XVIII IMEKO WORLD CONGRESS
Metrology for a Sustainable Development
September, 17 – 22, 2006, Rio de Janeiro, Brazil

ON THE MEASURAND DEFINITION

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Among the sources of measurement uncertainty, the GUM [ISO 1995] lists the incomplete definition of the measurand. Because of its relevance to the very concept of measurement uncertainty, a short analysis of *the problem of the definition of the measurand* is worth.

As a simple example of defining a measurand, let us consider the "diameter" of a bore, as it is presented by [Phillips et al. 2001]: "The simple definition as a diameter may be sufficient for a low accuracy application, but in a high accuracy situation imperfections from a perfectly circular workpiece may be significant". According to a radically operational definition, this sentence is fallacious: if the measurand is defined by the operation by which it is measured, the experimental evidence of different "diameter values" obtained for different positions on the same bore would lead to the conclusion that the bore has several diameters, not that the diameter is incompletely defined. On the other hand, the concept of "having several diameters" is admittedly inconsistent with the geometrical meaning of the term, to which the mentioned "simple definition" implicitly refers: on the other hand, diameters, as geometrically defined, cannot be physical properties, for the obvious reason that in the physical world no "perfect circles" exist at all. Radical operationalism is however seldom maintained: properties are a constitutive component of our knowledge, and we tend to assign them stable, and therefore transferable, meanings, to which any single operation only partially contributes. *It is this dependence to a model that can make a measurand definition incomplete*: in the example, if the common, geometrical, concept of diameter is maintained, then "in a high accuracy situation imperfections from a perfectly circular workpiece may be significant", and such imperfections typically lead to uncertainties in the diameter measurement. As the observation accuracy grows, the fact that diameter is not a single-valued quantity must be recognized as depending on the discrete structure of the matter, not on manufacturing imperfections anymore: in this situation, the remaining uncertainty is therefore intrinsic to the measurand definition (it is interesting that the VIM [ISO 1993], which in its first and second editions defines the measurand as "the particular quantity subject to measurement", in the (currently draft) third edition is switching to "the quantity intended to be measured", thus emphasizing the fact that what is declared to be the property under measurement could differ from the actually measured property. Incidentally, it can be noted that the VIM does not define the concept of measurand definition at all).

On the other hand, it is precisely the dependence on a model (instead of ontological roots) which allows the alternative option of defining measurands on ad hoc bases. For example, again [Phillips et al. 2001] notes that "some standards (...) have further defined the diameter of a bore to be the maximum inscribed diameter" (or, more plausibly, the maximum distance between points on the edge of the bore, to avoid defining a concept in terms of itself). But this conventionality can result in arbitrariness: why not to define the diameter as the average distance between opposite points? or as the difference between the maximum and the minimum of such distances? If properties are methods to associate information entities to objects, as I have sustained elsewhere, it is possible to arbitrarily define always new properties. *Conventionality in the definition of properties avoids arbitrariness if grounded on pragmatic bases*. For example, in a system constituted of a piston and a cylinder, the internal "diameter" of the cylinder could be defined as the minimum inscribed distance and the external "diameter" of the piston as the maximum inscribed distance. Were the internal "diameter" of a cylinder *c* ascertained to be greater than the external "diameter" of a piston *p*, the passage of *p* through *c* could be inferred:

given v_1 = internal_diameter(*c*) and
v_2 = external_diameter(*p*), then:
if $v_1 > v_2$ then passage(*p*,*c*)

being passage(*p*,*c*) related to a fact which can be experimentally established, thus leading to confirm, or to confute, the inference.

This implication has basically the same form as, for example:

given v_1 = applied_force(*x*) and
v_2 = mass(*x*), then:
acceleration(*x*) = v_1 / v_2

a rather lengthy expression of the known physical principle commonly written $F=ma$: the two inferences differ because of the implied algebraic structure on the sets of the values of the involved properties, i.e, because of the scale type in which such properties are evaluated: while the property "passage" only depends on an ordinal comparison and is in itself boolean, the property "acceleration" is defined in a ratio scale type. Both cases express a law stating a relation among properties that in principle are defined independently of each other (in [Mari 1999] I have analyzed the information conveyed by this relation: I called it "information-from-connection" and I suggested its pragmatic nature). These relations contribute to a complex concept of definition of properties, according to which each property is partially defined by means of other properties, in a network of mutual connections which expresses the available knowledge on such properties and therefore limits the conventionality of their definition [Mari 2005].

The network connecting the measurand to other properties guarantees the pragmatic meaningfulness of the measurand evaluation but, at the same time, is a further source of complexity for the definition of the measurand itself. Indeed, part of this network are the so called *influence quantities*, i.e., according to the definition of the VIM, those properties of the object under measurement or the environment (thus including the measuring system) that are not the measurand and that nevertheless affect the measurement result. As a simple example, it is known that metallic bodies expand as their temperature increases, so that the variations of the temperature of a metallic rod modify its length: if length is the measurand then, in this case, temperature is an influence quantity. The experimental evidence that repeated measurements on the same object produce different results because of variations of its temperature can be modeled according to two strategies:

- temperature is maintained as a “hidden variable” in the definition of the measurand, whose intrinsic uncertainty should be increased correspondingly to keep into account this under-determination;
- temperature is explicitly included in the definition of the measurand, which is then denoted for example as “length at 20 °C”.

The greater specificity of this second strategy offers the potential condition of obtaining measurand values of lower uncertainty, and therefore of higher quality, but requires the measurement of two properties, length and temperature, together with a control / correction system (which can be empirical or symbolic) for dealing with the situations in which the measured temperature is different from the specified one. Moreover, in this case temperature becomes a measurand in its turn, with the consequence that its value could depend on further influence quantities, for which the problem of choosing a strategy for dealing with the influence quantities is iterated. Once more, this shows the model dependence of the measurand definition.

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