applications the integrals that appear in the (4a,b) are valued through a sampling process so that a substitution is operated as:

$$\int_0^T y(t)\cos(\omega t)dt \Rightarrow \frac{1}{f_s}\sum_{i=0}^{N=T\cdot f_s} y\left(\frac{i}{f_s}\right)\cos\left(\omega\frac{i}{f_s}\right)$$
$$\int_0^T y(t)\sin(\omega t)dt \Rightarrow \frac{1}{f_s}\sum_{i=0}^{N=T\cdot f_s} y\left(\frac{i}{f_s}\right)\sin\left(\omega\frac{i}{f_s}\right) \quad (8)$$

being f_s the sampling frequency.

The (8) causes two uncertainties in the integral calculation

- the chosen method to approach the integral value
- the sensibility, in number of bits, of the analogical-to-numerical conversion

that can be considered at the right moment.

2.1 The frequency compatibility

Operating as the α and β parameters, is possible to valuate the ω_{cal} that can be supported by the best-fit procedure in front of the nominal angular frequency ω.

$$\frac{\partial O}{\partial\omega}=\int_0^T[y(t)-A\sin(\omega t+\varphi)][-A\cdot t\cdot\cos(\omega t+\varphi)]dt=0 \quad (9)$$

Resolving with respect to ω, it is possible to obtain:

$$\omega_{cal}=\frac{Num}{2Den} \quad (10a)$$

with *Num* expressed by the

$$Num=\left[\int_0^T y(t)\cos(\omega t)dt\right]^2-\left[\int_0^T y(t)\sin(\omega t)dt\right]^2 \quad (10b)$$

and *Den* expressed by the

$$Den=\left[\int_0^T y(t)\cdot t\cdot\cos(\omega t)dt\right]\cdot\left[\int_0^T y(t)\sin(\omega t)dt\right]- \quad (10c)$$
$$-\left[\int_0^T y(t)\cdot t\cdot\sin(\omega t)dt\right]\cdot\left[\int_0^T y(t)\cos(\omega t)dt\right]$$

In the (10c), will be considered the substitution of integrals that involve the *y(t)*, practically sampled at f_s frequency, in generalized form, with the same consideration about the accuracies.

$$\int_0^T y(t)t^j\cos(\omega t)dt \Rightarrow \frac{1}{f_s^j}\sum_{i=0}^{N=T\cdot f_s} y\left(\frac{i}{f_s}\right)i^j\cos\left(\omega\frac{i}{f_s}\right)$$
$$\int_0^T y(t)t^j\sin(\omega t)dt \Rightarrow \frac{1}{f_s^j}\sum_{i=0}^{N=T\cdot f_s} y\left(\frac{i}{f_s}\right)i^j\sin\left(\omega\frac{i}{f_s}\right) \quad (11)$$

Actually in (10c) j=1

The values of ω_{cal} and ω must be compatible in the limit of the computing accuracy.

The more natural way to obtain this result, is to repeat recursively the calculation that means normally since five to thirteen time, in the Power Quality applications.

2.2 The Taylor Series approach

Starting from (13a) formula that furnishes the ω_{cal}, we can consider that the *Num* and the *Den* re-valuation, starting from the respective derivative, to obtain directly:

$$\omega+\delta\omega=\frac{\sum_{k=1}^{\infty}\left(\frac{\partial^k Num}{\partial\omega^k}\right)\frac{(\delta\omega)^k}{k!}}{2\cdot\sum_{k=1}^{\infty}\left(\frac{\partial^k Den}{\partial\omega^k}\right)\frac{(\delta\omega)^k}{k!}} \quad (12)$$

in which

$$\omega_{calc}-\omega+\Delta\omega=\delta\omega \quad (13)$$

In (13) $\Delta\omega$ represents the adjustment to the difference between ω_{cal} and ω, obtaining directly, the $\delta\omega$ from the nominal angular frequency ω.

The estimate of $\delta\omega$ is possible only limiting the upper value of the summation to g in (12).

The determination is the solution of algebraic equation

$$\sum_{i=0}^{g}a_i(\delta\omega)^i=0 \quad (14)$$

with

$$a_i=\frac{2\omega}{i!}\frac{\partial^i Den}{\partial\omega^i}+\frac{2}{(i-1)!}\frac{\partial^{n-1}Den}{\partial\omega^{n-1}}-\frac{1}{i!}\frac{\partial^i Num}{\partial\omega^i} \quad (15)$$

2.3 The Num and Den successive derivative

The *Num* and *Den* are functions of integrals calculated from the generic forms

$$S_i=\int_0^T y(t)\cdot t^i\cdot\sin(\omega t)dt \quad ; \quad C_i=\int_0^T y(t)\cdot t^i\cdot\cos(\omega t)dt \quad (16)$$

Considering the general derivative with respect to ω of relations reported in are

$$\frac{\partial S_i}{\partial \omega} = \int_0^{\frac{2\pi}{\omega}} y(t)\cdot t^{i+1} \cdot \cos(\omega t)dt = C_{i+1} - 2\pi \frac{y(T)\cdot T^i}{\omega^2} \qquad (17)$$
$$\frac{\partial C_i}{\partial \omega} = -\int_0^{\frac{2\pi}{\omega}} y(t)\cdot t^{i+1} \cdot \sin(\omega t)dt = -S_{i+1}$$

it is easy to verify that we are in front of recursive relations all based on the (15) relations that make very simple the software.

