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Modelling of Non-Ideal Measuring Processes – Prerequisite for Error and Uncertainty Analysis

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Abstract: The determination of the uncertainty of measurement results is one of the most demanding but unpopular tasks in metrology. One of several reasons for this fact is the lack of well-structured models of non-ideal measurement processes on one hand and of qualification processes, which determine measurement uncertainties, on the other hand. Easily available and universally valid models would greatly help to improve measurement quality. But since process modelling is very laborious, practitioners are additionally confronted with cost-benefit questions. This paper starts with the most popular but rather incomplete error model and, on the basis of Signal and System Theory, derives a quantitative structure of the non-ideal measurement process. This structure is comprehensive and open for further and more challenging extensions.

Keywords: non-ideal measurement process, error quantities, erroneous measurement results

1. Introduction

The "Guide to the Expression of Uncertainty in Measurement" (GUM) [1] provides extensive advice for a coherent declaration of measurement uncertainties. The authors of hundreds of papers and guidelines have been interpreting GUM, trying to apply it to their specific needs. Discussions about this topic tend to be endless.

Of course the procedure of uncertainty analysis in terms of measurement quality takes place at the very end of the measurement chain and – metaphorically speaking – so do most discussions.

Though nobody questions the importance of the manifold procedures occurring on the different paths within a measurement process, even measurement experts seldom take the time to go beyond summary qualitative cause and effect considerations (Figure 1). The question should arise as to where errors originate. Reliable quantitative results concerning measurement quality are therefore rare.

On the other hand, GUM makes it quite clear that the starting point for every uncertainty analysis is always some kind of model, however simple it may be. Of course GUM itself cannot be expected to deliver models for any imaginable measurement task. Their number would be infinite.

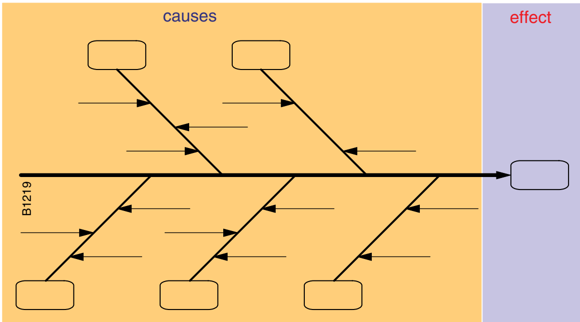


Figure 1. Qualitative cause and effect (Ishikawa) diagram.

The question thus arises whether there might be any fundamental and quantitative models available, which are based on general considerations of measurement science and which are independent of practical applications. And indeed, this is the case.

2. Analysis

The following sections base on the simple and most commonly used model of error analysis. A more promising, but also more demanding concept of signal and system theory will be presented then. It will be shown that both models can be merged for further development.

2.1 The Most Commonly Used Error Model

One of the most famous structures concerning measurement error quantities $\mathbf{e}_y(t)$ is quite simple. Initially the measurement process M is assumed to be ideal so that, according to the "Fundamental Law of Metrology" [2], the transfer behaviour is represented by the ideal transfer function matrix $\mathbf{G} = \mathbf{I}$ (unit matrix):

$$\hat{\mathbf{y}}(t) = \mathbf{G}\,\mathbf{y}(t) = \mathbf{I}\,\mathbf{y}(t)$$

Therefore, error quantities have to be zero in an ideal case. The equation thus is rewritten as:

$$\mathbf{e}_y(t) = \hat{\mathbf{y}}(t) - \mathbf{y}(t) \stackrel{!}{=} \mathbf{0}$$

But for the general non-ideal case, the error equation in its standard form remains:

$$\mathbf{e_y}(t) = \hat{\mathbf{y}}(t) - \mathbf{y}(t)$$

Because of its assumed ideal behaviour the measurement process M can even be omitted in the signal impact diagram (Figure 2), because the output is identical to the input. This simplification is quite common, especially in control engineering.

However, since all measurement results $\hat{\mathbf{y}}(t)$ are erroneous in practice, as we know, we have to add a place, where error quantities $\mathbf{e_y}(t) \neq \mathbf{0}$ may after all interfere. For simplicity reasons, we assume that this happens linearly.

