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MEASUREMENT *PLUS* OBSERVATION – A MODERN STRUCTURE OF METROLOGY

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Abstract – Diverse fields of Metrology reveal manifold perceptions concerning the meaning of the classical procedure «measurement» and of the trendy procedure «observation». No common consensus has become visible so far. The following systematic investigations of metrological tasks and structures develop and propose a self-evident definition of both procedures and the resulting analytical relationship. This is feasible by modelling and visualising dynamic subsystems by means of Signal and System Theory. In particular, one can consistently demonstrate and justify the complementary tasks of measurement *and* observation. Thus, we gain an extended and modern version of basic metrological structures.

Keywords: measurement, observation, observer, simulation, extended measurement, metrological structures

1. INTRODUCTION

Measurement is not the only source of quantitative perception in *Metrology*. We are familiar with other, but strongly related principles like observing, monitoring, testing, inspecting, calibrating, surveying, acquiring, collecting, diagnosing, simulating, estimating, predicting, human sensing, and so on.

The following presentation considers the two most notable terms, *measurement* and *observation*, our gates to the world of of information. Their mutual relationship will be the main issue. See *Symbols* and *Terminology* in the Appendix.

Metrology is an autonomous discipline of enormous importance. Few people are aware of this fact, as Metrology serves most discreetly, but effectively, in the background. Moreover, it is nearly impossible to consistently sketch and analyse its supporting influence on all the other disciplines like for example science, technology and sociology. Metrology uses instruments (Figure 1) to, hopefully, acquire objective, quantitative and accurate information about quantities of the real world.

Figure 1. Measurement Process for Blood Pressure (dreamstime)

Observation is not as manifest and tangible, though everybody knows, what observing does and what its aim is: an active acquisition of any type of information. That´s what the famous newspaper *Observer* does, that´s, what is done in hospitals, traffic junctions, climate networks, police activities, politics, and so on. Every living being, human, animal and plant, observes while utilising its senses.

Figure 2. Observation Process for Geostationary Satellites (Japan Meteorological Agency (JMA))

Should we distinguish between an *everyday observation procedure* on the one hand and a *scientific observation procedure* on the other hand? Does observation mean measurement without instruments, the results thus being subjective, qualitative and inaccurate? Is the prime observer a human being? Is the well-known *observer influence* typical in particle physics and effective in observer procedures only?

We cannot answer these questions by a simple *yes* or *no*. For example, we know of many observation procedures, which deliver objective, quantitative and accurate observation results (Figure 2). If they were inaccurate after all, we would at least be able to discuss observer errors and uncertainties scientifically. Two outstanding representatives of this observer category are the *Luenberger observer* and the *Kalman observer*, also called *Kalman Filter*.

To get better acquainted with our promising endeavour, we will compare the characteristics of measurement and observation procedures more in detail. Obviously, it will not be possible to combine all aspects we get to know from everyday life in one common framework. Most contributions of the widely spread field have to be disregarded. In consequence, a consistent definition for the sought after dual network *measurement and observation* may run against our feeling for language und may encounter some reserve from different directions.

Since we foster Metrology, particularly committed to the acquisition of objective, quantitative and accurate information, we brush aside all definitions, which base on subjective, qualitative and inaccurate observation results. This precondition in fact establishes a boundary line, but, as often with compromises, this will help a lot. Of course, we do not question the importance of subjective, qualitative and inaccurate everyday observation procedures.

One could assume that at least Metrology is well defined; however, this is not the case. Philosophers and scientists have been trying to pin down the essentials of Metrology for centuries. Seldom, engineers get involved in such discussions, a questionable fact and a serious deficiency indeed. Actually, a global definition within a larger context should be feasible and could provide a general orientation concerning apparent structures of seemingly complicated relationships.

Those recurrent remarks affect the two terms *measurement* and *observation* alike. The meaning of the term *measurement* is grossly and globally clear. However, what does the old and dignified term *observation* mean? Is there a relation between measurement and observation? Do we have a synonymic pair, one term scientific and one term colloquial? What are common and what are self-contained tasks and properties? What is a *measurement equation* and what is an *observation equation*? In Metrology, we experience an ever-increasing use of the term observation; is it a buzzword?

Wikipedia provides some general aspects concerning observation [8]. *The International Vocabulary of Basic and General Terms in Metrology* (VIM) [1] does not recognise the term *observation* and its mutations at all. On the other hand, the *Open Geospatial Consortium* (OGC), discusses together with ISO in its Standard *Geographic Information – Observations and Measurements* both terms [9]. But, the distinction between the two terms remains in this Standard still rather arbitrary and hardly convincing. Obviously, a concept for a concise common consensus would be most welcome. Moreover, a compromise seems unavoidable.

Browsing across disciplines, we find another field concerning observation, the *Signal and System Theory*, which is deemed to be the *Extended Theory of the Cause and Effect Principle*. Here, we come across concepts like *observer* and *observable canonical form*, with well-defined topological structures. Or, we come across quantitative properties like *observability*, with a precise mathematical definition concerning the realisability of observation procedures. Or, we come across implications like *observer error*, with a concise definition for quantitative observation performance. Again the question, what is observation in comparison to measurement, what do the not-yet-defined commonalities, complements and distinctions look like? Still a challenge to tackle!

The following sections will investigate and comment on such diverse questions, which can be encountered in rather confusing statements to be found anywhere: Structural perspectives with emphasised relational concepts seem to be the only promising framework. *Signal and System Theory* is the proper and reliable tool for this purpose. At the end, an interesting result will show up: We have to deal with measurement *plus* observation in most cases, and not with measurement *or* observation. Therefore, we state that *Metrology as Measurement Plus Observation*, can be denoted as *Extended Metrological Procedure* (EMP). Certainly, this claim has to be proved and to be backed up by convincing examples.

Additionally, a clear distinction between *measurement errors and uncertainties* and *observation errors and uncertainties* will result.

Before starting the central investigation, we establish some assumptions and conditions (Section 2) in order to enable general definitions. Then, a short recapitulation of the main structure of the *measurement procedures* is advisable (Section 3): Special consideration will focus on the so-called *reconstruction process*, in which the important *inference procedure* from sensor result quantities onto the quantities of interest (measurands) takes place. This exemplifies the significance of the already mentioned *measurement equation*, which always bases on the results of empirical and / or analytical *calibration procedures* (traceability issue). Under favourable conditions, this measurement model (equation) reveals the structure of measurement errors and uncertainties.

In parallel, we have to focus on the structure of *observation procedures* in science for, hopefully, objective, quantitative and exact results (Section 4). The proposition states that the observation procedures are tight-fitting extensions of the regular *measurement procedure*. Again, such a definition necessarily narrows the everyday meaning of the term *observation*. However, a main conclusion will be that, in addition to structural conditions (*observability*), observation in science cannot exist without any conventional measurement procedure. On the contrary, a measurement procedure *alone* will be viable, as soon as quantities to be measured are properly defined and technologically acquirable (*measurability*).

