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MEASURING BOTTLENECK TIMES IN PRODUCTION LINES WITH DISCRETE EVENT SYSTEMS FORMALISMS

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Abstract: This paper presents a theoretic approach to measurement of the bottleneck times in production lines. The mathematical formalism utilized is characteristic for Discrete Event Systems approaches, and is about the Markov chains. Using this approach, we introduce definitions of bottlenecks, and we discuss their implications for production automation and preventative maintenance. The bottleneck of a production line is a machine that impedes the system performance in the strongest manner. The size of the buffer between the two machines is assumed to be finite. The approach is based on the sensitivity of the system production rate to machine reliability parameters.

Keywords: production systems, bottlenecks, up-time, down-/time, discrete - event systems.

1. Introduction

Production lines are sets of machines arranged so as to produce a finished product or a component of a product. Machines are typically unreliable and experience random breakdowns, which lead to unscheduled downtime and loss of production. Breakdown of a machine affects all other machines in the system, causing blockage of those upstream and starvation of those downstream. To minimise such perturbations finite buffers separate the machines. The empty space of buffers protects against blockage and the full space against starvation, separate machines. Thus, production lines may be modelled as sets of machines and buffers connected according to a certain topology. From a system/theoretic perspective, production lines are discrete/event systems. Two basic models of machines reliability are mentioned in the literature: Bernoulli [1] and Markovian [2], [3]. Bernoulli model assumes that the process of Bernoulli trials determines the status of a machine in each cycle (i.e., the time necessary to process a part). In Markovian model the state of a machine in a cycle is determined by a conditional probability, with the condition being the state of the machine in the previous cycle. Both model Bernoulli and Markovian reflect practical situations: Bernoulli reliability model is more appropriate when downtime is small and comparable with the cycle time. This is often the case in assembly operations where the downtime is due to quality problems. Markovian models reflect operations where the downtime is due to mechanical failures, which could be much longer than the cycle time. In this paper we address the Markovian model. Intuitively, bottleneck (BN) of a production line is understood as a machine that impedes the system performance in the strongest manner. Some authors define the BN as the machine with the smallest isolation production rate (i.e. the production rate of the machine when no starvation and blockages are present). Other call the BN the machine with the largest inventory accumulated in front of it. Both may not identify the machine that affects the bottom line, i.e. the system production rate because the above definitions are local in nature and do not take into account the total system properties, such as the order of the machines in the production line, capacity of the buffers, etc. Identification of BNs and their optima capacity for to avoid the machines downtime is considered as one of the most important problems in manufacturing systems.

2. System Model and Definitions

The following model of a production line is considered:
1) The system consists of N machines arranged serially and N+1 buffers separating each consecutive pair of machines.
2) Each buffer B_i is characterised by its capacity $C_i < \infty$, $2 \leq i \leq N$, the first and the last buffer are considered to be of an infinite capacity.
3) Each machine has two states: up and down. When up, the machine produces with the rate 1 part per unit of time (cycle); when the machine is down, no production take place.
4) The uptime and the downtime of each machine M_i are random variables distributed exponentially with parameters λ_j and μ_i respectively.
5) Machine M_i is starved at time t if buffer B_{i-1} is empty at time t, machine M_1 is never starved.
6) Machine M_i is blocked at time t if buffer B_{i-1} is full at time t, machine M_N is never blocked.

The isolation production rate of each machine (i.e. the average number of parts produced per unit time if no starvation or blockage takes place) is:

$$\eta_i = \frac{Tup_i}{Tup_i + Tdown_i} = \frac{1}{1+\frac{\mu_i}{\lambda_i}} \qquad (1)$$

Machine M_i is the uptime bottleneck if:

$$\frac{\partial \eta}{\partial Tup_i} \rangle \frac{\partial \eta}{\partial Tup_j}, j \neq i \qquad (2)$$

and is the downtime bottleneck if:

$$\left|\frac{\partial \eta}{\partial Tdown_i}\right| \rangle \left|\frac{\partial \eta}{\partial Tdown_j}\right|, j \neq \mathrm{i} \qquad (3)$$

Machine M_i is the bottleneck (BN) if is both uptime bottleneck and downtime bottleneck.
Let M_i be the bottleneck machine. Then it is referred to as the uptime preventive maintenance bottleneck if :

$$\frac{\partial \eta}{\partial Tup_i} \rangle \left|\frac{\partial \eta}{\partial Tdown_i}\right| \qquad (4)$$

If the inequality is reversed, the bottleneck is referred to as the downtime preventive maintenance bottleneck.

Notice: a) The absolute values of $\frac{\partial \eta}{\partial Tdown_i}$ is used because otherwise this number is negative: increase in *Tdown* leads to a decrease of η.

b) In some instances, the downtime of a machine is due to lapses in the performances of manual operators, rather than machine breakdown, the identification of downtime bottlenecks provides guidance for development of production automation.

c) Preventive maintenance, as part of the total production maintenance, leads to both an increase uptime and a decrease of downtime of automated machines. Some of the preventive maintenance measures affect more the uptime and the others the downtime. We refer to them as uptime preventive maintenance and downtime preventive maintenance. Thus, the classification of the bottleneck in either uptime bottleneck or downtime bottleneck has an impact on planning actions that leads to the most efficient system improvement.

