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THE APPLICATION OF MONTE CARLO SIMULATION TO EVALUATE THE UNCERTAINTY OF CONTROL CHART PERFORMANCE INDICES

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Abstract: This paper presents the preliminary results of a research aimed to evaluate the effect of the measurement process on the effectiveness of control charting. To achieve this goal, the authors propose propagating the uncertainty of measurement through the control charting process using the Monte Carlo simulation technique (MCS).
Results are shown, evidencing that measurement uncertainty affects significantly the effectiveness of control charts an that the influence is different depending on the nature of the uncertainty contribution.

Keywords: Measurement Uncertainty, Control Charts, Monte Carlo Simulation, Average Run Length.

1. INTRODUCTION

Quality assurance involves organizing all industrial efforts to satisfy customers' demands for better products. Nowadays, an effective quality assurance system is characterized by the correct use of data in different feedback cycles inside the company, by giving priority to preventing defects or imperfections on the final product [1]. Preventive actions are necessary since the very first stages of process planning, to assure that the processes will operate in target with minimum variance. In this context, the importance of statistical processes control (SPC) is justified because of its applicability on both manufacturing and management processes. SPC uses control charts or process behavior charts (as denominated by Wheeler [2]), to identify special causes of variation and keep processes under statistical control. When applied according to the continuous improvement philosophy, SPC allows the reduction of process variability, minimizing the quality loss for the specific production.

Regardless the type of control chart, the effectiveness of SPC is influenced by sampling variations. Indeed, any control chart produces false alarms, as well as it omits signals of process change. Furthermore, the effectiveness of SPC also depends on the quality of the data used to plot the chart.

Several authors studied the effect of measurement errors on Shewhart´s control charts [3 - 7]. Most of these authors focused on the average and range or average and standard deviation charts, passively operated to detect process disturbances by means of the comparison of each sequential reading with 3σ control limits. The detection of an out-of-control condition is usually made by the classic rule "one point beyond the control limits". The probability of intervention or its inverse, the average run length (*ARL*), are adopted as performance indices. Measurement errors are represented by the normal distribution, which parameters are assumed constant in time and independent of the value of the measurand. The authors reported that, under this set of assumptions, measurement errors affect relevantly the performance of control charts: they increase the probability of false alarms and reduce the sensitivity of the control chart to detect process disturbances.

The main objective of the research reported in this paper is to evaluate the effect of the measurement process on the effectiveness of control charting. To achieve this goal, the authors propose propagating the uncertainty of measurement through the control charting process using the Monte Carlo simulation technique (MCS). The use of this technique in metrology is not new, but only recently the Supplement 1 of the Guide to the Expression of the Uncertainty in Measurement [8] has been issued, establishing good practices for the application of MCS in the evaluation of measurement uncertainty.

The key hypothesis of the reported research is that the indices of control chart performance, like *ARL*, are metrics to which the GUM concept of measurement uncertainty [8, 9] can be successfully applied. Results of simulated process analyses and real industry applications are presented and discussed.

2. METHODOLOGY

2.1 Method of propagation of uncertainty

Computer simulation can be considered a tool of great value when a complex mathematical model is used to describe a technical system or when the model does not allow an analytical solution. The evaluation of mathematical models by random sampling of probabilistic distributions is known as the Monte Carlo simulation technique.

The evaluation measurement models using the MCS technique is carried out in two steps [10]. The first one consists of establishing the measurement model, while the second involves the model evaluation. The fundamental differences between the classic method and the MCS are the type of information that describes the inputs quantity and the manner in which the information is processed to evaluate the measurement uncertainty. In the classic method, each input variable must be characterized by its probability density function (PDF), average, standard deviation and degrees of freedom. In MCS, the degrees of freedom are not involved in the calculations, but the knowledge about how many sampling variation is present in the input data is relevant for the analysis of the results.

In MCS, the output distribution will be obtained from the evaluation of the mathematical model through the combination of random samples of the input variables, respecting their probability distributions. Therefore, MCS produces the propagation of the PDFs of the input quantities through the mathematical model of the measurement. The result is also a PDF, that describes the measurand values which are consistent with the available information. For this reason, it is known as "method of the propagation of distributions".

2.2 Propagating measurement uncertainty through control charting

The objective of control charting is to detect signals of process change. Typically, estimates of process position and process dispersion are charted. These statistics are calculated using small samples, called subgroups, which are drawn from the process output periodically.

