Another argument to consider the reliability of the uncertainty Type B similar to Type A

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Abstract: The ISO-GUM express that the standard deviation of the experimental standard deviation of the mean $\bar{q}$ obtained for ratio $\sigma[s(\bar{q})]/\sigma(\bar{q})$ has a not negligible value for practical values of n, and therefore that Type A evaluations of the standard uncertainty are not necessarily more reliable than Type B evaluations. Although this correct comparison either, the assertion based on the value of the $s(\bar{q})$ is the not best, in way that this work to consider other relation as argument for this comparison.

Key words: uncertainty of the uncertainty, standard deviation distribution.

INTRODUCTION

In order to evidence that Type A evaluations of the standard uncertainty are not necessarily more reliable than Type B evaluations the ISO-GUM [1] argues that the standard deviation of mean can have an uncertainty considerable:

Number of observations, n	$\sigma[s(\bar{q})]/\sigma(\bar{q})$ (Percent)
2	76
3	52
4	42
5	36
10	24
20	16
30	13
50	10

Table 1- $\sigma[s(\bar{q})]/\sigma(\bar{q})$, the standard deviation of the experimental standard deviation of the mean $\bar{q}$ of n independent observations of a normally distributed random variable q, relative to the standard deviation of that mean.

Table 1 indicates that $\sigma(\bar{q})$ estimated by 10 observations will be known within approximately plus or minus 48% (for 1,96 standard deviations) at a 95 % confidence level. [2] Even using large data sets, the experimental determination of the standard deviation remains with a considerable amount of uncertainty.

The value of the $s(\bar{q})$ is the estimator of the $\sigma(\bar{q})$, that in turn it is the standard deviation of the sample distribution of the $\bar{q}$. The sample distribution of $s(\bar{q})$ it has as mean

$$E[s(\bar{q})] = c_4\sigma(\bar{q}) \qquad (1)$$

and standard deviation, [3]:

$$\sigma[s(\bar{q})] = \sigma(\bar{q})\sqrt{1 - c_4^2} \qquad (2)$$

, where the factor c_4 corrects the biased value of the $E[s(\bar{q})]$, being that for great samples this standard deviation can be approached by

$$\sigma[s(\bar{q})] = \sigma(\bar{q})/\sqrt{2(n - 1)} \qquad (3)$$

The sample distribution of the $s(\bar{q})$ is not symmetrical, being that this asymmetry is accented in the measure that the size of the sample diminishes:

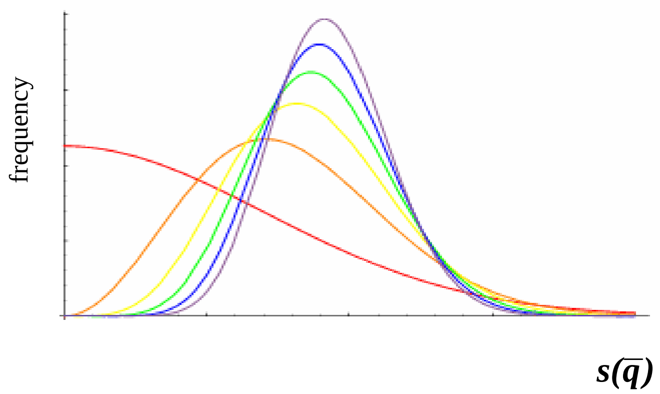

Figure 1- Sample distributions of the standard deviation. For more skewed, $n = 2$. [4]

The standard deviation nor always is a good pointer of variability:
"The standard deviation is a natural measure of spread for normal distributions but not for distributions in general. In fact, because skewed distributions have unequally spread tails, no single numerical measure does a good job of describing the spread of a skewed distribution. In summary the standard deviation is not always a useful parameter, and

even when it is (for symmetric distributions), the results of inference are not trustworthy." [5]. Therefore $\sigma[s(\overline{q})]$ and the ratio $\sigma[s(\overline{q})]/\sigma(\overline{q})$ are not the best pointers of variability to argue that the Type A evaluations of the standard uncertainty are not necessarily more reliable than Type B evaluations.