3. Bottleneck Indicators

We are seeking bottlenecks identification tools that are based on either the data available on the factory floor through real time measurements (such as average up - and down - time, starvation and blockage time, etc.), or on the data that can be constructively using the machines and buffers parameters (λ_i, μ_i, N_i). We refer to these tools as bottleneck indicators.

3.1. *A single machine case*

A single machine defined by assumptions made in the second paragraph is uptime bottleneck if $Tup < Tdown$ and it is downtime bottleneck if
$Tdown < Tup$.

Proff: Follows immediately from (1) since:

$$\frac{\partial \eta}{\partial Tdown} = \frac{Tup}{(Tup + Tdown)^2} \qquad (5)$$

and

$$\frac{\partial \eta}{\partial Tup} = \frac{Tdown}{(Tup + Tdown)^2} \qquad (6)$$

We may say that the smallest average uptime or down-time of a machine defines its nature as bottleneck: if $Tdown < Tup$, the primary attention of the preventive maintenance and automation should be given to the further decrease of the downtime; if $Tup < Tdown$, the attention should be concentrated on the increase of the uptime. Since in most practical situations $Tdown < Tup$, the above indicator, states that reduction of the downtime is more efficient than a comparable increase of the uptime.

3.2. *Two machine case*

It is well known that, given a constant ratio between Tup_i and $Tdown_i$, the machine with the longer up - and down - time is more detrimental to the system's production rate than with a shorter up - and down - time.
In view of this property, one might think that the bottleneck is the machine with the longer up - and down - time. This is not true. The reason is that an improvement of the machine with a shorter up - and down - time leads to a better utilization of the disturbance attenuation capabilities of the buffer than a comparable improvement of the machine with a longer up - and down - time. Therefore, an improvement of the "better" machine is the best for the system as a whole.
In a production line with two machines of equal efficiency (i.e., $\frac{Tup_1}{Tdown_1} = \frac{Tup_2}{Tdown_2}$), the machine with the smaller downtime is the bottleneck [4]. If the downtime of this machine is smaller than its uptime, preventive maintenance and automation should be directed toward the decrease of the downtime. If the downtime is sufficiently longer than the uptime, preventive maintenance and automation should be directed toward the increase of the uptime.
In the most practical situations, the isolation production rate of the machines (i.e., the faction $Tup/(Tup+Tdown)$ is greater than 0,5. Therefore, the most usual bottleneck is the downtime bottleneck. To identify the downtime bottleneck in the case of machine with unequal efficiency (i.e. $\frac{Tup_1}{Tdown_1}$

$\neq \frac{Tup_2}{Tdown_2}$) in [6] is given the following bottleneck indicator:

If $mb_1 Tup_1 Tdown_1 < ms_2 Tup_2 Tdown_2$, machine M_1 is the downtime bottleneck.

If $mb_1 Tup_1 Tdown_1 > ms_2 Tup_2 Tdown_2$, machine M_2 is the downtime bottleneck.

The probability of manufacturing blockage mb_i is defined as:

mb_i = Prob ({M_i is up at time t} $\cap$ {B_i is full at time t} $\cap$ {M_{i+1} fails to take parts from B_i at time t}).

The probability of manufacturing starvation ms_i is defined as:

ms_i = Prob ({M_{i-1} fails to put parts into B_{i-1} at time t} $\cap$ {B_{i-1} is empty at time t} $\cap$ {M_i is up at time t}).

4. Extreme status for buffers

In the sequel we'll try to determine the bottleneck behavior of the machines as a function of their efficiency correlated with buffer size. We'll also try to anticipate the events like buffers full or empty, which determine the bottlenecks. We consider a segment consisting of two machines M_i and M_{i+1} with intermediate storage B_i at any time between successive events. Let TA be the apparent time of an event occurrence at B_i. This event may occur or not if, in the mean time, another canceling event takes place.

Let P_i be the number of parts, which are scheduled in process by M_i until the occurrence of the event. We examine two different situations, which result in a buffer event.

We define the following:

pr_i The nominal production rate of machine M_i, $i = 1,...,N$

$BL(j,t)$ Level of buffer B_j, $j = 2, ..., N-1$

$T_{1j}(t)$ Delay time until the next arrival to B_j

$T_{2j}(t)$ Delay time until the next departure from B_j

BC_j The capacity of buffer B_j, $j = 2, ..., N$

4.1. Buffer - full event

Although the buffer B_i has enough space to accept the parts produced by M_i during the transient time T_{2i}, since M_i produces at a faster rate than M_{i+1} (or the delay time T_{2i} is too long), buffer B_i will fill (see Fig.1).

