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GEOMETRICAL DEVIATIONS MEASUREMENT UNCERTAINTY ESTIMATION USING RANDOM FUNCTION

Edi Kulderknup[1], *Argo Pandis*[2]

[1]Estonian Accreditation Centre, Tallinn, Estonia, edi@eak.ee
[2]Tepso Laborid Ltd, Tallinn, Estonia, argo@tepso.ee

Abstract - Workpiece geometrical deviations have essential influence for quality of apparatus. Geometrical deviations of apparatus mechanical components have accuracy level better as 1μm.
For apparatus mechanical parts are used widely limits for flatness, cylindricity and position. By measurements of them shall be taken account additionally factors like autocorrelation between measurement points and influence factors variation over measurement area. Last can be described as random function which gives ground for the model of measurement and uncertainty estimation.

Keywords**:** uncertainty, geometrical deviations, random function, measurement model.

1. INTRODUCTION

Workpiece geometrical deviations have significant influence for quality of the machinery and apparatus products. Now-a-days apparatus products, including measuring instruments, integrates more widely electronical parts which allow more exact presentation of measurement results and to have small measures of device. Above required that the mechanical components would be also small and with minimal values of tolerance limits including limits for geometrical deviations.

Main task of this work was to set up measurement model suitable for use in ordinary apparatus production practise where low cost of production is essential. Such measurement model should allow to choose optimal measurement process, to have higher measurement capability and accuracy of assemblies and to take account mate detail in work. Model gives possibility to estimate which uncertainty components are representative and how they act in practice.

Geometrical deviations of measuring instruments components have accuracy level better than 1 μm. High accuracy requires estimation of measurement uncertainty taking account influence factors which generaly have modest importance. More often are by designer prescribed geometrical deviations limits for workpiece straightness, parallelism and flatness, roundness and cylindricity and position.

By measurements of above geometrical deviations shall be taken account additional factors like the autocorrelation between measurement points and influence factors and its values variation over the measurement area. Values variation can be described as random function which gives ground for the measurement and uncertainty estimation model.

In this paper, mainly are concentrated to the straightness and parallelism geometrical deviations which are basic and more simple deviations. Autocorrelation effect is estimated for distance and not at different times.

This study work is further development of work [1] which handled statistical aspects of geometrical deviations measurements. This study work gives as novelty the model of the geometrical deviations measurements based on random function.

2. APPARATUS COMPONENTS GEOMETRICAL DEVIATIONS

Workpiece geometrical deviations are developed by the designer and have standardized normative values. Geometrical deviations for the real workpiece or part of devices are realized through production process and controlled through measurements [2]. Measurements of the geometrical deviation have difficulties by main reason that there exist problems to present the datum surfaces. The datum itself has deviation and often can't be given as real physical object.

Geometrical deviation Δ_{GD} can be presented by general model:

$$\Delta_{GD} = \max \{B + a_i + c_i\} \qquad (1)$$

where a_i and c_i are the measuring instrument minimal and maximal indications for deviation from reference value B.

Geometrical deviations have influence for both contacted parts of the device and so the measures have dependence from each other. In this study geometrical deviations are limited for less modest. Simplest are straightness and parallelism and roundness deviations which have two dimensional directions and in initial stage are more easy to handle.

Measurement of straightness and parallelism deviations can be performed when moving a workpiece under measuring instrument (for example dial gauge with stand appliance) and taking measuring instrument indications Δ in the measurement points. Indication values shall be placed on graph and through those values points draw up the reference datum line. Straightness deviation Δ_{STR} can be found calculating difference of measurement point values from datum line value (see Fig 1). Calculation is easy to automatized if datum line is presented through mathematical model. On a similar way can be found measurement result for roundness deviation but then shall be used equipment allowing rotation of the workpiece.

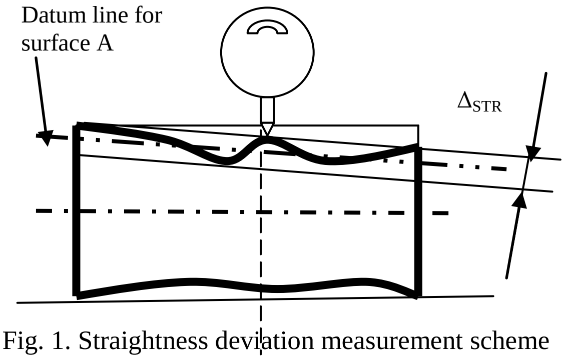

Fig. 1. Straightness deviation measurement scheme

For the parallelism deviation Δ_{PAR} estimation shall be found datum line of the other surface of workpiece. Using similar to straightness measurement procedure, this gives additional influence factor from datum line of the other surface and the measurement result shall be corrected taking account this (see Fig 2).