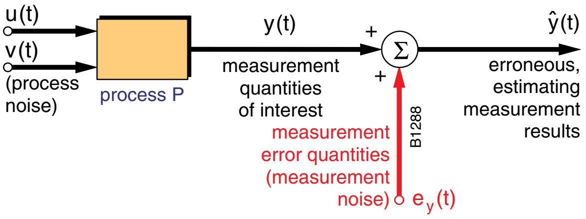


Figure 2. Most commonly used error model.

A further rearrangement of the error equation mentioned above confirms this fact:

$$\hat{\mathbf{y}}(t) = \mathbf{y}(t) + \mathbf{e_y}(\mathbf{t})$$

This leads to the conclusion that erroneous measurement results $\hat{\mathbf{y}}(t)$ consist of the measurement quantities of interest $\mathbf{y}(t)$ and the measurement error quantities $\mathbf{e}_y(t)$.

In this model all imaginable error contributions have been gathered which could have influenced the measurement results, whether they are of systematic or random nature and whether they are known or unknown.

Sometimes we speak of measurement noise $\mathbf{e}_y(t)$ (random errors) assuming that deterministic parts of the overall measurement error quantities $\mathbf{e}_y(t)$ have already been omitted.

In addition we are normally confronted with a second source of noise, the so-called process noise $\mathbf{v}(t)$ of the process P. Since this one does not deliver contributions to the measurement error quantities $\mathbf{e}_y(t)$, we need not consider it here.

One of the disadvantages of this simple and favoured model – which is moreover mathematically correct – is its apparent lack of the (ideal) measurement process M_0 (Figure 3).

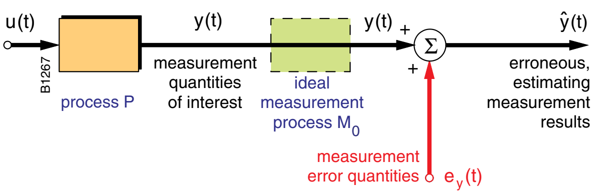


Figure 3. The apparent lack of the ideal measurement process M_0.

And still, the question remains open: Where do error quantities $\mathbf{e}_y(t)$ come from?

This conventional error model may help in simple cases, for instance if graphs of systematic errors of a non-ideal measurement process M are already available, delivered for example by a calibration process C. As soon as we take measurements with this now well-known measurement process M, we correct the non-ideal situation by subtracting known systematic error contributions from the measurement result in the qualification process Q.

But this structure is less helpful in more demanding cases, where we do not have quantitative error specifications and where we urgently need information as to where and how these error quantities emerge. Especially the influence and amount of the so-called disturbance quantities $\mathbf{v}_M(t)$ and loading quantities $\mathbf{z}_M(t)$ concerning the non-ideal measurement process M are not explicable at first sight.

Though it is clear that such a structure does not represent a physical reality, since error quantities $\mathbf{e}_y(t)$ are the *result* of non-ideal situations around and within the measurement process M, the mathematical definition of errors sanctions this structure, although admittedly only in a formal way.

2.2 Extended Error Model

Physical procedures within a measurement process M should be discussed according to the cause and effect principle. This is necessary to be able to describe its non-ideal behaviour. And we want to handle disturbance quantities imposed *on* and loading quantities resulting *from* the measurement process M.

This requires a tight integration of the process P in front of the measurement process M on the one hand and of the measurement qualification process Q behind the measurement process M on the other hand.

We start with process P, especially with the measurement quantities of interest $\mathbf{y}(t)$, if we can acquire them directly.

Although the definition of the measurement quantities of interest is of utmost importance, especially for the evaluation of measurement errors and uncertainties, that subject is beyond the scope of this paper. But note: Process quantities $\mathbf{y}(t)$ of interest need not be constant! They may change deterministically and / or randomly with time and space. By definition this utterly normal fact alone is never the cause of measurement errors and uncertainties. An ideal measurement process M_0 would map those quantities ideally.

Now, we rearrange the error equation in a way that provides an answer to the question as to where the error quantities originate:

$$\mathbf{e_y}(t) = \hat{\mathbf{y}}(t) - \mathbf{y}(t)$$

While this may look trivial from the point of view of the most commonly used error model the answer may be less obvious: The errors stem from the non-ideal measurement process M, as outlined in the following signal impact diagram (Figure 4).