The proposal will present four main concepts of observer procedures, which are performed by corresponding *observer processes*, or in short, by *observers*. Their evidence will be justified by structural relations.

Note that Metrology *describes* quantities of the real world. In diverse fields, *quantities* are information, properties, parameters, attributes, facts, states, phenomena, events, patterns, features, amounts, magnitudes, intervals, sequences, trajectories, transitions, triggers, errors, uncertainties, performance indices, and so on.

However, Metrology does not *explain*, *reason*, *comment*, *interpret*, *evaluate*, *validate*, *diagnose*, *justify* or *predict* anything. Such procedures belong to quite a different field of tasks, though following measurement and observation procedures directly. They base on given and accepted measurement result quantities, but need and use additional information, like references, demands, specifications, bounds, scopes, interests, and so on.

The following investigations concerning measurement and observation do not account for technological and instrumental principles. This by no means disregards *real-world aspects*, but rather concentrates on basic and essential concepts within the *abstract field* of the metrological world.

A few words concerning the two historical, but rather weak terms *direct measurement* and *indirect measurement* [1; 9]. They will be mentioned later on, since they allude directly to our topic. In short, direct measurement is measurement of *intended quantities* by sensors directly. Indirect measurement is measurement of *proxy quantities* by sensors and determination of finally intended quantities by *observation procedures* (observer). Hopefully, we will be able to do without these terms in the future.

2. ASSUMPTIONS AND CONDITIONS

To proceed in this endeavour, we will assume particular distinctions, assumptions and definitions. In plus, to restrict ourselves to as few terms as possible, we will omit some popular terms of everyday usage and redefine others.

Although we cover general principles of Metrology here, we assume that *quantities of interest* in the real world, whether measurable and / or observable or not, are defined and specified properly according to well-established codes of practice. This includes that these quantities are traceable to respective reference quantities.

Without loss of generality, we presume time and space to be continuous quantities.

We assume ideal procedures in the first place, knowing that errors and uncertainties will appear in the real and abstract world. We know that deviations from ideal postulations require an error and uncertainty analysis. They are mentioned, but not dealt with here.

In mathematical modelling procedures, we consider *relations between signals*, which are models of the real-world quantities of interest. Therefore, relations between signals are always abstract relations and we define them as *systems*, and thus as simplified and reduced models of *real-world, dynamic processes* [10] (item, device, artefact, article, product, processor, object, subject, individual, target, body, matter, asset, constituent, element, particle, field, organisation, reality, cosmos, and so on).

In order to enable an interdisciplinary understanding right from the beginning on, we assume multivariate (multiple input, multiple output; MIMO), linear, time invariant (LTI) dynamic systems. Therefore, all defined quantities and parameters are multivariable too, described by signal vectors. Consequently, *Linear Algebra* is the tool to choose for any quantitative description, solution and simulation.

Dealing with *dynamic systems* evokes discussions on time dependent (varying) *states* of such systems and their justification. Here again a general tool is provided by the versatile State Space Description (SSD) of linear dynamic systems.

That we assume *linear* systems throughout, is a constraint indeed. However, basic concepts and structures remain the same for nonlinear dynamic systems. Just the mathematical solutions become awkward.

Admittedly, these assumptions suggest a rather high complexity. This may be true for the final design of large systems. However, the enormous advantage is that the underlying structures always remain the same, and that they are rather simple and universally applicable.

Thus, understanding and comprehension of these basic principles, once retrieved, can be used in situations that are more complicated: Any following project can be handled the same way.

We will avoid an exclusive use of *mathematical models* in this paper, which describe the structures of measurement and observation procedures. They are necessary in the first place for a final description and concise definition of the Extended Metrology Concept. However, since *structures* are of primary interest here, *graphical representations* are much more convenient. They will dominate the paper as *Signal Relation Diagrams* (SRD).

Finally, powerful software is available for all requirements mentioned until now and later on. These software tools are easily adaptable to the model structures concerned.

3. MEASUREMENT CONCEPT

The normal measurement process M always consists of a *sensor process* S and a *reconstruction process* R (Figure 3). It is actively interconnected with the *process* P of interest.

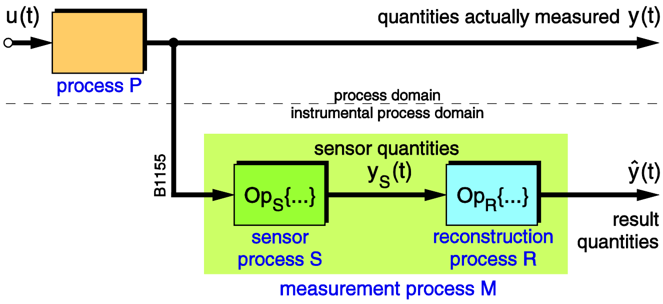

Figure 3. Signal Relation Diagram:
Structure of the Ideal Measurement Process M

Though the term *reconstruction* will play its role together with observation procedures too, we do not comment on this classification here. At present, we consider the *measurement process* M as a whole and we postulate its ideal model, which we call *nominal model* MN. It obeys the simple, but extremely important *Fundamental Axiom of Metrology*, which states that the models of the *resulting quantities* $\hat{\mathbf{y}}(t)$ of a measurement process M have to equal numerically the models of the *measurement quantities* $\mathbf{y}(t)$ of interest at any time and at any location: $\hat{\mathbf{y}}(t) = \mathbf{I} \times \mathbf{y}(t)$. This represents the ideal *measurement equation*.

We readjust this relation so that it includes possible measurement errors $\mathbf{e}_y(t)$, which are zero for the fictitious ideal measurement process MN: $\hat{\mathbf{y}}(t) - \mathbf{y}(t) = \mathbf{e}_y(t) = \mathbf{o}$ and are unequal zero in the *error equation* of the *nonideal measurement process* M (Figure 4).

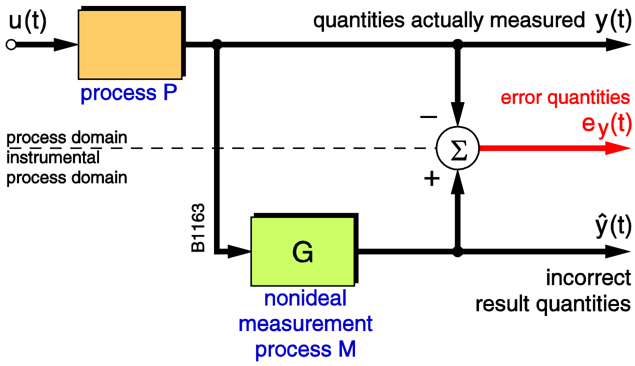

Figure 4. Signal Relation Diagram:
Structure of the Nonideal Measurement Process M

For every measurement process M, the reconstruction process R has to be designed so it can fulfil the requirements of the Fundamental Axiom of Metrology. Thus, systematic errors $\mathbf{e}_y(t)$ in the sensor process S can be compensated. This remains true, when an observation process O is added.