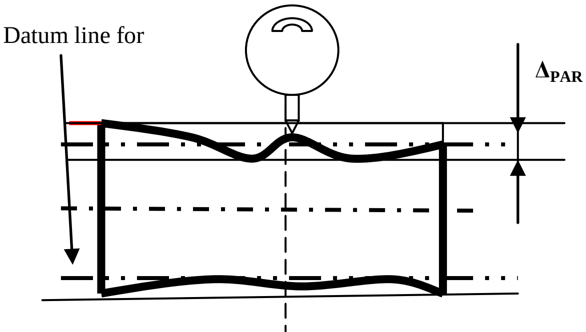

Fig. 2. Parallelism deviation measurement scheme

In practice more importance have the flatness and the cylindricity deviations which are complex deviations and have three dimensional directions. Flatness and cylindricity geometrical deviations can be presented through straightness and roundness, but then must be included additionally specific influence factors caused by the third length dimension.

Based on general model (1) the measurement model can be expressed taking account influence factors as follows:

$$y = x + K_{MI} + K_{RE} + K_{MF} + K_{DA} + K_{ENV} + K_{OBJ} \quad (2)$$

where x is the indicated value for geometrical deviation Δ_{GM}, K_{MI} is the correction from the measuring instruments based on calibration, K_{RE} is the correction from the reading of indication, K_{MF} is the correction from measurement method which includes large number of components as measurement force, velocity, probe parameters and others, K_{DA} is the correction from the datum deviations, K_{OBJ} is the correction from the workpiece characteristics and K_{ENV} is the correction from the environment conditions.

Equation (2) is ground for the uncertainty estimation.

For ordinary measurements having accuracy level up to 0,010 mm and less, main influence factors raised from the measuring instrument and reading.

For exact measurements more influence obtain measurement method and object characteristics. For straightness deviations measurement uncertainty components related to measurement method are measurement force, velocity, distance between measurement points, gauges probe parameters, datum estimation and measuring instrument position on datum during measurement process.

Uncertainty components related to measurement object is surface roughness and systematic variation of surface texture caused mainly by production process. Workpiece production process causes components, including trends, which acting more like the systematic factors such as vibration and production tools specific characteristics.

If measured shall be 3D geometrical deviation then uncertainty altered over 3 axes.

More exact measurements required more exact model of the uncertainty estimation and useful is application of wide complex of statistical principles [3].

3. ADDITIONAL STATISTICAL TOOLS FOR UNCERTAINTY ESTIMATION

3.1 Autocorrelation effect

According to GUM the correlation effect shall be taken account by uncertainty estimation. The correlation coefficient is a measure of the relative mutual dependence of two variables, equal to the ratio of their covariances to the positive square root of the product of their variances. The covariance associated with the estimates of two input quantities *Xi* and *Xj* shall be included for the calculation formula of combined uncertainty. Main source of variables correlation by the geometrical deviations measurement are use of the same physical measurement standard, measuring instrument and reference datum.

Geometrical deviations measurements performed on ordinary production conditions there is insufficient information to evaluate the covariance associated with the estimates of *Xi* and *Xj*. Also, if the accuracy level in ordinary plant conditions measurements is low, then correlation coefficientt is taken to be minor and treated as insignificant.

If accuracy level of the workpiece deviations has value ca 1 μm then correlation effect has importance. One of the factors which is not often taken account by uncertainty estimation is the autocorrelation effect. It also includes on some extend other correlation quantities. Autocorrelation is the cross-correlation of an input signal with itself [4]. Autocorrelation has specific importance for the measurements which have automatized process and are performed rapidly. There is no time to recover of the measuring instrument sensor to the correct position.

If real technical processes has continuous function $f(t)$ the autocorrelation is suitable to present by Equation (3):

$$R_f(t) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} f(t_i) f(t_{i+1}) dt \quad (3)$$

where T is time value.

