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SENSOR FAULT DETECTION BY TESTING THE GENERALIZED VARIANCE OF THE INNOVATION COVARIANCE

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Abstract - A new method for testing the covariance matrix of the innovation sequence of the Kalman filter is proposed. The generalized variance (determinant) of the random Wishart matrix is used in this process as a monitoring statistic, and the testing problem is reduced to determination of the asymptotics for Wishart determinants. In the simulations, the longitudinal and lateral dynamics of the F-16 aircraft model is considered, and detection of sensor failures, which affect the covariance matrix of the innovation sequence, are examined.

Keywords**:** sensor, fault detection, Kalman filter, innovation sequence, generalized variance

1.. INTRODUCTION

Many fault detection filters have been developed to detect and identify sensor and actuator faults by using analytical redundancy. In [1] an analytical redundancy-based approach for detecting and isolating sensor, actuator, and component (i.e., plant) faults in complex dynamical systems, such as aircraft and spacecraft is developed. A statistical change detection technique based on a modification of the standard generalized likelihood ratio (GLR) statistic is used to detect faults in real time. The GLR test requires the statistical characteristics of the system to be known before and after the fault occurs. As this information is usually not available after the fault, the method has limited applications in practice.

In [2,3] the algorithms for detection and diagnosis of multiple failures in a dynamic system are described. They are based on the Interacting Multiple-Model (IMM) estimation algorithm, which is one of the most cost-effective adaptive estimation techniques for systems involving structural as well as parametric changes. The proposed algorithms provide an integrated framework for fault detection, diagnosis, and state estimation. In [4] multiple model adaptive estimation (MMAE) methods have been incorporated into the design of a flight control system for the variable in-flight stability test aircraft (VISTA) F-16, providing it with the capability to detect and compensate for sensor and control surface/actuator failures. In methods described in [2-4], the faults are assumed to be known, and the Kalman filters are designed for the known sensor/actuator faults. As the approach requires several parallel Kalman filters, and the faults should be known, it can be used in limited applications.

In the references [5-7] the neural network based methods to detect sensor, control surface/actuator failures are developed and discussed. However, the methods based on artificial neural networks and genetic algorithms do not have physical bases. Therefore according to the different data corresponding to the same event the model gives different solutions. Thus, the model should continuously be trained by using the new data.

One of the diagnosis approaches based on Kalman filtering is the analysis of the innovation sequence [8, 9]. These approaches do not require a priori statistical characteristics of the faults, and the computational burden is not very heavy. If the system operates normally, the normalized innovation sequence in a Kalman filter is a Gaussian white noise with a zero mean and with a unit covariance matrix. Faults that change the system dynamics by causing surges of drifts of the state vector components, abnormal measurements, sudden shifts in the measurement channel, and other difficulties such as decrease of instrument accuracy, an increase of background noise, reduction in control surface/actuator effectiveness etc., effect the characteristics of the normalized innovation sequence by changing its white noise nature, displacing its zero mean, and varying unit covariance matrix. Thus, the problem is how to detect as quickly as possible any change of these parameters from their nominal value. Methods of testing the agreement between the innovation sequence and white noise, and the detection of any change of its mathematical expectation have been discussed in detail in the literature [8, 10], therefore, it shall not be concentrated on testing these characteristics.

Testing, in real time, the covariance matrix of the innovation sequence of the Kalman filter turns out to be very complicated and not well developed, since there are difficulties in the determination of the confidence domain for a random matrix. Moreover, the existing methods of high-dimensional statistical analysis [11,12] usually lead to asymptotic distributions; this sharply diminishes the operativeness of these methods. In practice, therefore, one makes use of a scalar measure of this matrix such as the trace, the sum of the matrix elements, generalized variance (determinant), the maximal eigenvalue of a matrix, etc., each characterizing one or another geometrical parameter of the correlation ellipsoid.

In [8] the trace of the matrix is used for the scalar measure of the tested covariance matrix. Although the trace of the sample covariance matrix is the easiest to check, it might lead to incorrect decisions at detection of faults, because it disregards the off-diagonal matrix elements.

In [9] a method is described for checking the sum of all elements of the inverted covariance matrix of the innovation sequence.

In Ref. [13] an operative method of testing the innovation covariance of the Kalman filter based on the Tracy–Widom distribution is proposed. The maximal eigenvalue of the random Wishart matrix is used as the monitoring statistic, and the testing problem is reduced to determine the asymptotics for the largest eigenvalue of the Wishart matrix. An algorithm for testing the largest eigenvalue based on the Tracy–Widom distribution is proposed.