4. OBSERVATION CONCEPT

Proposition: In many metrological applications, an *observation process* O extends the *ordinary measurement process* M and its capability (Figure 5):

Extended Measurement Process *=*
Measurement Process plus ***Observation Process***

Observation processes O offer effective and efficient metrological methods for information extraction from real-world processes, which are not accessible for ordinary sensor processes S. In a first approach, results are estimates of the immeasurable, time dependent internal state quantities $\mathbf{x}(t)$ of process P (state reconstruction). Awkwardly enough, such observation processes are often called «Soft Sensors» or «Virtual Sensors».

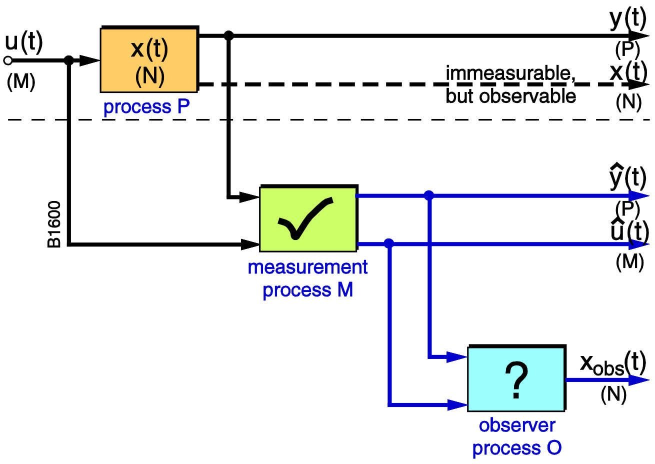

Figure 5. Signal Relation Diagram:
Structure of the Extended Measurement Process with
Process P, Measurement Process M, Observation Process O

Unlike the measurement process M, with its interconnecting and interacting sensor process S, the observer process O is a *data processor* without any direct interconnection and interaction with the real world. It uses available *sensoric data* and performs *real-time operations* according to given models or sub-models of the actual process P. So, the sought-after information about process P is available synchronously.

It does not matter how this processor is realised, whether as an analogue electronic circuit in very simple situations, or as a Digital Signal Processor (DSP), integrated within an arbitrarily complex computer.

In the following analysis of *observation procedures*, we will be able to establish the corresponding *observer equations* for the *observation process* O.

Why is this method almost unknown in measurement practice? There are three main reasons:

- On the one hand, many observer structures have been successfully realised in practice for a long time, without having been recognised or described as such.
- On the other hand, prerequisites for observer solutions may be demanding insofar, as process models are absolutely vital. How complete and how exact they have to be, depends on the respective demands and tasks.
- Although some observer processes can be realised by analogue electronic circuits in a simple way, the "grown-up" measurement device "observation process" O needs the computer, on which it can be implemented. This means that observers were of no practical importance before the arrival of recent computer facilities.

4.1. History of the Observation Process

The history of the scientific observation procedure is quite young although we know of early appearances. As recently as 1963, D. G. Luenberger published a report on the observation of *variable states* in linear systems [2; 3; 4]. Thereby, he continued the distinguished concept of State Space Description (SSD) of dynamic systems set up by R. E. Kalman [5]. Up to now, *observation theory* has confined itself mainly to the time domain. Applications in the frequency domain occasionally appear too [6; 7].

Gradually, several observer structures have appeared. Four standard structures, associated with the basic idea, are presented here for the first time:

- Simulating Observer SO
- Open-Loop Observer OLO
- Reconstructing Observer RO
- Closed-Loop Observer CLO

They depend on the availability of *sensoric results*, are designed in a simple way, and readily lead to suitable results.

There are several substructures of these four observers, which are more complex, but serve additional practical needs:

- Least Squares Observer LSO (Kalman Filter KO)
- Minimal Order Observer MOO
- Unknown Input Observer UIO
- Nonlinear Observer NLO

Without exception, they ask for a deeper understanding of theoretical prerequisites. Their design is demanding in most cases [6]. They are not considered here.

Finally, the development of observation processes has led to *optimal filters*. For example, the Kalman Observer KO [5] has exactly the same structure as the *closed-loop observer* CLO. Optimisation criteria specify the free tunable *observer parameters*, which depend on the properties of all the random quantities involved. However, the free choice of such parameters makes the *design* of this so-called optimal closed-loop observer or optimal filter an artistic skill.

4.2. Concept of an Observation Process

Under normal circumstances, we hope that the input quantities $\mathbf{u}(t)$ and / or the output quantities $\mathbf{y}(t)$ of a multivariable, dynamic process P should be measurable. However, this is often not the case for several reasons. Additionally, state (inner) quantities **x**(t) of a system *cannot be measured per definition*, which is a theoretically useful curiosity of the State Space Description (SSD). Therefore, specified, immeasurable input quantities and output quantities, as well as specified state quantities, *have to be observed*, if we depend on them.

The idea of an observation process O suggests that we *calculate* by *estimating* properly defined, immeasurable quantities of interest. On the one hand, we use *sensoric results*, which are at our disposal, delivered by measurement. On the other hand, we invoke *modelling results* about essential *interrelations to measurable quantities*, which at first sight may seem of no concern or may be useless. In other words, we develop an *extended model* of the dynamic process concerned, preferably, a model of the State Space Description type (SSD). Often, this top-down axiomatic approach of an extended model can be simplified in special situations by a *model reduction procedure*. Such a *sub-model* will enable for example the so-called reduced order or *minimal order observer process* (MOO).

Actually, this indicates that we must hold the *structure* of a process model in the designing phase of the observer. In the following implementing phase we additionally need the decisive numerical *parameters*.

In continuation, we have to respect a famous criterion: The *observability* of a system has to be fulfilled. Any model can be tested in this regard by mathematical means.

There are further mathematically defined constraints, which imply special properties of a process model, like *measurability*, *invertibility*, *controllability*, and so on [12].

Since we discuss observation procedures, we assume, without loss of generality, ideal, error free *measurement procedures* with the *measurement equations* $\hat{\mathbf{u}}(t) = \mathbf{I} \cdot \mathbf{u}(t)$ and $\hat{\mathbf{y}}(t) = \mathbf{I} \cdot \mathbf{y}(t)$ respectively. And we signify the multivariate measurement processes MU and MY graphically by tiny quadratic blocks (Figure 6).

In accordance with the estimates of measurement results, we call the results of the observation procedure *observer estimates* and apply the index "obs", e.g. $\mathbf{x}_{obs}(t)$.

Under such circumstances, the observation process O delivers estimates $\mathbf{x}_{obs}(t)$ of time dependent state quantities **x**(t), and sometimes of other, immeasurable quantities **z**(t).