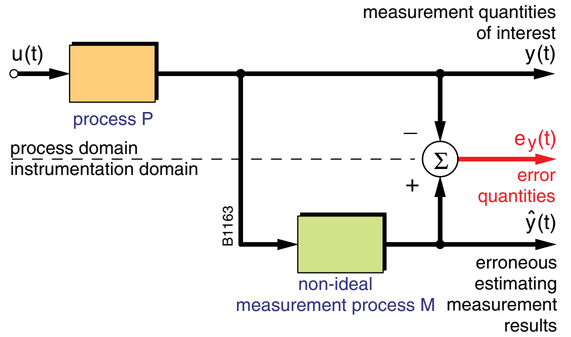


Figure 4. Extended error model [2].

Here the measurement quantities of interest **y**(t) are assumed to be physical quantities and to be well defined. But their temporal and spatial values are unknown. This is also true for the error quantities $\mathbf{e}_y$(t): Their temporal and spatial values are unknown as well, due to their dependency on the unknown measurement quantities **y**(t).

Nevertheless, the structure of this measurement model forms the basis for error analyses in any direction. We are able to discuss, to calculate, and to simulate as if all quantities were well known. And we look from the process P in the direction from cause to effect.

Practitioners tell us that the most critical region concerning the quality of measurements is the sensor process S attached to the process P. But where exactly is this sensor process S attached (Figure 5)?

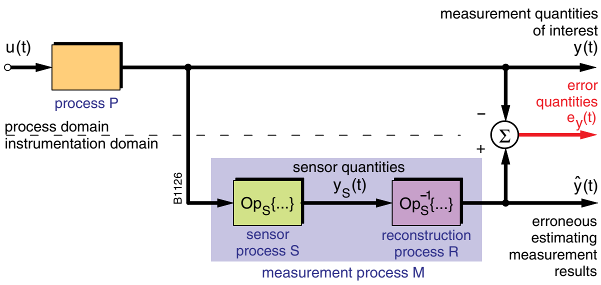


Figure 5. Structured measurement process M [2].

Each sensor in this multi-sensor net S delivers an electrical signal. But normally we are not interested in those additional physical quantities $\mathbf{y}_S$(t), since we need the estimate of the acquired quantities **y**(t) of interest. Therefore we provide a second sub-process, the reconstruction process R, which delivers the estimates of those quantities of interest **y**(t). The reconstruction process R always represents the mathematical inversion of the sensor process S. This inversion considers all possible, but unwanted cross effects among all sensors and thus corrects them. Though theoretically possible, dynamic inversion is restricted in practice to very simple cases of low order and to low signal-to-noise (SNR) situations.

That means that in principle all non-ideal phenomena – like nonlinearities – in the sensor process S can be corrected within the reconstruction process. In consequence, error quantities $\mathbf{e}_y$(t) (Figure 5) will only appear if for various reasons the reconstruction process R is not able to settle his task completely.

Very often not all quantities and parameters of interest of a process can be measured directly by sensors. But if there are causal relations between the desired, but immeasurable quantities **z**(t) and the measurable substitutes **y**(t) in the sense of cause and effect, described by models, it is possible to determine quantities and parameters of a process by so-called model-based measurement.

By extending the basic structure of Figure 5 we obtain for example the open-loop observer OLO, which estimates immeasurable quantities using the model of sub-process SP (Figure 6). It is obvious that in addition a new type of errors will appear, the observer errors $\mathbf{e}_z$(t).

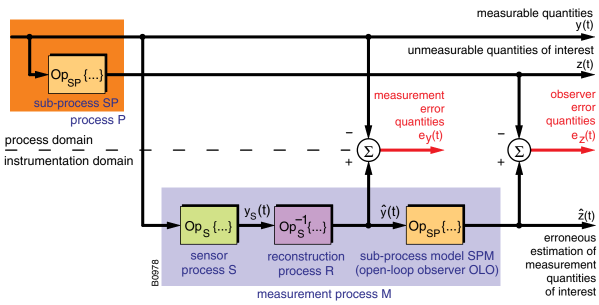


Figure 6. Model-based measurement by an open-loop observer

2.3 Comparison between Error Models

Now, where is the difference to the first, simpler error model? Both models claim to use the same error equation. But equations are unrelated to questions of cause and effect.

If we assume that a non-ideal measurement process M may be divided into a parallel connection of the ideal nominal measurement process M_0 and the deviation process ΔM which is assumed to be responsible for the error quantities, we see a new type of structure in the signal impact diagram (Figure 7).