In a real industrial situation, an initial process run is made and provisional control limits are calculated from the collected data. If the process shows a reasonable degree of control, these control limits are used for process supervision. If the process is not under control, corrective actions are executed to eliminate special causes and new limits are calculated. All the actions that are performed until the control limits are defined characterize the so called “Phase 1” of SPC. When the concept of measurement uncertainty is accepted and applied to the data used to calculate the control limits, it becomes clear that an uncertainty of the control limits exists, that results from the propagation of measurement uncertainty though the “Phase 1” of SPC. The higher the uncertainty of measurement, the larger will be the region within which the true value of the control limits could be.

A simulation was performed to show the effect of random and systematic contributions to measurement uncertainty on the control limits. The manufacturing process model and its parameters have been obtained from a real machining operation, i.e. μ=50,698 mm and σ=5 μm. The uncertainty of measurement also describes the measurement process that is in fact performed for control charting, i.e. $U_{95\%}$=4.5 μm.

Figure 1 shows that random effects increase the data variation and enlarge the distance between control limits, with almost no effect on the position of the process. On the other hand, unknown and residual systematic errors could produce a shift on the process position chart, as shown in figure 2. In these figures, the subgroups’ averages, grand average and control limits drawn in continuous lines correspond to the process as the operator can perceive it by measurement. In dashed lines the grand average and limits calculated using values of the measurand that are consistent with the measured values and the information that is available on measurement uncertainty.

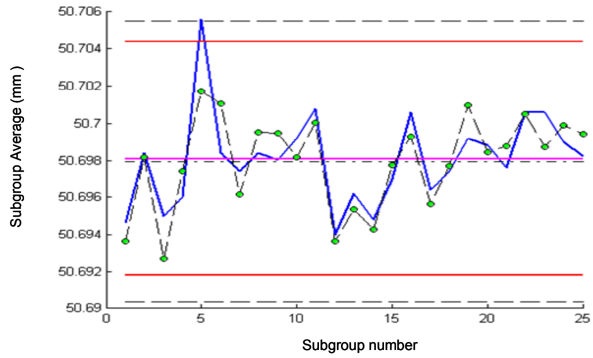

Figure 1: Effect of random measurement errors on the average control chart.

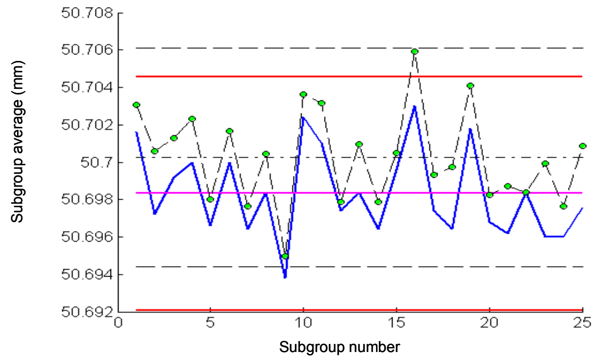

Figure 2: Effect of systematic measurement errors on the average control chart.

In figure 3, histograms depicting the distribution of the values of the grand average and control limits obtained by repeated simulation can be observed (1000 simulation runs). In this simulation, the original measured values have been preserved, but different values of the measurand that are consistent with the declaration of measurement uncertainty have been generated.

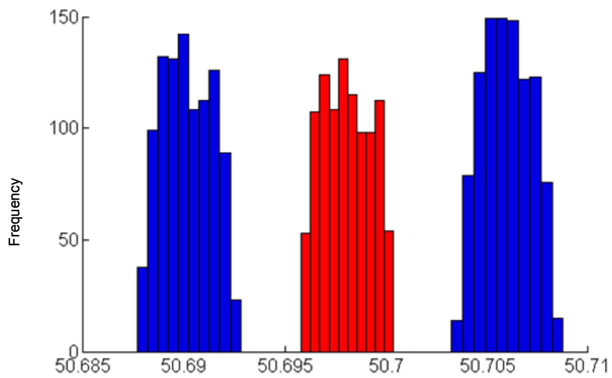

Figure 3: Empirical distribution for grand average (middle) and control limits (right and left side) due to the effect of measurement uncertainty.