In [14] a new approach based on the generalized Rayleigh quotient for testing the innovation covariance of the Kalman filter is proposed. The generalized Rayleigh quotient of the vector under the sample and theoretical innovation covariance matrices are used in this study for the monitoring statistics. The optimization process of testing quality is reduced to the classical problem of maximization of the generalized Rayleigh quotient.

In this study, an algorithm for checking the generalized variance of the innovation sequence, characterizing the volume of the correlation ellipsoid and application to an aircraft dynamics, is investigated. This is the overall measure in terms of the volume of the error ellipsoid, which is proportional to the determinant of covariance matrix of the innovation sequence.

The structure of this paper is as follow. In Section II, the fault detection problem in multidimensional dynamic systems based on the Kalman filter innovation is formulated. New results for testing the generalized variance (determinant) of the innovation covariance is proposed in Section III. The AFTI/F-16 aircraft model description and Extended Kalman filter for the F-16 nonlinear dynamic model estimation are given in Section IV. In Section V some simulations are carried out for the sensor fault detection problem in the AFTI/F-16 aircraft flight control system. The changes that affect the innovation covariance have been considered. The fault detection algorithm based on construction of confidence intervals for the Wishart determinants via its asymptotics is examined in this section. Section 6 gives a brief summary of the obtained results and the conclusion.

2. STATEMENT OF THE PROBLEM

Let us consider a class of systems described by differential equations of the form

$$x(k+1) = \Phi(k+1,k)x(k) + G(k+1,k)w(k)$$

$$z(k) = H(k)x(k) + V(k)\,, \tag{1}$$

where $x(k)$ is the n-dimensional state vector of the system, $\Phi(k+1,k)$ is the transition matrix of order nxn of the system, $w(k)$ is the random n-dimensional vector of system noises, $G(k+1,k)$ is the transition matrix of system noises of order nxn, $z(k)$ is the s-dimensional measurement vector, $H(k)$ is the measurement matrix of the system of order sxn, $v(k)$ is the random s-dimensional vector of measurement noises. It is assumed that the random vectors $w(k)$, $v(k)$, and $x(0)$ are mutually independent white Gaussian processes with zero expectations and covariance matrices defined by the relations:

$$E[w(k)w^T(j)] = Q(k)\delta(kj)\,,$$
$$E[v(k)v^T(j)] = R(k)\delta(kj)\,,$$
$$E[x(0)x^T(0)] = P(0)\,,$$

where $\delta(kj)$ is the Kronecker symbol.

Under the above-mentioned a priori information, the estimator $\hat{x}(k/k)$ of the state vector and the covariance matrix of errors $P(k/k)$ are found with the help of the optimal Kalman filter. Moreover, if the optimal filter is normally operating, then the normalized innovation sequence

$$\begin{aligned}\tilde{v}(k) &= [H(k)P(k/k-1)H^T(k) + R(k)]^{-1/2}\\ &\times[z(k) - H(k)x(k/k-1)]\end{aligned} \tag{2}$$

is a white Gaussian noise with zero mean and identity covariance matrix [8]:

$$E[\tilde{v}(k)] = 0\,,\ E[\tilde{v}(k)\tilde{v}^T(j)] = P_{\tilde{v}} = I\delta(kj)\,,$$

where $\hat{x}(k/k-1)$ is the extrapolation value by one step,

$$\begin{aligned}P(k/k-1) &= \Phi(k,k-1)P(k-1/k-1)\Phi^T(k,k-1)\\ &+G(k,k-1)Q(k-1)G^T(k,k-1)\end{aligned}$$

is the covariance matrix of extrapolation errors, $P(k-1/k-1)$ is the covariance matrix of estimation errors in the preceding step, I is the identity matrix.

The changes in the properties of the system or characteristics of perturbations (faults of measuring devices, abnormal measurements, changes in statistical characteristics of noises of the object or of measurements, etc) leading to a change in the covariance matrix of the innovation sequence (2) are considered.

It is of interest to develop an operative method of testing the determinant of the covariance matrix of sequence (2).

3. ALGORITHM OF SOLUTION

Two hypotheses are introduced:

γ_0 : no sensor fault occurs;

γ_1 : a sensor fault occurs.