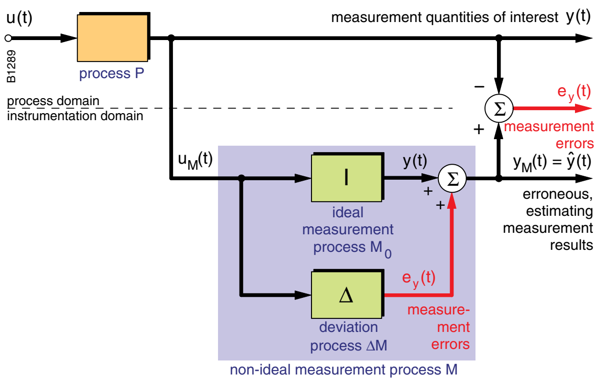


Figure 7. Ideal measurement process M_0, non-ideal deviation process ΔM and non-ideal measurement process M.

Interestingly, we find in this new structure both forms of the error equation in one diagram.

$$\mathbf{e_y}(t) = \hat{\mathbf{y}}(t) - \mathbf{y}(t)$$

and

$$\hat{\mathbf{y}}(t) = \mathbf{e_y}(t) + \mathbf{y}(t)$$

The main difference between the two concepts is that we now see where the errors come from and that we may start modelling the causes of these errors systematically.

For example, one cause might be a nonlinear static transfer response line, where the vertical differences to the ideal nominal line are the deltas (Δ) depending on the actual measurement quantity y.

Insofar no inconsistence between the two concepts has appeared.

In the following section we have to focus on details in order to be able to discuss the deviation process ΔM further. This will be done in the time domain, but it can be handled in the frequency domain likewise.

2.4 Extended Model of the Non-Ideal Measurement Process

As was shown in [2], there are only three different reasons for the non-ideal behaviour of any system:

- disturbing quantities $\mathbf{v}(t)$ affecting the system
- loading quantities $\mathbf{z}(t)$ of the system affecting the system in front of it
- non-ideal transfer response of the system

This is especially true for the measurement process M and may again be illustrated by a signal impact diagram (Figure 8).

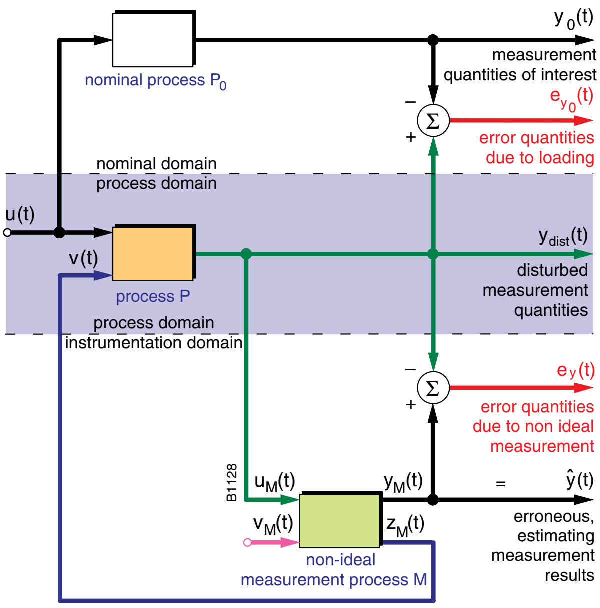


Figure 8. Two different types of error quantities

It shows a new error category, the "error quantities $\mathbf{e}_{\mathbf{y}_0}(t)$ due to loading". These quantities amount to the usual measurement errors $\mathbf{e_y}(t)$.

From this overall structure we open the multivariate non-ideal measurement process M for further inspection. To describe the transfer response behaviour in the time domain, we use the common state variable description shown in Figure 9. Nonlinear effects may be integrated in this structure.

$$\dot{\mathbf{x}}(t) = \mathbf{A}\,\mathbf{x}(t) + \mathbf{B}\,\mathbf{u}(t) + \mathbf{E}\,\mathbf{v}(t)$$
$$\mathbf{y}(t) = \mathbf{C}\,\mathbf{x}(t) + \mathbf{D}\,\mathbf{u}(t) + \mathbf{F}\,\mathbf{v}(t)$$
$$\mathbf{z}(t) = \mathbf{G}\,\mathbf{x}(t) + \mathbf{H}\,\mathbf{u}(t) + \mathbf{J}\,\mathbf{v}(t)$$

or

$$\begin{bmatrix} \dot{\mathbf{x}}(t) \\ \mathbf{y}(t) \\ \mathbf{z}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} & \mathbf{E} \\ \mathbf{C} & \mathbf{D} & \mathbf{F} \\ \mathbf{G} & \mathbf{H} & \mathbf{J} \end{bmatrix} \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \\ \mathbf{v}(t) \end{bmatrix} = \mathbf{P} \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \\ \mathbf{v}(t) \end{bmatrix}$$

where the partitioned matrix $\mathbf{P}$ is an overall transfer response matrix, serving as a short-form model of the whole measurement process M.