Once the control limits are defined, process supervision begins on a periodical basis. For this purpose, the operator draws new samples from the process output, calculates the statistics of process position and process dispersion and compares these statistics with the respective control limits. If the statistics fall within the control limits, it is said that the process remains in statistical control. If any of the statistics fall outside of the limits, a special cause of variation could be acting and a reaction plan should be triggered. This process supervision stage is known of “Phase 2” of SPC.

The average run length (*ARL*) is an accepted index to the performance of a control chart to detect process variations during “Phase 2”. It is the average of the values of run length (*RL*) that could be obtained from a large number of control charts plotted with random data sampled from the same process. The *RL* is the number of samples that are necessary for a given chart to detect a change in one of the process parameters, computed from the moment in which the change happens to the moment in which the chart produces the corresponding signal.

To characterize this phenomenon, the above-reported simulation has been extended to the “Phase 2”, introducing a disturbance of the process mean. After analyzing the control chart plotted with measured values, a *RL*=21 subgroups was obtained. For the 1000 charts simulated with values of the measurand, the *RL* presented high variability. The empirical distribution of these run lengths is strongly asymmetric where 50 % of the obtained values are equal or less than 32 and 95 % are less than 370 (see figure 4).

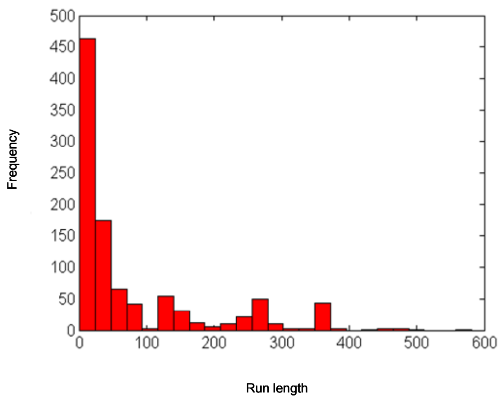

Figure 4: Empirical distribution of possible values of run lengths due to measurement uncertainty.

These preliminary results show that a single control chart obtained by sampling a given manufacturing process can be analyzed using MCS to evaluate the effect of measurement uncertainty. Nevertheless, the analysis of a unique control chart can not consider the effect of sampling variation, relevant in SPC. Because of this, two different cases have been addressed in this research. The first analysis has been performed on real measurement data, obtained during the long-term operation of an industrial machining process. In the operation interval several out of control signals have been detected by means of an individual values and moving range control chart, generating several corrective actions and process adjustments. Thus, in this case, the values of *RL* can not be calculated, because the instant in which the process disturbance occurs is unknown. Instead, the dispersions of the number of points identified below the lower control limit (*LCL*) and above the upper control limit (*UCL*) have used as index to the effect of measurement uncertainty on the control chart performance.

The second analysis has been made using only simulated data. An average and range chart with subgroup size equal to 5 has been used to study the same process under the effect of different measurement uncertainties. In this analysis, the magnitude and timing of the process disturbance is controlled by the mathematical model. Then, the value of the *RL* can be calculated and its statistics used as indices to the effect of measurement uncertainty.

3. SIMULATION RESULTS

3.1 Analysis with real industrial data

The manufacturing process distribution model and its parameters have been obtained from a real machining operation (μ=50.6958 mm and σ=4.4 μm). These estimators have been calculated from a set of 2948 observations, obtained during three months. The normal probability plot (figure 5) shows that the long-term process distribution is acceptable for conventional control charting.

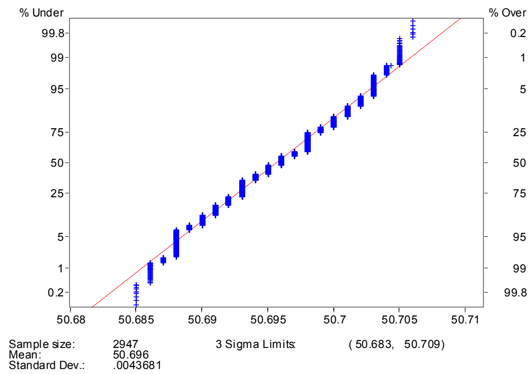

Figure 5: Process normal probability plot.

The control limits of the chart for individual values have been calculated using the first 100 observations. Using these control limits, out of control situations have been identified and counted, stratifying the signals below the *LCL* and above the *UCL*, respectively 350 and 8 points. Unfortunately, in a real SPC operation it is not easy to know how many of these signals are actually due to special causes; indeed, several of them can be false alarms.