Let us write the expression for the sample covariance matrix of the sequence $\tilde{\nu}(k)$:

$$\hat{S}(k) = \frac{1}{M-1}\sum_{j=k-M+1}^{k}[\tilde{\nu}(j) - \bar{\tilde{\nu}}(k)][\tilde{\nu}(j) - \bar{\tilde{\nu}}(k)]^T \qquad (3)$$

where

$$\bar{\tilde{\nu}}(k) = \frac{1}{M}\sum_{j=k-M+1}^{k}\tilde{\nu}(j) \qquad (4)$$

is the sample mean; M is the number of realizations used (the width of the sliding window).

As is known [12], under the validity of the hypotheses γ_o, the random matrix

$$A(k) = (M-1)\hat{S}(k) \qquad (5)$$

has the Wishart distribution with M degrees of freedom and is denoted by $W_s(M, P_{\tilde{\nu}})$:

$$A \sim W_s(M, P_{\tilde{\nu}}), \qquad (6)$$

where s and $P_{\tilde{\nu}}$ dimension and covariance matrix of the normalized innovation sequence $\tilde{\nu}$ respectively. In testing statistical hypotheses, the testing of the Wishart statistics (6) is complicated and not well developed in view of the difficulty of constructing the confidence domain for a random matrix. In practice, one of the scalar measures of the above-mentioned matrix is usually applied for testing random matrices. The choice of one or another scalar measure as the monitoring **s**tatistic for a particular problem being solved depends on the basic indicators of supervision (the sensitivity, the inertia, the volume of computational expenditures, etc) The experience and intuition of a researcher is also of importance**.** In this paper, construction of confidence intervals for the Wishart determinants via its asymptotics is considered. In [15] the following asymptotic result for determinants of Wishart matrices is obtained.

Theorem. Let $A \sim W_s(M, P_{\tilde{\nu}})$, $M = 1,2,...$ Then

$$\left(\frac{\det(A)}{\det(P_{\tilde{\nu}})(M-1)!}\right)^{\frac{1}{\sqrt{2\log(M)}}} \xrightarrow{d} e^N, \qquad (7)$$

where $\xrightarrow{d}$ stands for convergence in distribution and N is a standard normal random variable.

Taking the logarithm of the left and right hand sides of the formula (7), the following expression can be written as:

$$\frac{1}{\sqrt{2\log(M)}}\log\left(\frac{\det(A)}{\det(P_{\tilde{\nu}})(M-1)!}\right) \xrightarrow{d} N \qquad (8)$$

Let us denote the left hand side of the expression (8) as

$$WD = \frac{1}{\sqrt{2\log(M)}}\log\left(\frac{\det(A)}{\det(P_{\tilde{\nu}})(M-1)!}\right) \qquad (9)$$

and call it the statistic of the Wishart determinants. Taking into account that the covariance matrix of the normalized innovation sequence in the case of normally operating of system is the identity matrix, e.d. $P_{\tilde{\nu}} = I$, and combining the formulas (8) and (9) the following expression can be written:

$$WD = \frac{1}{\sqrt{2\log(M)}}\log\left(\frac{\det(A)}{(M-1)!}\right) \xrightarrow{d} N \qquad (10)$$

If the confidence probability β is selected as,

$$P\left\{|WD| \le u_{\frac{1+\beta}{2}}\right\} = \beta\,;\ 0 < \beta < 1 \qquad (11)$$

the threshold value, $u_{\frac{1+\beta}{2}}$ can be determined ($u_{\frac{1+\beta}{2}}$ is the quantile of the standard normal distribution). Hence, when the hypothesis γ_1 is true, the absolute value of the statistic WD will be greater than the threshold value $u_{\frac{1+\beta}{2}}$. Then the decision rule on the current state of the system of estimation with respect to the introduced hypotheses will be written in the form:

$$\begin{aligned} &\gamma_0 : |WD(k)| \le u_{\frac{1+\beta}{2}}, \forall k \\ &\gamma_1 : |WD(k)| > u_{\frac{1+\beta}{2}}, \exists k \end{aligned} \qquad (12)$$

Consequently, by comparing the above-defined statistic of the Wishart determinants $WD(k)$ with the confidence limits for the corresponding standard normal distribution, it is possible to detect sensor faults using decision rule (12).