OBJECTIVE

We need one another comparison that not, $\sigma[s(\overline{q})]/\sigma(\overline{q})$ to conclude that the Type A evaluations of the standard uncertainty are not necessarily more reliable than Type B evaluations.

The number N of standard deviations $\sigma[s(\overline{q})]$ from the interval of the probabilities, of the sample distribution of the $s(\overline{q})$ is obtained through the distribution of the sample variance that is proportional to chi-square distribution, because $\sigma^2[(\overline{q})]/\nu$ is a constant. Therefore the confidence interval for $s^2(\overline{q})$ is :

$$\frac{\sigma^2[(\overline{q})]\ \chi^2_{1-\alpha/2}}{\nu} \leq s^2(\overline{q}) \leq \frac{\sigma^2[(\overline{q})]\ \chi^2_{\alpha/2}}{\nu} \qquad (4)$$

Where ν stands for the degrees of freedom of the sample distribution.

Therefore:

$$N = \frac{\sqrt{\dfrac{\sigma^2[(\overline{q})]\ \chi^2_{\alpha/2}}{\nu}} - \sqrt{\dfrac{\sigma^2[(\overline{q})]\ \chi^2_{1-\alpha/2}}{\nu}}}{\sigma[s(\overline{q})]} = \frac{\sqrt{\dfrac{\sigma^2[(\overline{q})]\ \chi^2_{\alpha/2}}{\nu}} - \sqrt{\dfrac{\sigma^2[(\overline{q})]\ \chi^2_{1-\alpha/2}}{\nu}}}{\sigma(\overline{q})\sqrt{1 - c_4^2}} \qquad (5)$$

For $n = 2$, and from de median of the sample distribution of the $s(\overline{q})$, with +16% of the probability results $N = 0{,}784\,\sigma[s(\overline{q})]$ and with –16% , $N = 1{,}212\,\sigma[s(\overline{q})]$. These numbers become evident the asymmetry of the sample distribution of the $s(\overline{q})$ and this is another way to consider that $\sigma[s(\overline{q})]/\sigma(\overline{q})$ is not a good indicator to express the reliability of the uncertainty type A.

METHODOLOGY

Having the knowledge of the

$$\sigma(\overline{q}) = \sigma/\sqrt{n} \qquad (6)$$

does not exist a sample distribution of the $s(\overline{q})$. In the practical one, $\sigma(\overline{q})$, is estimate through of the

$$s(\overline{q}) = s/\sqrt{n} \qquad (7)$$

and we use the Student-t distribution. Thus, the uncertainty of the uncertainty originates from the use of this sample distribution, in way that the $t_p(\nu)/z_p$ that express the relation of the factor t of the Student-t distribution, for degree of freedom ν that defines an interval $-t_p(\nu)$ to $+t_p(\nu)$ that encompasses the fraction p of the distribution for the factor z of the normal distribution for the same fraction p express better the uncertainty of the uncertainty. In the table below we have this relation for some values of the n:

ν	90%(t)	90%(z)	95%(t)	95%(z)	t/z 90%	t/z 95%
1	6,314	1,645	12,71	1,96	3,8383	6,48469
2	2,92	1,645	4,303	1,96	1,77508	2,19541
3	2,353	1,645	3,182	1,96	1,4304	1,62347
4	2,132	1,645	2,776	1,96	1,29605	1,41633
9	1,833	1,645	2,262	1,96	1,11429	1,15408
19	1,729	1,645	2,093	1,96	1,05106	1,06786
29	1,699	1,645	2,045	1,96	1,03283	1,04337

Table 2- Factors t, z and relations t/z

This table becomes evident that for these data the uncertainty of the $s(\overline{q})$ can be up to 6,5 times the $\sigma(\overline{q})$, in other words, the uncertainty of the $s(\overline{q})$ increases when the sample size decreases.

CONCLUSION

Due **to** the statistical concepts presented in this work is evident that the relation $t_p(\nu)/z_p$ is more appropriate that the relation $\sigma[s(\overline{q})]/s(\overline{q})$ to show how great is the uncertainty of the Type A uncertainty for small samples.

REFERENCES

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