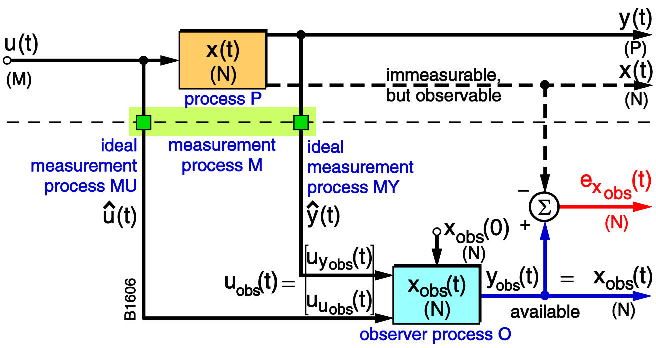

Figure 6. Signal Relation Diagram:
Nonideal Observation Procedure with Ideal
Measurement Processes MU and MY and Observer Process O

4.3 Errors and Uncertainties

The performance of any *observation procedure* is not ideal and we have to define, analyse and declare the *observation error vector* $\mathbf{e}_{\mathbf{x}_{obs}}(t)$ (Figure 6). It is an aggregation of diverse single errors. They arise due to deviations between input quantities of the process and the observer, due to deviations between initial values of the state quantities, due to model errors, and due to numerical errors.

Under certain favourable prerequisites of the time dependent estimation procedure, components of the observation error approach zero with time t asymptotically. For the characterisation of these errors, all common methods of *error theory* apply. Systematic and random errors as well as time dependent and time independent errors are to be expected.

Remark: Although the observation error vector $\mathbf{e}_{\mathbf{x}_{obs}}(t)$ is well defined, we cannot determine it experimentally, since the quantity vector to be observed **x**(t), will always remain unknown, as is the case with any measurement procedure. Nevertheless, this error model is extremely important, since we are able to investigate and discuss the design and behaviour of the observation process theoretically.

Uncertainties primarily stem from uncertainties of the numerical values of the model parameters.

Every observation process can be calibrated by usual procedures with error and uncertainty evaluation. But since the observer is based on models or sub-models of the process of interest, model errors can only be evaluated by a *parallel calibration* with the original process P.

5. THE FOUR STRUCTURES OF METROLOGICAL OBSERVATION

The following sections introduce the structures of the four basic observation processes and discuss prerequisites concerning their optimal behaviour. *Signal relation diagrams* (SRD) visualise the basic *structures of observation*. Preferably this is done by graphical tools rather than by corresponding *mathematical equations*, though these are not too complex. Yet, they are presented in a reduced manner to enable a basic overview. They are just cited, but neither derived nor explained.

We have already considered the general situation concerning an observation procedure in Figure 5. The question arises, what the intended model of the observer process should look like. There are two principal concepts:

First, the overall structure of the *observer equation* will be the same as for any other multivariable dynamic process, described by the well-known State Space Description (SSD):

$$\dot{\mathbf{x}}(t) = \mathbf{A}\,\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$$
$$\mathbf{x}(0) = \mathbf{x}_0$$
$$\mathbf{y}(t) = \mathbf{C}\,\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t)$$

Second, the internal structure of the *observer equation* and the *parameter matrices* **A** to **D** have to be chosen so that the desired transfer function of the respective observer can fulfil its prescribed task properly.

For example, a sophisticated arrangement of the elements in the matrices leads to the so-called *observable canonical form* of the set of equations with convenient structural properties.

The four observers to be presented will differ concerning the composition of the input signals $\mathbf{u}_O(t)$ and of the output signals $\mathbf{y}_O(t)$. This means that we consider, analyse and define properties of the *input structure* and *output structure* of the respective observer. They become typical features concerning their classification.

The output signals are the estimates of the state signal vector $\mathbf{x}(t)$, sometimes of the output signal vector $\mathbf{y}(t)$ and of the immeasurable output signal vector $\mathbf{z}(t)$.

Note that the mentioned description of multivariable dynamic systems by the State Space Description (SSD) always includes the description and structures of *dynamic systems in equilibrium (static) state* and of *nondynamic systems* as special cases. This fact is very important and convenient for the four types of observer processes to be defined next.

Our thought experiments each assume a natural or man-made arrangement (process P), whose model PM is given by a set of differential and arithmetic equations in State Space Description (SSD).

5.1. Simulating Observation Process

The first observer type is the dynamic *simulating observation process* SO (Figure 7). If there should be no measurement process M available at all, we will have to do without any sensoric data about process P as an observer input.

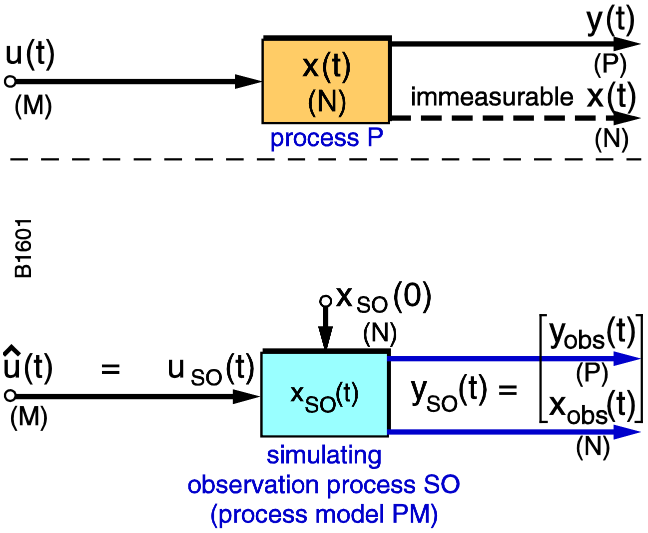

Figure 7. Signal Relation Diagram:
Simulating Observation Process SO

In this situation, it is obvious that the model of this type of observer has to be the exact *process model* PM, which we get from an elaborate modelling procedure. First of all the model represents the process, and hence, the emerging *simulation results* from the model represent the *behaviour* of the process under the same circumstances.

The set of *observation equations* is based on the standard State Space Description (SSD).

In principle, the input quantities $\mathbf{u}_{SO}(t) = \hat{\mathbf{u}}(t)$ should be the exact models of the input quantities $\mathbf{u}(t)$ of the process to be simulated. They are produced artificially by some Function Generator FG according to specifications and tasks $\mathbf{y}_{FG}(t)$. Then they are fed to the observer: $\mathbf{y}_{FG}(t) = \hat{\mathbf{u}}(t) = \mathbf{u}_{SO}(t)$.

Here, the extended output quantity vector $\mathbf{y}_{SO}(t)$ consists of two sub-vectors, the estimated output quantity vector $\mathbf{y}_{obs}(t)$ and the estimated state quantity vector $\mathbf{x}_{obs}(t)$. This defines the definite *observer equation*:

$$\dot{\mathbf{x}}_{SO}(t) = \mathbf{A}\,\mathbf{x}_{SO}(t) + \mathbf{B}\,\hat{\mathbf{u}}(t)$$
$$\mathbf{x}_{SO}(0) = \mathbf{x}_{SO_0}$$
$$\mathbf{y}_{SO}(t) = \begin{bmatrix} \mathbf{y}_{obs}(t) \\ \mathbf{x}_{obs}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{C} & \mathbf{D} \\ \mathbf{I} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{SO}(t) \\ \hat{\mathbf{u}}(t) \end{bmatrix}$$

Thereby, compared to the standard State Space Description (SSD), the extended observer equation shows a second, very simple output equation for the estimated state quantity vector $\mathbf{x}_{obs}(t)$.