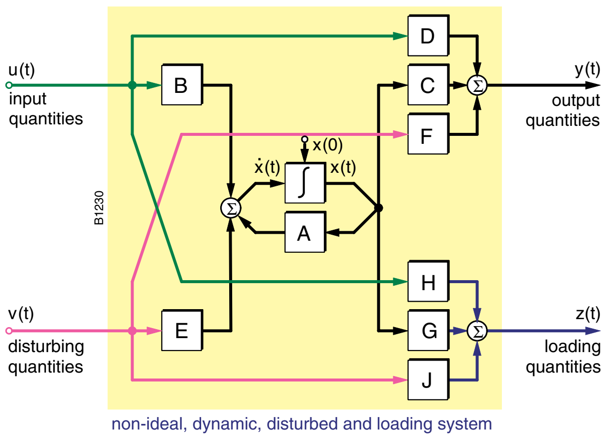


Figure 9. Common model of a non-ideal process in the time domain description

Although this is a very simple structure even for a large number of different sensors (sensor net), we have to keep in mind that the main practical difficulties lie in the determination of the correspondingly large number of matrix elements. The determination of these elements has to be done by time-consuming calibration (parameter identification).

Since we want to compare this model with the conventional error model (Figure 7), we have to redesign it in order to obtain isolated error quantities with separate error paths.

We show the isolation of the influences of the dynamic behaviour of a measurement process M by the example of a non-ideal transfer response and by the influences of the disturbing quantities $\mathbf{v}_M(t)$ including their dynamic behaviour (Figure 10). This leaves two main error quantities, namely the errors $\mathbf{e}_{dyn}(t)$ due to the dynamic behaviour and the errors $\mathbf{e}_{dist}(t)$ concerning disturbing influences. This procedure is correct at least for linear or linearised systems.

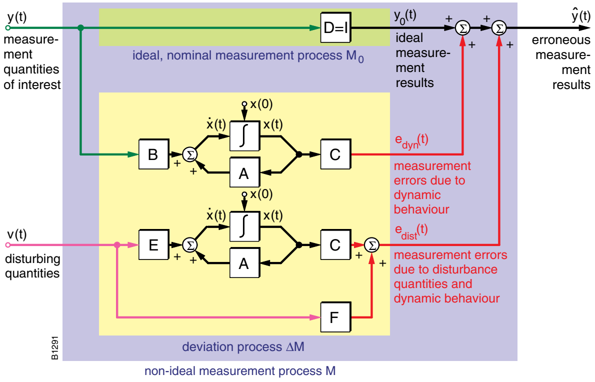


Figure 10. Example of a non-ideal measurement process M with non-ideal dynamic transfer response behaviour and disturbance quantities

The same procedure would deliver error quantities according to loading effects and parameter deviations.

Each single hardware component in a measurement chain or in a sensor array may be individually modelled by this concept.

It is worth mentioning that the structures introduced above do not only apply to deterministic signals, but to random signals as well. Since random input information cannot be described analytically by equations, we have to use instead its characteristic values and characteristic functions, such as arithmetic means, variances, distribution functions, correlations functions, spectral density functions etc. Consequently, the transfer response of a system is discussed with regard to its characteristic values and characteristic functions.

The same ideas apply to errors, error bars, uncertainties, uncertainty limits etc. which may accompany quantities at any point within the measurement chain.

In this regard the transfer response laws are identical with the laws of error or uncertainty propagation. There is only one transfer response model of processes.

3. Synthesis

After these different analysing and refining steps we return to a general view concerning the integration of a non-ideal measurement process M between process P and qualification process Q.

