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THE METROLOGICAL SYSTEM AS A SMALL-WORLD NETWORK: SOME NOTES FOR THE ANALYSIS OF ITS STRUCTURAL EFFICIENCY

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Abstract: The metrological system is presented in its structure as a network, and as such analyzed in view of its efficiency. It is shown that the general strategies adopted at this regards in the small-world networks can be usefully exploited in this case, and some empirical solutions are derived.

Keywords: metrological system, traceability, measurement science.

1. INTRODUCTION

Measurement is aimed at expressing the information obtained on the empirical comparison between the measured object and a second object, or scale of objects, chosen as standard. The condition of intersubjectivity that characterizes measurement requires the widespread availability of such standards, so to ensure that two formally identical measurement results have the same meaning (i.e., report the same information on the comparison of different objects to the same standard) even if acquired in different times and places. This highlights the critical role, from both the epistemological and the operative point of view, of the metrological system, whose goal is to guarantee that measurement results are actually related to a given standard even in the case the empirical comparison with it has been performed only in indirect way, i.e., by comparing the object under measurement with an intermediary standard, which had been compared in its turn with the standard or, recursively, with another intermediary standard (the fact that the comparison is possibly performed with an instrument which realizes the standard instead of with the standard itself is immaterial for the present analysis). In short, measurement is based on the comparison with entities (objects, instruments, ...) which are calibrated by means of a procedure that make them traceable to a standard.

Given its globally hierarchical organization, the metrological system is mainly maintained and developed by means of inter-laboratory comparisons:

- performed "vertically" across the system, when aimed at transferring the traceability from the National Metrological Institutes, which stand at the top of the hierarchy, towards the field laboratories;
- performed "horizontally" across the system, when aimed at allowing laboratories at the same level in the hierarchy to mutually control their standards, and make them compatible with each other.

The total costs required for running this system are surely socially significant, and this justifies, in our opinion, an analysis aimed at identifying the critical factors of its efficiency and, in conclusion, at reducing such costs while maintaining the same quality of service.

2. THE METROLOGICAL SYSTEM AS A NETWORK

In this analysis, the metrological system can be interpreted in a purely topological way, as a network whose nodes are the standards or the instruments which realize them and edges are the traceability relation between the entities associated with nodes. Since, at least as a first approximation, the costs of the system can be thought of as depending only the structure of this network, we suggest that its topological analysis can lead to some useful indications on the strategies to follow for a global governance of the system.

This network is at the same time:

- *connected*, since each node is directly or indirectly linked to all other nodes, thus making the structure a single system;
- *large*, since the number n of its nodes is high; the value n expresses the pragmatical importance of the system: when the number of the mutually traceable standards or instruments increases, the importance of the whole system increases in its turn;
- *sparse*, since the number k of its edges is far less than $n(n-1)/2$, the number of edges of a complete network of n nodes (i.e., a network in which each node is connected with all other nodes); for our previous hypothesis the value k (or the correlated $\bar{k}=2k/n$, mean number of edges per node) is directly related to the total costs of the system, and therefore it should be minimized.

The basis of the metrological system is currently guaranteed by the *Mutual Recognition Arrangement* (MRA). Recognizing that "*confidence in measurements is an essential prerequisite to international trade and facilitates almost every task in the industrialized world*", the goal of the MRA is to "*extend and consolidate pre-existing worldwide confidence in measurements*" [1]. This highlights once more the importance of the global efficiency of the system, that requires the minimization of the index k for the given value of the parameter n. On the other hand, it is clear that k expresses only one component of a proper efficiency criterion for the system:

- in a network made of a single chain the value k is actually minimized, but at the same time the mean geodesic, i.e., shortest, distance d between nodes is high (the distance of a pair of nodes is defined as the minimum number of edges that must be crossed to reach one node from the other one); since each traceability relation implies a propagation of uncertainty, the index d should also be minimized;
- in a network with a single hub node, to which all other $n-1$ nodes are connected, both the values k and d are actually minimized, but at the same time the maximum degree c of nodes is high (the degree of a node is defined as the number of connected edges; in this case $c=n-1$, precisely because of the hub node); since the global efficiency of the system depends also on the operative reliability of its nodes (in terms of both fault tolerance and reachability), the index c should also be minimized.

Therefore, given the number n of the nodes in the metrological system the minimization of the indexes k, number of edges, d, mean distance between nodes, and c, maximum number of connections to a single node, leads to a global maximization of the efficiency of the system. On the other hand, there seems to be a general trade-off between such three indexes.

A great deal of attention has been recently drawn towards a family of networks having, at the same time, a low value for the three indexes, and therefore being globally efficient. They are called *small-world networks* [2]. While the prototype of such structures is the network of acquaintance between people (such that persons are associated with nodes and an edge between two nodes is present if the two persons are in relation of acquaintance with each other: despite of the high number n of nodes in this network and the relatively low value $\bar{k}$ of mean number of acquaintance relations between people, it seems that the mean distance d between any two nodes of this network is around 6), small-world networks have been recognized also, for example, in the structure of neural connections in the human brain and of Internet hosts.

We suggest that the metrological system can be significantly interpreted as a small-world network, and such analyzed in the perspective of its efficiency. Interestingly, the structure of several networks exhibiting the small-world effect is indeed close to the one of the metrological system: a weakly hierarchical structure, with some "central nodes" around whom clusters of nodes are aggregated, and some further edges, more or less randomly drawn between nodes belonging to different clusters. Given this organization for the network, the small-world effect becomes mathematically obvious: if the number of nodes within a distance r of a central node grows exponentially with r, then the value of d will increase as $\log(n)$ [3].

3. THE PROBLEM OF TRACEABILITY

In this context, peculiar to the metrological system is what we suggest to call the *structural problem of traceability*: given the fact that the metrological traceability is the "property of a measurement result relating the result to a stated metrological reference through an unbroken chain of calibrations of a measuring system or comparisons, each contributing to the stated measurement uncertainty" [4], it is crucial that the chain is actually unbroken, i.e., that the network is connected. It should be noted that the condition of being unbroken is, both conceptually and operatively, defined only on a local basis, i.e., for a single pair of nodes: the connection between two entities is unbroken if the measurement results obtained in their comparison are compatible with each other. If, for example, each measurement result is expressed, as recommended by the GUM [5], by an estimated value x for the measurand and its expanded uncertainty $U(x)$, two measurement results are compatible if $|x_1-x_2|<U(x_1-x_2)$ [6]. The mentioned problem of traceability derives then from the non-transitivity of such compatibility relation, an issue that we have analyzed in terms of a relational formalization of measurement [7]. The peculiarity with respect to the standard semantics of the small-world networks is therefore that in this case the local traceability does not guarantee the global one, a problem that must be dealt with together with the search of a global maximization of the system efficiency.

We consider that at least two general and complementary solutions are available to the problem:

- a first solution is based on the adoption of a stronger and asymmetric compatibility relation (it could be called a "transferable compatibility") for "vertical" comparisons, requiring not only the non-null intersection between the coverage intervals, but also that the "upper" interval in the traceability chain is included the "lower" one;
- a second solution suggests to perform also some "horizontal" comparisons, such as those on which the MRA is based, but performed at all the levels of the traceability chain and between nodes belonging to different clusters, with the effect of shorten the distance between the clusters themselves.

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