There are a lot of examples in practice for nondynamic systems, where the differential equations in the State Space Description (SSD) disappear and only the nondynamic (algebraic) *feedthrough equations* remain: $\mathbf{y}_O(t) = \mathbf{D}\,\mathbf{u}_O(t)$.

We call the presented procedure *offline simulation*. It may run in real time, but generally, this is unusual and unnecessary.

Simulation is a widely used and extremely helpful tool in many fields. It provides understanding and knowledge about the operation and functionality of natural and man-made processes [15]. Normally, simulations are less elaborate and costly than experiments with real-world processes. This becomes apparent, when we observe procedures, which simulate destructive experiments. Simulating procedures can be repeated almost endlessly.

As we all know, many simulating procedures serve as *educational* tools in diverse fields. Especially important are simulators as *training* facilities with trainees in the loop.

If the quantities to be observed are space and not only time dependent, we need a multidimensional model. This requirement is the base of the Finite Element Method (FEM), which develops multidimensional results, often assisted by coloured visualisation (Figure 8).

Figure 8. Multidimensional Estimation by a Simulating Observation Process SO: Temperature Distribution on a Brake Disc. (SOLIDWORKS)

Moreover, if the input quantities are random quantities, the simulation process is called a *Monte Carlo Simulation Process* MCSP. The Function Generator FG will deliver random quantities, each with defined probabilistic properties, especially with defined *probability density functions* (pdf). Normally, the process model PM is deterministic. The simulation results will be random quantities. However, in addition, we need their statistical characteristics (mean values, variance values, probability density functions, correlation functions, spectral power density functions and so on). Hence, a main task of a Monte Carlo Simulation Process is the determination of the *relations* between the statistical *input* characteristic values and functions and the statistical *output* characteristic values and functions.

This procedure is not an *analytical simulation procedure* by means of Stochastics, but an *empirical simulation procedure* by means of Statistics. Therefore, it requires a huge amount of processing time and of computer memory, to deliver reliable results. The theoretically infinite limits of *expected values* and *expected functions* will never be reached in practice.

5.2. Open-Loop Observation Process

The next observer type is the dynamic *open-loop observation process* OLO (Figure 9). Only the input quantity vector $\mathbf{u}(t)$ of the process P is acquired by the measurement process MU. We assume ideal measurement. The output quantity vector $\mathbf{y}(t)$ and the state quantity vector $\mathbf{x}(t)$ are immeasurable.

The result $\hat{\mathbf{u}}(t) = \mathbf{u}_{OLO}(t)$ of the measurement is transferred as input quantity vector to the process model PM, which now works parallel and synchronous to process P.

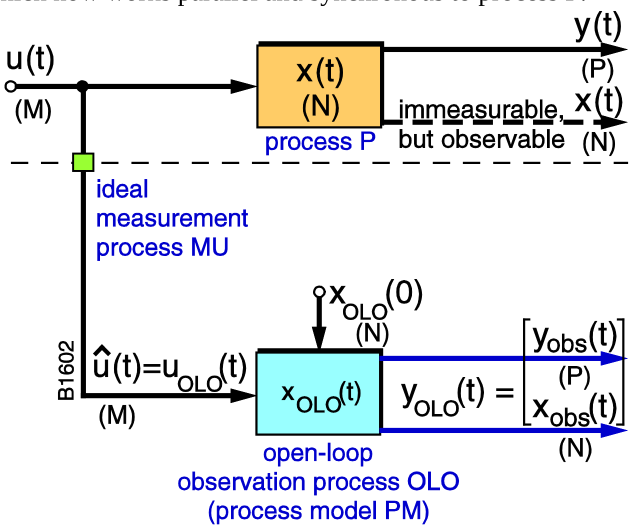

Figure 9. Signal Relation Diagram:
Open-Loop Observation Process OLO

The two sub-vectors of interest, the estimated output quantity vector $\mathbf{y}_{obs}(t)$ as well as the estimated state quantity vector $\mathbf{x}_{obs}(t)$ constitute the extended output quantity vector $\mathbf{y}_{OLO}(t)$ of the observation process.

$$\dot{\mathbf{x}}_{OLO}(t) = \mathbf{A}\,\mathbf{x}_{OLO}(t) + \mathbf{B}\,\hat{\mathbf{u}}(t)$$

$$\mathbf{x}_{OLO}(0) = \mathbf{x}_{OLO_0}$$

$$\mathbf{y}_{OLO}(t) = \begin{bmatrix} \mathbf{y}_{obs}(t) \\ \mathbf{x}_{obs}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{C} & \mathbf{D} \\ \mathbf{I} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{OLO}(t) \\ \hat{\mathbf{u}}(t) \end{bmatrix}$$

Therefore, compared to the standard State Space Description (SSD), the extended observer equation shows a second, very simple output equation for the estimated state quantity vector $\mathbf{x}_{obs}(t)$.

There are a lot of examples in practice for nondynamic systems, where the differential equations in the State Space Description (SSD) disappear and only the nondynamic (algebraic) *feedthrough equations* remain: $\mathbf{y}_{obs}(t) = \mathbf{D}\ \hat{\mathbf{u}}(t)$ (see the following example and Figure 10).

Since at least some input quantities $\mathbf{u}_{OLO}(t) = \hat{\mathbf{u}}(t) = \mathbf{u}(t)$ are at disposal, the open-loop observer process OLO is not a simulating observation process type.

Sometimes, this type of observation procedure is considered as a *reconstruction in parallel connection*.

Some difficulties seem obvious: Since the initial values $\mathbf{x}_{OLO}(0)$ of the inner process quantities $\mathbf{x}_{OLO}(t)$ at instant time t = 0 are unknown, we have to provide the best possible, but still incorrect initial value vector $\mathbf{x}_{OLO}(0)$. Obviously the model will provide erroneous results $\mathbf{x}_{obs}(t)$ and therefore time dependent systematic observation errors $\mathbf{e}_{\mathbf{x}}(t)$. It can be shown that for an *asymptotic stable process* P the influence of the incorrect initial values will have disappeared by time t [11] during the initial operation:

$$\mathbf{e}_{\mathbf{x}}(\infty) = \lim_{t\to\infty} \mathbf{e}_{\mathbf{x}}(t) = \mathbf{o}$$

Then, the *ideal* open-loop observation process will be synchronised with the process from that time on. It will never again deliver wrong estimates! Such an open-loop observation process is called an *asymptotic open-loop observer*.

Dynamic observers in equilibrium (static) state and *non-dynamic observers* have nothing to do with initial values.

The speed (rate) of the estimates approaching the steady-state condition depends only on the dynamic behaviour of the process P. It is not controllable by an operator within the observer anyhow. This may be considered a disadvantage of the open-loop observer. However, there is an advantage too: We do not get stability problems with this structure.

The open-loop observation process is used in everyday practice. This procedure is frequently called *indirect measurement*, because intended quantities cannot be measured directly by sensors, but only *model-based* by an observer.