2.4 The Taylor Series truncation accuracy

The choice of a value of *g*, involves the valuation of truncation error of (15) that can be written as

$$\frac{\Delta(\omega+\delta\omega)}{\omega+\delta\omega} = \left|\frac{R_{num}}{Num_{extensive}}\right| + \left|\frac{R_{Den}}{Den_{extensive}}\right| \qquad (18)$$

being R_{Num} and R_{Den} convergence ranges of the Taylor series in which *Num* and *Den* are expressed, considered as relative uncertainties.
Starting from the generic Schlömilch expression:

$$R_{g_{\frac{Num}{Den}}}(\omega+\delta\omega) = \frac{\delta\omega^p(\omega+\delta\omega-\xi)^{g+1-p}\frac{\partial^{g+1}}{\partial\omega^{g+1}}\left[\frac{Num(\xi)}{Den(\xi)}\right]}{g!\cdot p} \qquad (19)$$

knowing that, it is possible to have information of the uncertainty with an arbitrary p and where the ξ value depends by p because $R_n(\omega)$ have to assume always the same value. Choosing p = 1, for the Cauchy form and removing the ξ dependence in the interval $\omega \leftrightarrow \omega+\delta\omega$ the previous became:

$$R_{g_{\frac{Num}{Den}}}(\omega+\delta\omega) = \frac{\delta\omega(\omega+\delta\omega-\xi)^{g}\frac{\partial^{g+1}}{\partial\omega^{g+1}}\left[\frac{Num(\xi)}{Den(\xi)}\right]}{g!\cdot} \qquad (20)$$

Deriving the previous with respect ξ variable, we obtain the maximum value:

$$\frac{\partial}{\partial\xi} R_{g_{\frac{Num}{Den}}}(\omega+\delta\omega) = 0 \qquad (21)$$

that gives:

$$(\omega+\delta\omega-\xi) = \frac{g\cdot\frac{\partial^{g+1}}{\partial\omega^{g+1}}\left[\frac{Num(\xi)}{Den(\xi)}\right]}{\frac{\partial^{g+2}}{\partial\omega^{g+2}}\left[\frac{Num(\xi)}{Den(\xi)}\right]} \qquad (22)$$

To valuate the derivative's values we can use the following approximation always true in the interval ξ-ω:

$$(\omega+\delta\omega-\xi) = \delta\omega - (\xi-\omega) \qquad (23)$$

So we can write:

$$\frac{\partial^{g+1}}{\partial\omega^{g+1}}\left[\frac{Num(\xi)}{Den(\xi)}\right] = \frac{\partial^{g+1}}{\partial\omega^{g+1}}\left[\frac{Num(\omega)}{Den(\omega)}\right] + \frac{\partial^{g+2}}{\partial\omega^{g+2}}\left[\frac{Num(\omega)}{Den(\omega)}\right](\xi-\omega) \qquad (24)$$

and, for the first derivative, it is possible to write the next:

$$\frac{\partial^{g+1}}{\partial\omega^{g+1}}\left[\frac{Num(\xi)}{Den(\xi)}\right] = \frac{\frac{\partial^{g+1}}{\partial\omega^{g+1}}\left[\frac{Num(\omega)}{Den(\omega)}\right] + \delta\omega\frac{\partial^{g+2}}{\partial\omega^{g+2}}\left[\frac{Num(\omega)}{Den(\omega)}\right]}{1+g\frac{\frac{\partial^{g+2}}{\partial\omega^{g+2}}\left[\frac{Num(\omega)}{Den(\omega)}\right]}{\frac{\partial^{g+2}}{\partial\omega^{g+2}}\left[\frac{Num(\xi)}{Den(\xi)}\right]}} \qquad (25)$$

For the subsequent derivative, the value is similarly and easy to find. Considering, in the previous, the third derivatives, calculated with respect to ω and ξ, equals, we can write:

$$\frac{\partial^{g+2}}{\partial\omega^{g+2}}\left[\frac{Num(\xi)}{Den(\xi)}\right] = \frac{\frac{\partial^{g+2}}{\partial\omega^{g+2}}\left[\frac{Num(\omega)}{Den(\omega)}\right] + \delta\omega\frac{\partial^{g+3}}{\partial\omega^{g+3}}\left[\frac{Num(\omega)}{Den(\omega)}\right]}{1+g} \qquad (26)$$