4. EKF FOR THE F-16 AIRCRAFT MODEL ESTIMATION

The above covariance matrix testing algorithms are applied to multi-input multi-output model of an AFTI/F-16 fighter aircraft. The nonlinear mathematical model for the longitudinal and lateral motion of F-16 fighter is as follows:

$$x(k+1) = Ax(k) + Bu(k) + F(x(k)) + Gw(k) \qquad (13)$$

where $x(k)$ is the 9-dimensional state vector of the aircraft, A is the transition matrix of the aircraft with the order of 9x9, B is the control distribution matrix of the aircraft with the order of 9x6, $u(k)$ is the 6-dimensional control input vector, $F(x(k))$ is the 9-dimensional vector of nonlinear elements of the system, G is the transition matrix of the system noises, and $w(k)$ is the random 9-dimensional system noise vector with zero mean and the correlation matrix of $E\left[w(k)w^T(j)\right] = Q(k)\delta(jk)$.

The aircraft state variables are:

$$x = \left[v, \alpha, q, \theta, \beta, p, r, \varphi, \psi\right]^T,$$

where v is the forward velocity, α is the angle of attack, q is the pitch rate, θ is the pitch angle, β is the side-slip angle, p is the roll rate, r is the yaw rate, ϕ is the roll angle, and ψ is the yaw angle.

The fighter has six control surfaces and hence six control inputs. These are:

$$u = \left[\delta_{HR}, \delta_{HL}, \delta_{FR}, \delta_{FL}, \delta_C, \delta_R\right],$$

where δ_{HR} and δ_{HL} are the deflections of the right and left horizontal stabilizers, δ_{FR} and δ_{FL} are the deflections of the right and left flaps, δ_C and δ_R are the canard and rudder deflections.

The measurement equations can be written as:

$$z(k) = Hx(k) + V(k), \tag{14}$$

Here H is the measurement matrix, which is 9×9 unit matrix in case, and $V(k)$ is the 9-dimensional vector of measurement noises, where its mean and covariance matrix respectively are:

$$E\left[V(k)\right] = 0; E\left[V(k)V^T(j)\right] = R(k)\delta(jk).$$

By using quasi-linearization method, let us linearize the equation (13):

$$\begin{aligned} x(k) &= A\hat{x}(k-1) + B\hat{u}(k-1) + F(\hat{x}(k-1)) \\ &+ A\left[x(k-1) - \hat{x}(k-1)\right] + F_x(k-1)\left[x(k-1) - \hat{x}(k-1)\right] \\ &+ B\left[u(k-1) - \hat{u}(k-1)\right] + Gw(k-1) \end{aligned} \tag{15}$$

where $F_x = \left[\dfrac{\partial F}{\partial x}\right]_{\hat{x}(k-1)}$.

The following recursive Extended Kalman Filter (EKF) algorithm for the state vector estimation of the F-16 fighter motion is obtained in [16]:

$$\begin{aligned} &\hat{x}(k) = A\hat{x}(k-1) + B\hat{u}(k-1) + \\ &F(\hat{x}(k-1)) + P(k)H^T R^{-1}(k) \times \nu(k) \\ &\nu(k) = z(k) - H\left[A\hat{x}(k-1) + B\hat{u}(k-1) + F(\hat{x}(k-1))\right] \\ &P(k) = M(k) - M(k)H^T\left[R(k) + HM(k)H^T\right]^{-1} HM(k) \\ &M(k) = AP(k-1)A^T + BD_u(k-1)B^T + \\ &F_x(k-1)P(k-1)F_x^T(k-1) + GQ(k-1)G^T \end{aligned} \tag{16}$$

where $P(k)$ is the covariance matrix of the estimation error, $M(k)$ is the covariance matrix of the extrapolation error, and D_u is the covariance matrix of the control input error.

5. SENSOR FAULT DETECTION SIMULATION RESULTS

Let us show that, on the basis of the algorithm for testing the covariance matrix of the innovation sequence proposed in this paper, one can in a timely manner detect the faults appearing in the measuring channel. Measurements were processed using the Extended Kalman filter (16). The expressions for the innovation sequence and the normalized innovation sequence of EKF respectively are:

$$\nu(k) = z(k) - H\left[A\hat{x}(k-1) + B\hat{u}(k-1) + F\left(\hat{x}(k-1)\right)\right] \tag{17}$$

$$\tilde{\nu}(k) = \left[R(k) + HM(k)H^T\right]^{-1/2} \nu(k) \tag{18}$$

To detect failures changing the covariance matrix of the innovation sequence the above generalized variance of the random Wishart matrix $A(k)$ is used. In the simulations, $M = 20$, s=9, and $\beta_2 = 0.99$ are taken, and the threshold value $u_{\frac{1+\beta}{2}}$ is found as 3.