Many procedures of this type can be found everywhere. Nobody talks about observers though. A well-known example may illustrate this situation:

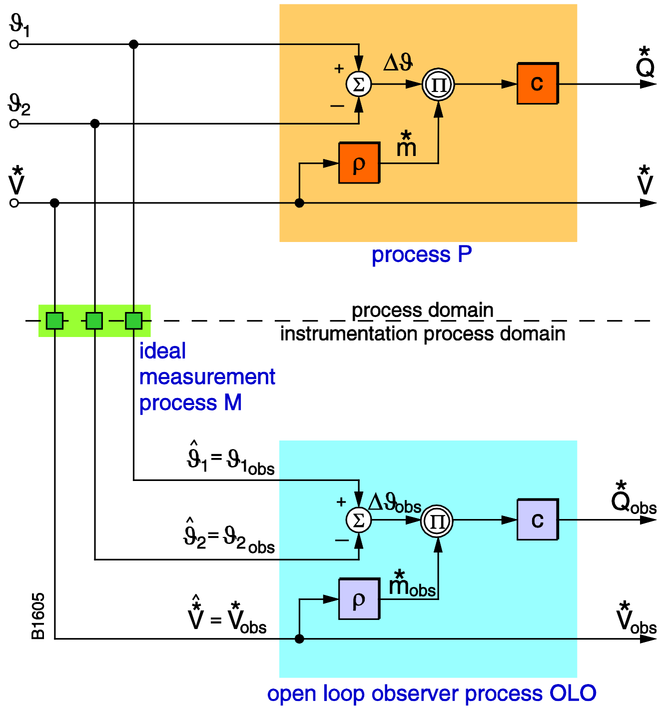

Figure 10. Signal Relation Diagram:
Open-Loop Observer OLO for the Determination of the
Heat Consumption within a Process

Nowadays, the energy flow to factories, plants, machines, buildings, or flats is determined by an open-loop observation process, because the quantity heat-flow $\overset{*}{Q}$ cannot be measured directly, as there are no *heat flow sensor processes* available. So instead, we measure the following input quantities to the heat exchanging process by ideal sensors: volume flow $\overset{*}{V}$ to the process and two temperatures, ϑ_1 at the input and ϑ_2 at the output of the process. The parallel process model PM (open-loop observer OLO) of the heat exchange process will estimate the overall heat consumption of the process by interrelating the models of the measured quantities according to the process model. (Figure 10): $\overset{*}{Q} = \rho c (\vartheta_1 - \vartheta_2) \overset{*}{V}$.

5.3. Reconstructing Observation Process

If *output quantities* $\hat{\mathbf{y}}(t)$ are determined by means of measurement procedures, we arrive at a new situation. While we have been trying to estimate the state (inner) quantities $\mathbf{x}(t)$ and the output quantities $\mathbf{y}(t)$ by looking forward, we now try to get information about all or certain state (inner) quantities $\mathbf{x}(t)$ quasi looking backwards in a suitable model by means of inferring procedures. We call this procedure a *reconstructing observation process* RO (Figure 11).

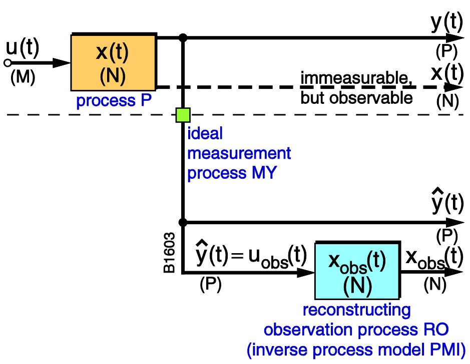

Figure 11. Signal Relation Diagram:
Reconstructing Observation Process RO

Again, the model of process P will play its role. But now its *nondynamic output equation* $f_y\{\mathbf{x}(t), \mathbf{u}(t)\}$ in the State Space Description (SSD) is requested and will be prominent. Looking backwards asks necessarily for inverting, inferring, reconstructing, retrospecting. Therefore, this type of observation procedure is sometimes considered a *reconstruction in series connection* after the process P.

Depending on the structure of process model PM and on the state quantities to be observed, the reconstructing observation process RO has to be the mathematical *inverse sub-model* PMI of process P.

And here, constraints come in. The main problem is the *invertibility*, also called *observability* of the model, a well-known task in Signal and System Theory.

The most important precondition says that structures to be inverted must have an equal *number of output quantities as* there are *input quantities*. This means that matrix $\mathbf{C}$ in the output equation has to be quadratic, in order to enable an inversion procedure and to get hold of the state quantities $\mathbf{x}(t)$ of process P: $\mathbf{x}_{obs}(t) = \mathbf{C}^{-1}(\hat{\mathbf{y}} - \mathbf{D}\,\hat{\mathbf{u}}(t))$. As soon as the input has a direct link to the output, the input quantity $\mathbf{u}(t)$ has to be measured, too.

Figure 12 presents two versions of a reconstruction trade-off, which is found frequently in practice. Note the symmetries in the structure of the process and observation processes. They belong to recurrent incidents in Signal and System Theory and in Mathematics respectively.

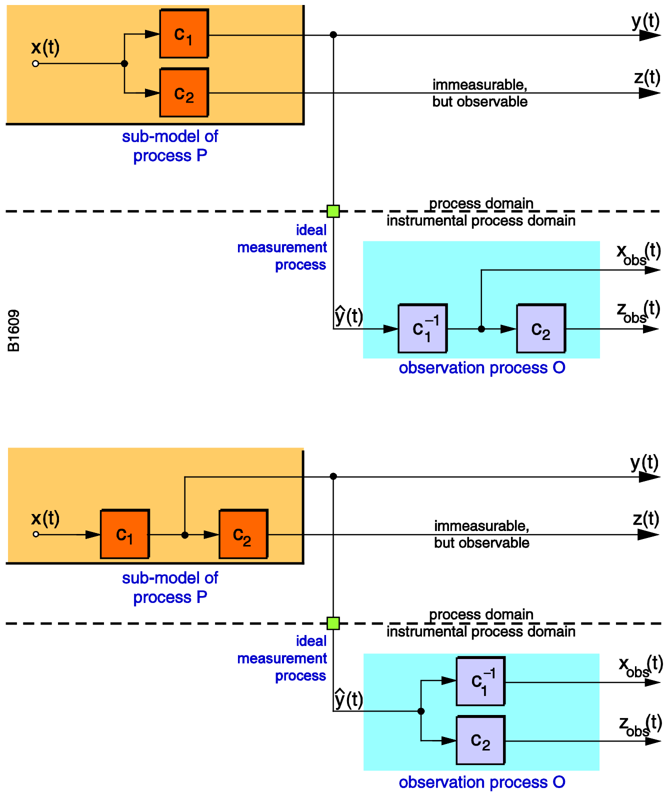

Figure12: Signal Relation Diagram:
Reconstructing Observation Process RO of two particular structures concerning immeasurable quantities

Many reconstruction tasks fail because of the tight condition of invertibility. Normally the number of state quantities $\mathbf{x}(t)$ is higher than the number of the output quantities $\mathbf{y}(t)$. Of course, there are special constellations and detours, like independent or uncoupled signal paths, which allow direct access to certain state quantities.