3.1 Process and Measurement Process

The strong interconnections between measurement process M and process P (Figure 8) establish a special series connection. If the models of both processes are given in the state space representation, the two parameter matrices $\mathbf{P}_M$ and $\mathbf{P}_P$ are "multiplied" by the so-called Redheffer-Star-Product Matrix **S** (Figure 11), delivered by Signal and System Theory and supported by adequate software-packages.

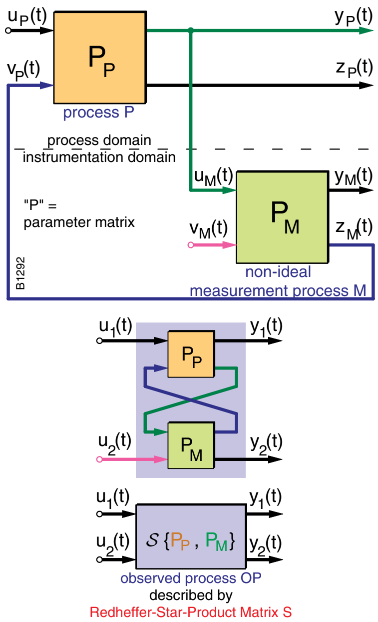


Figure 11. Redheffer-Star-Produkt for the multivariable series connection of two systems

3.2 Qualification Process

Until now, we have not discussed error correction nor uncertainties $\boldsymbol{u}_{\mathbf{y}}(t)$ of the measurement results. The qualification process Q is responsible for this important task (Figure 12).

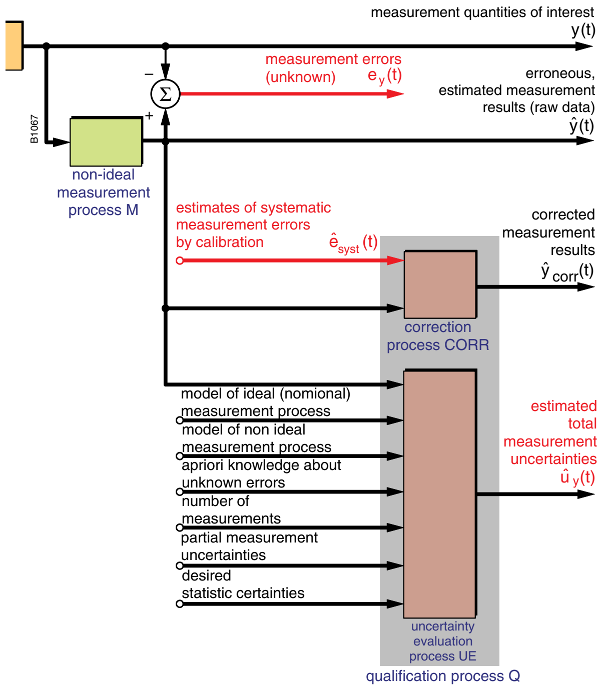


Figure 12. Qualification Process

The qualification process Q estimates the measurement errors $\mathbf{e}_{\mathbf{y}}(t)$ and the measurement uncertainties $\boldsymbol{u}_{\mathbf{y}}(t)$ of the non-ideally acquired quantities $\mathbf{y}(t)$ as well as the immeasurable quantities $\mathbf{z}(t)$ (Figure 6). The immeasurable quantities have been non-ideally estimated via models of the process P within the reconstruction process R.

The qualification process Q depends on the following information:

- target specifications $\mathbf{w}_Q(t)$ concerning the required quality of the result quantities $\mathbf{y}(t)$ and $\mathbf{z}(t)$
- models of the process P of interest
- models of the measurement quantities $\mathbf{y}(t)$ and $\mathbf{z}(t)$ of interest
- models of the non-ideal transfer behaviour of the measurement process M
- models of the non-ideal measurement environment
- models of the non-ideal measurement procedures

The qualification process Q has to be considered a system, too. Signal and System theory as well as Stochastics and Statistics describe it without restrictions.

The qualification process Q may run like any self-contained process, either continually in real-time or discontinuously as a batch process. Again, all necessary information has to be provided at the proper time as an input for signal processing.

Considering system theoretic structures the qualification process Q is an "Open Loop Observer" (OLO) with well-known laws and properties.

A discussion of the details of the qualification process Q would exceed the scope of this paper. They are treated within GUM [1] and in numerous other papers.