The reason of the asymmetry in the number of signals above and below the control limits can be understood when considering some details about the operation of the process and its variation:

- The process target is 50.698 and its variation (6σ) is considerably smaller than the product tolerance.
- The control limit has been set to the target.
- The characteristic is an internal diameter and the part is scrapped if the value is bigger than the upper tolerance limit.

- The diameter has a natural trend to diminish because of tool wear and the process presents structural variation, as depicted in figure 6.

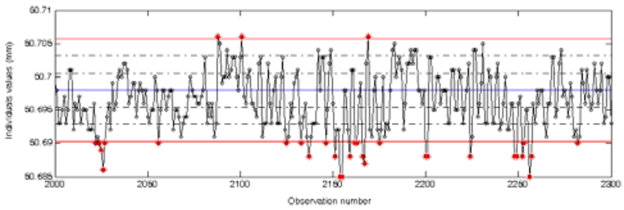

Figure 6: Control chart for individual measurements

Because of this context, the operator was predisposed to let the process run, even when the chart already produced a point below the *LCL*. There is no loss of quality, the life of the tool is extended and he feels safe, because it is less probable to produce parts with diameters exceeding the upper tolerance limit.

Afterwards, 10.000 control charts were derived from the original one applying MCS. Each chart was built with a sequence of values of the measurand that are consistent with the original measured values and the declaration of measurement uncertainty. The "Phase 1" produced different control limits for each simulated chart. Each pair of limits was extended to identify out of control signals during the corresponding "Phase 2". The signals of out of control in each chart were counted and stratified, in a similar manner than above.

The measurement model used in these simulations included repeatability, residual offset and resolution. The standard uncertainty due to repeatability was σ_e=2 µm (normal pdf), the interval within which the residual offset could be was a= ±1 µm (uniform pdf) and the resolution was Res=1 µm (uniform pdf). These values were obtained during the uncertainty assessment of the bore gage actually used for process control.

The histograms in figures 7 and 8 show the dispersion of the number of points due to the effect of measurement uncertainty, as well as the corresponding number of signals found in the chart built with measured values (red line).

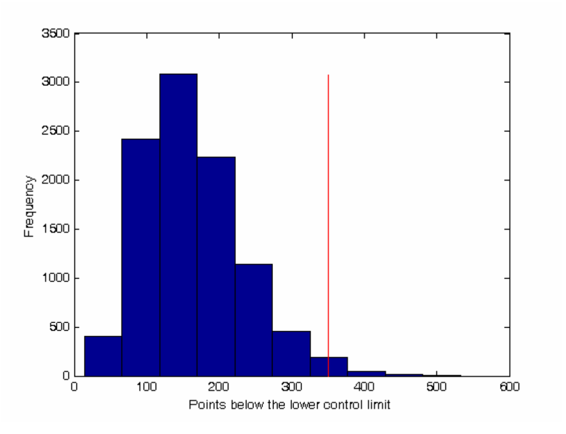

Figure 7: Distribution of signals below the *LCL*.

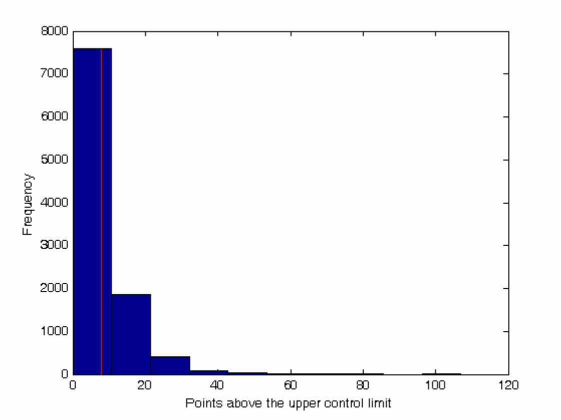

Figure 8: Distribution of signals above the *UCL*

3.2. Analysis with simulated process data

In this analysis the process parameters remained the same, but the measured values were simulated using a standard routine for the generation of normally-distributed numbers. Thus, the simulated data does not represent a process as described in the section before, but a process that is under control in the classic sense, i.e., it presents purely random variation. An average and range control chart was applied, being the subgroup size n=5. The control limits were estimated using 100 subgroups to reduce the impact of the sampling variation on the control limits.