A nice example from the ancient world: Eratosthenes estimated the terrestrial circumference at about 200 B.C. From our present-day understanding of structures and perception, he faced a nondynamic reconstructing observation problem. He considered local properties and relations (geometrical quantities and rules) and combined them in quite a simple model (triangulation) concerning his "process". He measured different quantities like *time, arc lengths* and *angles* respectively in an appropriate setting. He inverted his model to infer on the immeasurable quantity of interest, on the circumference of the Earth. The result proved astonishingly accurate. Of course, this is a modern description from an up-to-date point of view.

5.4. Closed-Loop Observation Process

The most capable observation process, the *closed-loop observation process* CLO (Figure 13), measures the input quantities $\mathbf{u}(t)$ *and* output quantities $\mathbf{y}(t)$. All previous structures provide distinct elements. We now compare the *measured* output quantities $\hat{\mathbf{y}}(t)$ (set point) with the *observed* output quantities $\mathbf{y}_{obs}(t)$. If there should be deviations $\mathbf{e}_{y_{obs}}(t)$ between them, which we call *output error signals*, we would assume that the model PM of the process P is incorrect and that we should correct it. As a measure for such a correction, we would use these output error signals, weigh them by the *observation controller matrix* **L** (control law) and use the results as inputs to the process model PM via the input summator (balance point). This procedure adds a *feedback path* to the observation structure, so that the open-loop observation process OLO becomes a *closed-loop observation process* CLO. System Theory handles this task in detail.

The main topic of this observer is the state reconstruction, the measurement of the inner quantities $\mathbf{x}(t)$.

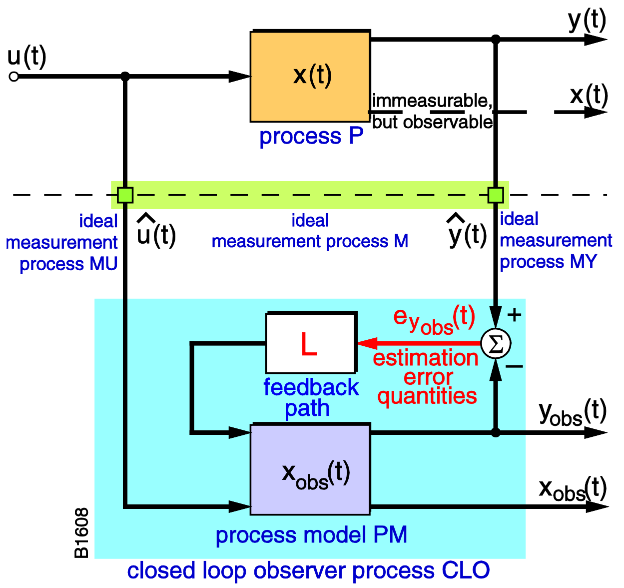

Figure 13. Signal Relation Diagram:
Closed-Loop Observation Process CLO

Behaviour and *stability* of this observation process have to be evaluated. Precondition of this concept is *controllability* as well as *observability*. Both criteria depend on the process model PM and can be checked and proved analytically.

The control law has to be designed by means of the free parameters (static transfer values) $\ell_{i,j}$ of the matrix **L**, in order to guarantee a quick disappearance.

From Control Theory we know that this procedure cannot be arbitrarily fast without endangering the *stability* of the loop. The main task of observer theory is the design and synthesis of an optimal control law for a trade-off between estimation error, robustness against nonideal circumstances and speed and general performance: The user must specify the criteria of optimality himself. Such an observer will be a new dynamic system, whose behaviour has to be judged by the usual criteria of System Theory.

The problem of unknown initial values still exists, defused however by the disappearance of the initial errors with time t, if the observer is asymptotically stable.

After reaching steady state condition, the observer will head for the intended trajectory $\mathbf{x}_{obs}(t)$, the output of process and observer will be equal and the error signal $\mathbf{e}_{y_{obs}}(t)$ will disappear.

Non acquired variables $\mathbf{v}(t)$, which disturb process P, cause dynamic errors between process and observer. If the disturbance disappears, the error will disappear too.

Observability and controllability of the process P are prerequisites for the application of an observer. This indicates that all information about the inner quantities $\mathbf{x}(t)$ will be included in some form in the output quantities $\mathbf{y}_{obs}(t)$ too and that we may influence all inner quantities $\mathbf{x}(t)$ via the balance point (summator) by the observer controller OC and its observer controller matrix **L** respectively. Since we must know the system anyway, observability can easily be tested. Suitable structures enable the determination of (N) inner quantities $\mathbf{x}(t)$ by one single output quantity y(t) only, if observability is guaranteed. This statement is at least surprising and not intuitive.

The principal benefit of the closed-loop observer stems from the fact that direct inversions are avoidable and that the output matrix **C** can adopt almost any arbitrary form. Moreover, we need fewer sensors to determine all quantities of interest. This was not possible with reconstruction processes in series and parallel connection, since the output matrix **C** had to be inverted.

5.5. Mixed Observation process Structures

It is obvious that there exist mixtures and / or combinations of these four basic structures. This is true especially for nondynamic systems, which are easy to handle, because only algebraic and transcendental equations of the Sate Space Description (SSD) remain. However, one should look for the simplest structures possible, which lead efficiently to the unknown quantities. Keep in mind that many solutions, calculated and / or estimated, are accessible by proper use of the three basic relations, series, parallel and circuit connection. Normally, any of them will deliver a solution.

Nonlinear models may be handled in the same way, with the same structures. However, mathematical difficulties may prevent efficient solutions.

The elegant solution of the closed-loop observer offers the following possibilities:

- in general: additional and improved information about the process of interest
- acquisition of delicate quantities and quantities, which are not measurable otherwise
- completion of incomplete measurements
- supervision of sensors (fault detection)
- creation and assignment of redundant information for diagnostic and failure purposes

The limits of observer applications are due to our limited knowledge about processes and signals involved. There are some areas, mechanics, electricity, propulsion technology for example, where we may count on detailed knowledge and where models are simple. However, there are many fields, where the quantitative knowledge is either rudimentary or the models are extremely complex, like in applied process engineering, aerodynamics, turbulence analysis, bio-technology, medicine, economy, astronomy.

Here too, the important question is, whether we are better off with inaccurate information than with none at all. Of course, the response depends on the respective process and the demands concerning the information of interest.

6. OBSERVABILITY

Controllability and *observability* are two basic terms in System Theory and in measurement practice. Originally, the control engineering community has developed the *observability* topic from current applicational needs.

If we want to determine internal quantities of a dynamic process, the property *observability* is the important prerequisite. It is a structural and parametric property, which we gain by an analysis of the process model.

We even solve an additional task concerning the term *observability*: How many *sensor processes* do we need to determine defined internal variables of the process and at which positions within the process structure do we locate them? The inversion of the question is equally interesting: May we omit certain sensor processes without losing the property of *observability* of a system.