Substituting the previous relation in the derivative with inferior order in the (25), it is possible to obtain the derivative with n+1 order. The ratio between the different orders derivatives is:

$$\frac{\frac{\partial^{g+1}}{\partial\omega^{g+1}}\left[\frac{Num(\xi)}{Den(\xi)}\right]}{\frac{\partial^{g+2}}{\partial\omega^{g+2}}\left[\frac{Num(\xi)}{Den(\xi)}\right]} = (g+1)\frac{\left\{\frac{\partial^{g+1}}{\partial\omega^{g+1}}\left[\frac{Num(\omega)}{Den(\omega)}\right] + \delta\omega\frac{\partial^{g+2}}{\partial\omega^{g+2}}\left[\frac{Num(\omega)}{Den(\omega)}\right]\right\}}{\left\{(g^2+g+1)\frac{\partial^{g+2}}{\partial\omega^{g+2}}\left[\frac{Num(\omega)}{Den(\omega)}\right] + \delta\omega\frac{\partial^{g+3}}{\partial\omega^{g+3}}\left[\frac{Num(\omega)}{Den(\omega)}\right]\right\}} \qquad (27)$$

So the value of $R_n(\omega)$ is given by:

$$R_{g_{\frac{Num}{Den}}}(\omega+\Delta\omega) = \frac{\Delta\omega(g+1)^g}{g!} \qquad (28)$$
$$\frac{\left\{\frac{\partial^{g+1}}{\partial\omega^{g+1}}\left[\frac{Num(\omega)}{Den(\omega)}\right] + \delta\omega\frac{\partial^{g+2}}{\partial\omega^{g+2}}\left[\frac{Num(\omega)}{Den(\omega)}\right]\right\}^{g+1}\left\{\frac{\partial^{g+2}}{\partial\omega^{g+2}}\left[\frac{Num(\omega)}{Den(\omega)}\right] + \delta\omega\frac{\partial^{g+3}}{\partial\omega^{g+3}}\left[\frac{Num(\omega)}{Den(\omega)}\right]\right\}}{\left\{(g^2+g+1)\frac{\partial^{g+2}}{\partial\omega^{g+2}}\left[\frac{Num(\omega)}{Den(\omega)}\right] + \delta\omega\frac{\partial^{g+3}}{\partial\omega^{g+3}}\left[\frac{Num(\omega)}{Den(\omega)}\right]\right\}^{g+1}}$$

Determination of convergence ranges is related to the upper bound valuation of Schlömilch inequality that furnishes:

$$R_{g_{\frac{Num}{Den}}}(\omega+\delta\omega) = \frac{\delta\omega(g+1)^g}{g!(g^2+g+1)^{g+1}}\frac{\left\{\frac{\partial^{g+1}}{\partial\omega^{g+1}}\left[\frac{Num(\omega)}{Den(\omega)}\right]\right\}^{g+1}}{\left\{\frac{\partial^{g+2}}{\partial\omega^{g+2}}\left[\frac{Num(\omega)}{Den(\omega)}\right]\right\}^{g}} \qquad (29)$$

The first applications of the (29) in real case, implies that for

- *g*=2; $(\Delta\omega_{cal}/\omega_{cal})_2$= 2%
- *g*=3; $(\Delta\omega_{cal}/\omega_{cal})_3$= 0.2%

The accuracy related to the integrals is normally negligible in both the cases, being of the order of few tens of ppm.

For the integral's uncertainty it is possible to see [12].

3. SOFTWARE CONSIDERATIONS

The modularity identified in the frequency determination of CFA using the Taylor series decomposition, makes the algorithm very speed with respect to the loop determination system.

There is the evidence that the more heavy part in the calculation of frequency, is related to the function shown in (16), related to the sampling of *y(t)*.

If each of these integrals require a computation time of τ, every cycle of a loop, in the classical determination requires 4τ multiplied by the number of loop to obtain the congruence in (10a) that means a global computation time $T = 20\tau \div 52\tau$.

The accuracy in this case approach the 0.1%

In the case of Taylor series algorithm, with g=3, the $T = 6\tau$ with an accuracy that is about the 0.2%.

It is evident an advantage from 4 to 13 time in computation time in front a lower accuracy that became the half.

4. CONCLUSIONS

In the CFA algorithm is easy to determine, by a low number of sampling, the parameters foreseen by the EN 50160.

The intrinsic problem of CFA are related to the correct identification of the true angular frequency, that is difficult and heavy to compute.

The paper demonstrate the possibility to cut drastically computation time with a loss in accuracy that is, in every case, sufficient for the requirement of EN 50160.

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