The measurement model was also the same used in the previous section, but the values of each contribution were changed according to a factorial analysis (2^3). The lower value of each contribution is set to zero; the upper, to the value reported in the previous section.

The perturbation of the process mean was introduced after the definition of the control limits. The simulation continues until a signal of out of control is generated by the averages' control chart (i.e. a point beyond the control limits). Thus, the value of *RL* is obtained by counting the number of subgroups between the perturbation and the signal. Two values of displacements have been studied: δ=0 and δ=1. In the first case no disturbance exists and the process remains in control during "Phase 2". This condition makes possible the study of the false alarm rate. The second condition corresponds to a process shift that is equal to the standard deviation of the process divided by the root of subgroup size ($\sqrt{5}$).

To evaluate the simultaneous effect of sampling and measurement uncertainty, 500 charts were built with random measured values sampled from a process with unchanging parameters. Afterwards, each of these charts was used to simulate 500 group of values of the measurand that are consistent with the uncertainty of measurement. The output of the complete simulation process is a matrix containing 500x500 values of *RL*.

Two statistics calculated from the values of *RL* have been used to characterize the effect of measurement uncertainty. The first one is the grand average of the run length across the entire matrix; the second one is the average of the standard deviation of the values of *RL* that are obtained across the effect of measurement uncertainty.

Figure 9 shows the results of these statistics (color maps have been used to represent the simulation results in the 3D domain of measurement uncertainty influence quantities).

The graphics in the top of the figure correspond to the grand average of *RL*; the graphics on the bottom, to the average standard deviation of *RL* values.

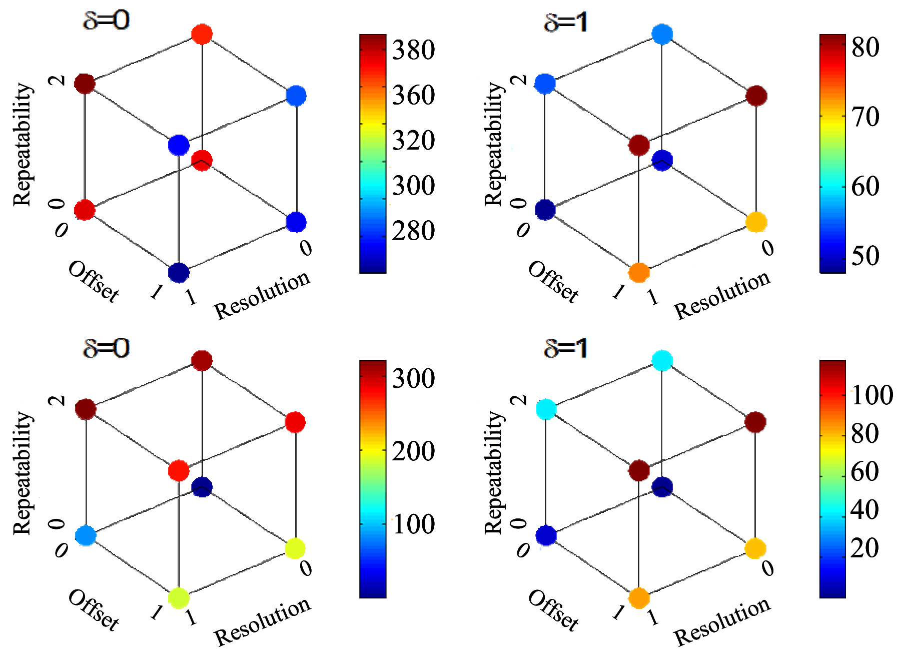

Figure 9: The grand average (top) and the average standard deviation (down) of the run length, studied for a process under control (left) and a process in which the mean was disturbed (right).

4. DISCUSSION

Both cases above-described contribute to understand the effect of measurement uncertainty on control charts, particularly, on process position charts like individual values chart or subgroup averages charts.

The first analysis is focused on a process of the real world, operated in such a way that the theoretical assumptions that rule the application of control charts are barely satisfied. In spite of that, the particular combination of statistical tool and measurement process was efficient enough to improve the stability of the process and also its capability from *Cp*=0.7 to *Cp*=2 in less than six months with almost no investment.