It is obvious that success concerning the observability of our model cannot be guaranteed, since some internal state quantities x(t) may have no direct or indirect influence on the intended measurable output quantity y(t). So we need criteria for the property *observability*, which will consider certain aspects of the inner model *structure* and its *parameters*.

Qualitative definition: A dynamic system is observable, if its state quantities $\mathbf{x}(t)$ can uniquely be determined for all times $t > 0$ by the measurement of the input quantities $\mathbf{u}(t)$ and the output quantities $\mathbf{y}(t)$. Quantitative definition: A linear dynamic system is observable, if the *observability matrix* $\mathbf{Q}_{obs}$ of the process model is regular, that means, has full rank (N), or has (N) independent rows respectively [12; 13; 14]:

$$\mathrm{rank}\{\mathbf{Q}_{obs}\} = (N)$$

with

$$\mathbf{Q}_{obs} = \begin{bmatrix} \mathbf{CA}^0 \\ \mathbf{CA}^1 \\ \vdots \\ \mathbf{CA}^{n-1} \\ \vdots \\ \mathbf{CA}^{N-2} \\ \mathbf{CA}^{N-1} \end{bmatrix}$$

Note that this matrix contains only parameters of the state section and the output section of the process model. The input section does not have to be considered.

In this version of definition the property *observability* is a binary criterion: *system observable* / *system not observable*.

If a system is not observable, this can be changed by an appropriate choice of measurement quantities and / or of additional sensors [16]. Sometimes the selection of alternative sensor locations may help.

Surprisingly, many systems of high order are observable by means of a single output signal y(t).

There is a dual version of the property *observability* in the measurement area, namely the term *controllability* in the control area. All statements in one area have a quantitative equivalent in the other area. Tools, developed in one area can be used in the other area too. So the metrology community benefits from the vast stock of tools of the control community.

7. CONCLUSION

MEASUREMENT PLUS OBSERVATION

First: A *measurement procedure* is *acquisition* and *disposal* of properly defined quantities using sensory interaction. We know of further terms for this type of activity. Some of them "sell" fairly well as catchwords:

- intelligent measurement
- smart measurement
- model-based measurement
- calculating measurement
- test measurement
- direct / indirect measurement
- logic-based measurement (expert system)
- learning measurement (neural network)
- robust measurement
- fuzzy measurement
- diagnosis measurement

All act on the same scientific basis, each with tiny applicational and individual endorsements.

Second: The terms *observation* and *observing* rarely appear in a systematic metrological context. More familiar are procedures like *reconstruction*, *state estimation*, *simulation*, *indirect measurement*, *filtering*, *calculating*, and so on. At least, they all are justified and summarized by the term *model-based measurement*. However, on closer inspection, as done in this paper, concerning the structural properties of those procedures, they are *observation processes* indeed.

It has been shown that the absence of an own sensory contact with the real world is a shared characteristic of all observing activities. However, and this is important, observation procedures *do* intrinsically depend on ongoing measurement results, gained and provided by some measurement procedures, including human sensoric activities.

Four standard types of observation structures have been defined. They all are tightly related via the State Space Description (SSD). The most versatile observer is the closed-loop observer (CLO), which is the initial source of pursuing extended structures, conventionally called filters.

Third: Both processes, *measurement process* M and *observation process* O, are readily linked in several ways. This happens within the *Extended Measurement Process* ME, which finally provides information of interest about the process to be investigated.

Thus terms like *to observe*, *observation*, *observer* and so on, get an unambiguous meaning in the scientific field of Metrology. This concept should be transferred to other fields too.

Caution: Models, which are used extensively in measurement and observation procedures are not exact, due to many reasons. These deficiencies appear as model errors and model uncertainties. This issue has not been handled yet.

APPENDIX

GLOSSARY OF TERMS

A selection of important terms, used in this presentation:

- Metrology – Measurement Science and Technology
- Fundamental Axiom of Metrology
- process / system
- quantity / signal
- dynamic / nondynamic system
- deterministic / probabilistic (random) signal
- Signal and System Theory SISY
- Signal Relation Diagram SRD
- State Space Description SSD
- Finite Element Method (FEM)
- Function Generator FG for stimulating quantities
- to measure, measurand, measurement, measurement procedure, measurement process M
- to observe, observation, observation procedure, observer, observation process O
- extended measurement process ME = measurement process M *plus* observation process O
- sensing, acquisition, mapping /sensor process S
- reconstructing, inversion, inferring, reconstruction process R
- intended quantity, measurand
- resulting quantity, measurement result
- simulation, simulation procedure, simulation process
- Monte Carlo Simulation Process MCSP
- model, sub-model, modelling
- model-based measurement / model-based observation
- calibration, identification
- measurement error, observation error, model error
- mathematical model / empirical model
- model structure, model parameters
- nominal model MN
- measurement equation / observation equation
- set of equations: system description
- solution of the set of equations: system behaviour
- observability, observability criterion
- measurement error, observation error, model error

LIST OF INDICES

P	process
PM	process model
R	reconstruction process
M	measurement process
S	sensor process
O	observation process
SO	simulating observation process
RO	reconstructing observation process
OLO	open-loop observation process
CLO	closed-loop observation process
obs	vector of observed quantities

LIST OF SYMBOLS

A selection of important symbols, which constitute a systematic and coherent framework:

$\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$, $\mathbf{x}$, $\mathbf{y}$, $\mathbf{z}$	general signal vectors of a process model PM concerning *input*, *disturbance*, *reference*, *state*, *output*, *immeasurable output*
$\mathbf{u}(t)$	vector of input quantities of measurement processes
$\hat{\mathbf{u}}(t)$	resulting input quantities by measurement
$\mathbf{u}_{obs}(t)$	vector of input quantities of observer processes
$\mathbf{x}(t)$	vector of state (inner) quantities
$\mathbf{x}_{obs}(t)$	vector of state (inner) quantities of the process model PM and intended result by observer process
$\mathbf{x}_{obs}(0)$	vector of initial values concerning the state quantities $\mathbf{x}_{obs}(t)$ of the process model PM
$\mathbf{y}(t)$	vector of output quantities of interest
$\mathbf{y}_S(t)$	vector of output quantities by sensor process
$\hat{\mathbf{y}}(t)$	resulting output quantities by measurement
$\mathbf{y}_{obs}(t)$	vector of output quantities by observer process
$\mathbf{z}(t)$	vector of immeasurable output quantities to be observed
$\mathbf{e}_y(t)$	vector of measurement errors concerning output quantities
$p^d(x)$	probability density function (pdf) of the quantity x
$\mathbf{e}_{\mathbf{y}_{obs}}(t)$	vector of observation errors concerning output quantities
$\mathbf{e}_{\mathbf{y}_{obs}}(t)$	vector of observation errors concerning output quantities
$\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$	parameter matrices of the process model PM in the State Space Description (SSD)
$\mathbf{I}$	unity matrix
$\mathbf{L}$	control matrix of the closed loop observer CLO
$\mathbf{O}_{obs}$	observability matrix

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