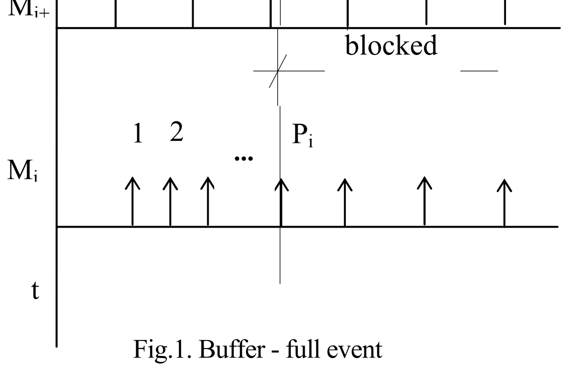

Fig.1. Buffer - full event

In Fig.1. the continuous line represents a machine operation on a work-part and the arrows represent arrivals to the succeeding buffer. Blank intervals indicate idle periods due to blockage or starvation of machines. The function, from Fig.1. is encountered when:

$$(pr_i > pr_{i+1}) \cap [pr_i (T_{2i} - T_{1i}) > BC_i - BL(i)] \quad (7)$$

The buffer - full event will occur when the P_i-th part leaves from M_i. The number of parts produced by M_i after t + T_{1i} is P_i -1. From Fig.1. the sequel relations hold:

$$P_i - P_{i+1} = BC_i - BL(i) \quad (8)$$

$$P_i = 1 + (TA - t - T_{1i}) \cdot pr_i \quad (9)$$

$$P_{i+1} = (TA - t - T_{2i} + T'_{2i}) \cdot pr_{i+1},\ i = 1,...,N \quad (10)$$

Time interval between a departure and the end of processing of the first blocked part of M_i, lies in an inter-departure interval of M_{i+1}:

$$\frac{1}{pr_i} \leq T'_{2i} \leq \frac{1}{pr_{i+1}} \quad (11)$$

From (8) - (11) we obtain:

$$(T_{1i} - T_{2i}) + \frac{BC_i - BL(i)}{pr_{i+1}} < P_i \left(\frac{1}{pr_{i+1}} - \frac{1}{pr_i}\right) \leq (T_{1i} - T_{2i}) + \frac{BC_i - BL(i)}{pr_{i+1}} + \frac{1}{pr_{i+1}} - \frac{1}{pr_i} \quad (12)$$

$$P_i = 1 + int\{[BC_i - BL(i) + pr_{i+1}(T_{1i} - T_{2i})] \cdot \frac{pr_i}{pr_i - pr_{i+1}}\} \quad (13)$$

4.2. Buffer-empty event

This event is dual to the blockage and analogous results will be derived. The buffer-empty event is encountered when buffer B_i is exhausted and its succeeding machine M_{i+1} has just transmitted a work-part downstream.

Although the buffer B_i has enough parts for the transient period T_{1i}, because machine M_{i+1} produces faster than M_i (see Fig.2.), or the delay time T_{1i} is too long, finally B_i becomes empty.

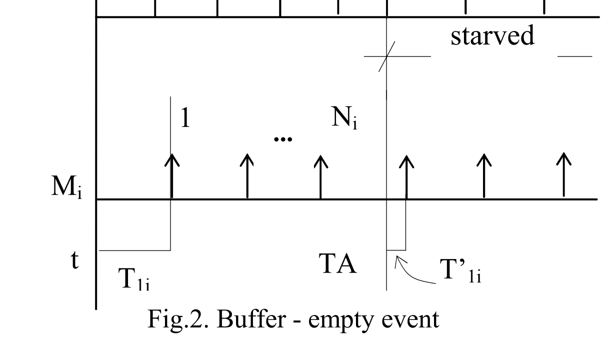

Fig.2. Buffer - empty event

The function of Fig.2. is encountered when:

$$(pr_i < pr_{i+1}) \cap [pr_{i+1}(T_{i1} - T_{2i}) > BL(i)] \quad (14)$$

The inter-departure interval of M_{i+1} just before occurrence of the empty buffer event satisfies:

$$0 \le T'_{i1} \le \frac{1}{pr_i} - \frac{1}{pr_{i+1}} \quad (15)$$

$$P_{i+1} - N_i = BL(i) + 1 \quad (16)$$

$$P_{i+1} = 1 + (TA - t - T_{2i}) \cdot pr_{i+1} \quad (17)$$

$$P_i = (TA - t - T_{1i} + T'_{1i}) \cdot pr_i \quad (18)$$

Analogous results with these of section 3.1 are obtained:

$$P_{i+1} = 1 + int\{[BL(i) + pr_i(T_{2i} - T_{1i})] \cdot \frac{pr_{i+1}}{pr_{i+1} - pr_i}\} \quad (19)$$

5. Conclusions

Identification and measurement of the bottleneck times in production lines has implications for both the natures of preventive maintenance and production automation. In this paper we address the Markovian model of production lines with bottlenecks. In lines where machines have identical efficiency the machine with the smaller downtime is the bottleneck. In two-machine lines, the downtime bottleneck is the machine with the smallest value of $p \cdot Tup \cdot Tdown$, where p is the probability of blockage for the first machine and the probability of starvation for the second. Anticipation of events like buffer full or buffer empty, which determine the bottlenecks, has also implications for the preventive maintenance. Future work in this area should focus on extensions of the results obtained in systems with high failure rates.

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