The real control chart run in the shop floor presented 350 signals below the *LCL* and 8 signal above the *UCL*. The distributions of the signals for the 10.000 charts built with simulated values of the measurand show that the effect of the measurement process could be relevant. In particular, the average of the number of signals below the *LCL* in the simulated charts is significantly smaller than the number of signal produced by the real chart (see figure 7). There is also a heavy dispersion between the minimum and maximum values found. As the measurement uncertainty contributions are reduced (not shown), the dispersion also diminishes and the average number of signals below the *LCL* approaches progressively the value of 350, obtained by the real chart. In the limit, when all the contributions to uncertainty are set to zero, the dispersion disappears and the number of signals repeats for all the simulations.

The behavior of the number of signals above the *UCL* is similar regarding the dispersion, but the effect on the average is less significant. The description above helps understanding how the results of the simulations have to be interpreted and provides the context to discuss the results of the simulated process.

The second analysis allowed the use of the statistical properties of the *RL* to evaluate the impact of different contributions to measurement uncertainty. For the chart used in the simulations, the theoretical values of *ARL* are 370 for $\delta = 0$ (i.e. a stable process) and 44 for $\delta = 1$ (i.e. shift of the mean equal to the standard deviation of the sample average). These theoretical values are obtained when the control limits are positioned exactly at $\pm 3 \cdot \sigma / \sqrt{n}$ of the grand average.

Given that the initial run was simulated with a large number of subgroups, the average of the *RL* approaches to the theoretical *ARL* when the measurement uncertainty contributions are set to zero (see graphics on the top of figure 9). In this case, the average standard deviation of the *RL* values is zero, because the variation is exclusively due to the manufacturing process (see graphics on the bottom of figure 9). In presence of measurement uncertainty, the grand average and the average standard deviation of the *RL* values are modified. In particular, it can be observed that:

- The grand average is more affected by the residual offset than by the resolution or the repeatability;
- The average standard deviation of the values of *RL* that are obtained across the effect of measurement uncertainty is more affected by the repeatability than by the resolution or residual offset.

When the process is under control (δ=0), the grand average of *RL* values tends to diminish, resulting in a control chart that is more prone to produce false alarms (graphic in the top-left of figure 9). When the mean of the process is disturbed (δ=1), the grand average increases, being so the chart less sensitive to the changes of the real process (graphic in the top-right of figure 9).

These results can now be analyzed from the viewpoint of the propagation of measurement uncertainty. First, it is necessary to define the measurand. The authors propose using the mean of *RL* as a measure of chart performance for each value of δ. Thus, its best estimate is the grand average of the *RL* values, computed across the effect of sampling and across the effect of measurement errors. The bigger the difference between the theoretical value of *ARL* and this estimate, the bigger will be the effect of measurement uncertainty on the performance of the control chart.

Then, it is necessary to associate an uncertainty to the estimate. In this case, two main contributions to uncertainty can be identified. The first one is natural the sampling variation of the average *RL*, as computed from the repeated simulation when the contributions to measurement uncertainty are set to zero. The second one can be obtained from the average standard deviation across the effect of measurement errors, as depicted in figure 9 (graphics on the bottom). Thus:

$$mean(RL)_{TV} = \overline{\overline{RL}} \pm k \cdot u_{RL} \qquad (1)$$

where *k* is an expansion factor. The understanding of this dual contribution to the uncertainty of the performance measure is important in the everyday use of control charts. It means that the performance of an SPC scheme depends on sampling decisions as well as from measurement decisions. Increasing the size of subgroups or changing the statistics used to supervise the process will add no value if the measurement uncertainty is excessive. On the other hand, high accuracy measurements will not improve the performance of a chart if poor sampling decisions are made.

5. CONCLUSIONS

In the previous sections it has been shown how the effect of measurement uncertainty can be propagated by Monte Carlo simulation, to quantify the performance of a given type of control chart, applied to supervise certain manufacturing process.

Two different simulations have been show. One performed using real industrial data and the other with simulated data, generated using parameters of a real process. In both cases the effects of measurement uncertainty were significant, despite the relationships between the tolerance and the measurement uncertainty were reasonable.

Finally, the grand average of the run length, obtained across sampling and measurement variations, has been defined as a measure to the performance of a control chart. It has been shown that the effect of measurement uncertainty can be isolated, processing properly the values of *RL* obtained by repeated simulations.

In opinion of the authors, the results here presented are only preliminary and more efforts are needed to make clear the complex interaction between measurement and manufacturing process variations in SPC schemes.

6. ACKNOWLEDGMENTS

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