3.3 Physical and Information Processes

We may look at processes in terms of physical as well as information processes. Let us state this more precisely:

- Process P,
  whose quantities and parameters have to be measured, is a physical process, even if informatics tools are involved.
- Non-Ideal Measurement Process M,
  which is attached to process P, contacting or contact-free, always consists of a non-ideal sensor process S and a reconstruction process R (Figure 5).
  Due to their attachment to process P, sensors within sensor process S are inevitably physical processes, too, even if some informatics tools are involved.
  Unlike the processes mentioned above, the reconstruction process R within the measurement process M is an information process, based on models of sensor process S and process P, even if some physical structures are involved.

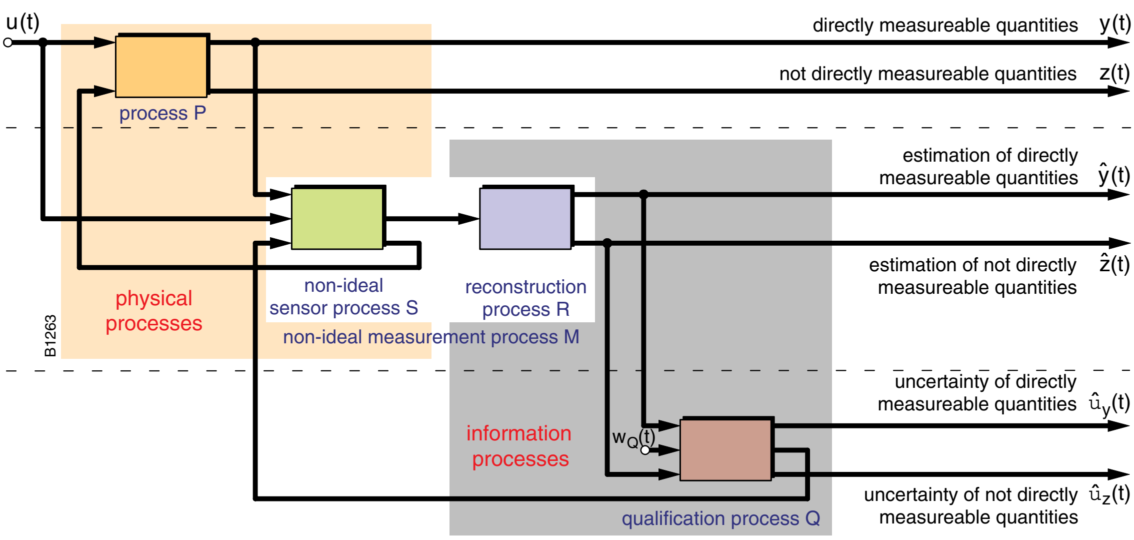
- Qualification Process Q,
  which determines the quality of the measurements according to given criteria $\mathbf{w}_Q(t)$, is an information process, even if the hardware running the process is of physical nature.

Thus we may state that a measurement procedure takes place simultaneously on two levels, on a real physical level and on a virtual information level. As shown in Figure 13 the separation line runs exactly between the sensor process S and the reconstruction process R.

Whereas the physical processes involved are considered obvious, the ideas about the information processes often remain vague. For instance: The simplest information process within the reconstruction process is a scaled gauge; it is the inverse model of the respective sensor element.

However, in certain applications, the information processes may escalate in number and become very voluminous. They are commonly called data mining processes. This is true, especially if the number of measurement quantities, of sensors, of disturbing quantities, of interdependencies, or of qualification criteria is very high.

Sometimes the information processes are called filter processes (e.g. observer, Kalman filter), because



they restore model-based measurement results of high quality out of noisy sensor information.

The information processes (reconstruction process R, qualification process Q) are based on three important information sources:

- delivered and stored signals of physical processes (real and virtual signals)
- models of physical processes
- target specifications for all member processes

As long as just the main measurement procedures and measurement quality are under consideration, no further discussions in the direction of concrete realisations of instrumental processes are needed.

4. Conclusion

Not surprisingly, the two error models are identical if we just consider the superposition of influences on the measurement results. But the extended model provides an insight necessary to understand and describe the quality of measurement procedures. The model of the measurement process and the measurement procedure is the core of any error and uncertainty analysis. But two problems arise in this respect: Models are not easily available and the identification of parameters by calibration is awesome, apart from very simple cases of everyday measurements.

Thus, Signal and System Theory as well as Statistics and Stochastics can provide powerful tools that are supported by appropriate software packages.

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