The proposed fault detection algorithm is based on the computation of the generalized variance of the matrix $A(k)$. The sequence of computations performed at each step of measurements has the following form:

- The estimates of the Extended Kalman filter and the value of the normalized innovation sequences are computed according to the (16)-(18);
- The random Wishart matrix $A(k)$ is determined via expressions (3)-(5);
- The generalized variance of the matrix $A(k)$ is found, and the statistic of the Wishart determinants $WD(k)$ is calculated via expression (9);
- The decision on the current state of the estimation system is made using the decision rule (12);
- The sequence of computations is repeated starting from item 1 for the next moment of time $k+1$.

Obtained results are presented in Fig.1-4. Figure 1 shows the admissible bounds of the statistic $WD(k)$ and the plots of their behaviors in the case of normal functioning of the all measurement channels. As is expected, at all points, $-3 < WD(k) < 3$. The corresponding normalized innovation sequence in the third measurement channel (pitch rate gyroscope channel) $\tilde{\nu}_q(k)$ is shown in Fig. 2. The graphs of the normalized innovation sequences in the other measurement channels are very similar to the ones in Fig.2.

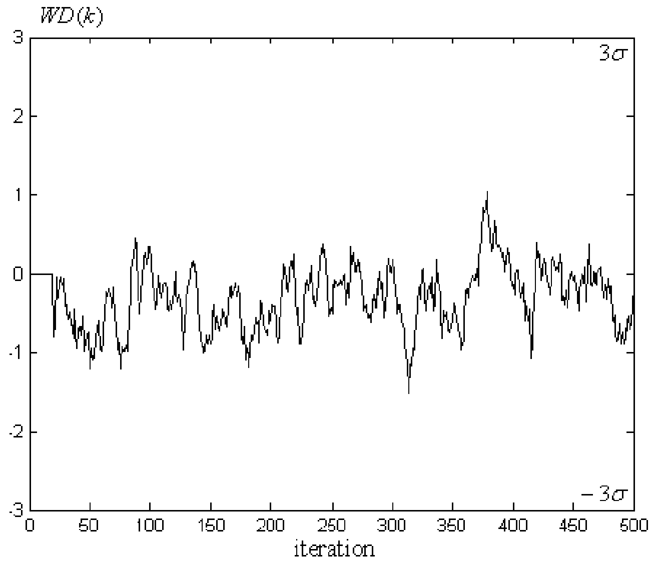

Fig.1. Graph of the statistic $WD(k)$ for normal operating of the measurement channels

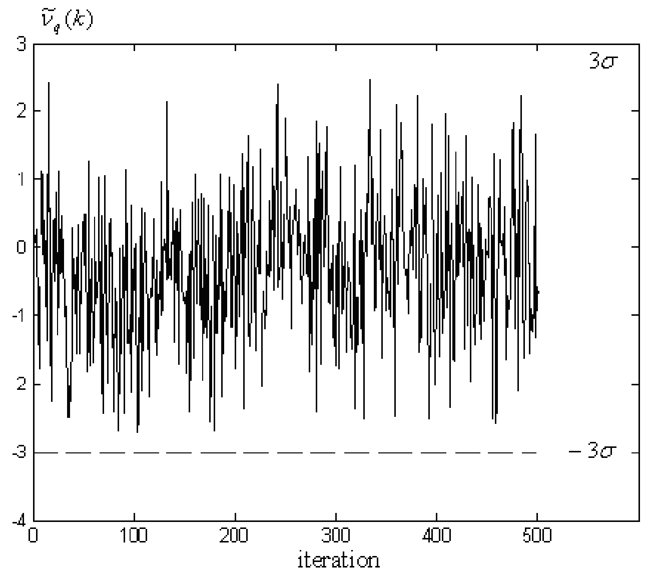

Fig.2. Behaviour of the $\tilde{\nu}_q(k)$ in the case of normal operating of the measurement channels

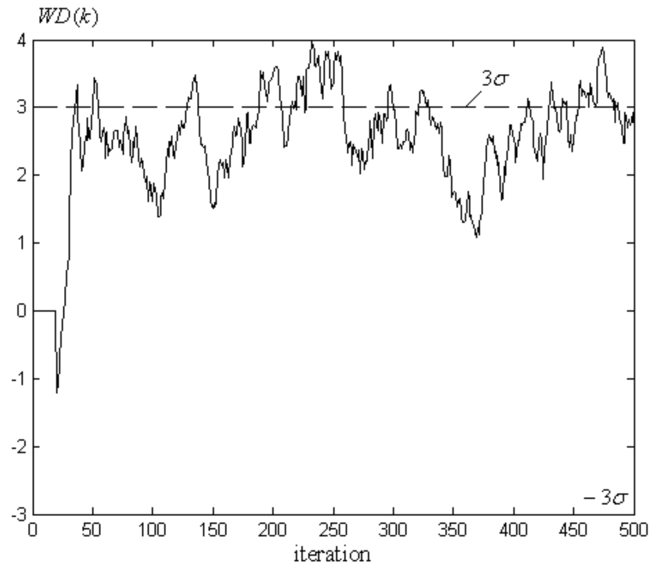

Fig.3. Behaviour of the statistic $WD(k)$ in case of changes in noise variance in the pitch rate gyroscope

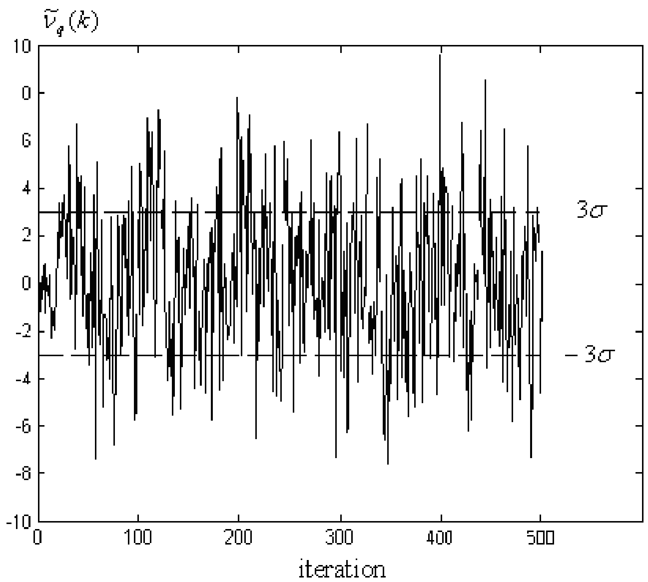

Fig.4. Behavior of the normalized innovation sequence $\tilde{\nu}_q(k)$ in case of changes in noise variance in the pitch rate gyroscope

To verify efficiency of proposed algorithms, beginning from the step k=30, a fault in the third measurement channel (pitch rate gyroscope fault) is simulated (the noise variance in the pitch rate gyroscope is increased by multiplying with 3). The simulation results corresponding to this case are presented in Fig.3-4. Figure 3 shows that the value of $WD(k)$ increases after the 30th step and exceeds its threshold at the step $k=42$ (0.36 s after the fault occurs).. As a result, based on the decision rule (12), sensor failure is noted. The behavior of the appropriate normalized innovation sequences $\tilde{\nu}_q(k)$ is presented in the Fig. 4.

The introduction of developed sensor fault detection algorithm does not distort the results of the estimates of the filter and has no influence on their accuracy. On the whole, the simulation results justify the theoretical calculation obtained and show the practical applicability of the proposed fault detection algorithms.

6. CONCLUSION

In this paper operative method for sensor fault detection based on testing the covariance matrix of the innovation sequence of the Kalman filter is proposed.

The generalized variance (determinant) of the random Wishart matrix is used in this process as a monitoring statistic, and the testing problem is reduced to determination of the asymptotics for Wishart determinants. An algorithm for testing the covariance matrix of the innovation sequence based on construction of confidence intervals for the Wishart determinants is proposed.

The generalized variance of the random Wishart matrix is the overall measure in terms of the volume of the error ellipsoid, which is proportional to the determinant of covariance matrix of the innovation sequence.

The sensor fault detection algorithm is based on the evaluation of the generalized variance of the Wishart matrix and its comparison with the confidence bounds of the standard normal distribution.

An extended Kalman filter has been developed for nonlinear flight dynamic estimation of an F-16 fighter. Failures in the sensors affect the characteristics of the innovation sequence of the EKF. The failures that affect the variance of the innovation sequence have been considered. The theoretical results are confirmed by the simulations carried out on a nonlinear dynamic model of the F-16 aircraft.

The algorithms proposed in this paper do not require a priori information on the change of the covariance matrices of the innovation sequences in the case of a fault and can be used in the problems of sensor fault detection and diagnosis of dynamic systems.

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