By straightness and parallelism deviations measurement the autocorrelation effect is caused mainly by the measurement instrument rigidness, where the sensor can`t follow exactly the profile of the workpiece surface. Important role for the autocorrelation effect has sensor movement velocity, distance between measurement points and measurement force. By practical measurements of geometrical deviations is better to use length L value rather than time T value in Equation 3 for variable t_i.

3.2 Random function as measurement model

One possible way to give more data for the geometrical deviations measurement and for the measurement uncertainty is to use more widely statistical tools including random function. Random function application allows to give statistical model for the whole measured line or area. Model can be estimated separately for the two (2D) directions which gives simplified 3 dimensional picture if were combined together.

Random function is function with non-systematic argument t having on each value of t random quantity R. Argument t of random function is given as $X(t)$ and random function is expressed as $X(t)= R\,f(t)$. For concrete case one value t is expressed as $x_i(t)$ which has estimates for expectation $E_x(t)$ and standard deviation $\sigma_x(t)$. For random function is important correlation function which characterise link between the following to each other values of argument t.

Mathematical expectation for random function $X(t)$ on fixed argument values of $t=t_i$ gives random quantity $X(t_i)$ with mathematical expectation $E[X(t_i)]$. For various t_i values exist various $E[X(t_i)]$ which represents random function $E_x(t)$. Practically $E_x(t)$ is an average line around which situates other lines of random function.

Variance of random function $X(t)$ on fixed argument values of $t=t_i$ gives random quantity $X(t_i)$ with variance $D[X(t_i)]$. For various t_i exists various variances which represents random function $D_x(t)$. Practically $D_x(t)$ characterise line variation around average line of function $E_x(t)$.

Correlation between the following to each other values of argument t is expressed through correlation moment $K_x(t_1, t_2)$ which can be presented by Equation (4):

$$K_x(t_1, t_2) = E[\hat{X}(t_1)\ \hat{X}(t_2)], \quad (4)$$

where $\hat{X}(t_i) = X(t_i) - E_x(t_i)$ is average centralised random function.

More innovative is to give random function for 3 dimensional geometrical deviations as flatness and cylindricity measurement but this is difficult for the practical use in industry. Functions can be developed separately for 2 dimensions and based on those give simulated 3 dimensional picture.

3.3 Cumulative probability function CPF as measurement model based on random function

If random function is found, then this can be develop as cumulative probability function $F(x)$. $F(x)$.gives possibility for simulation of measurement, for example using Monte-Carlo model as $F(x) = P(x)$, where $F(x)$ is cumulative probability function and $P(x)$ is probability of geometrical deviation various values. On the other hand probability $P(x)$ can be linked also to the expanded uncertainty U of geometrical deviation Δ_{GD} measurement. Above is presented as shown on Fig. 3. Curves Meas 1, 2,…,.i present measurement results by various parameters of the concrete measurement process.

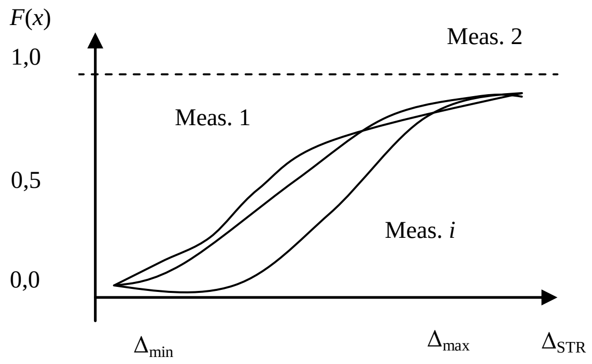

Fig. 3. CPF for various distances between measurement points and velocity (Meas. 1, 2,.., i) of Δ_{STR} measurements.

3. PRACTICAL MEASUREMENT RESULTS

In an initial stage measurements were simplified and task was to find main directions. Calibrated digital dial gauge with indication interval value 1 µm and with expanded uncertainty U=2 µm, k=2, was used as measurement instrument. Datum line (reference) was found using regression analyse based on the measurement results in measurement points. Measured were 10 workpieces produced in short-time cycle using the same production process. This allows to have less influence of long time trends from the production process. Measurement results were found in the measurement points with constant interval h between points. Measurements were carried out with two various velocity v_i of movement of the workpiece, $v_1 \approx 0{,}5$ cm/s and $v_2 \approx 1{,}0$ cm/s and with three various intervals h_i between measurement points, h_1 = 6 mm, h_2 = 3 mm and h_3 = 2 mm. As measurement result was used mean value of results around the eleven measurement points. Those were used for the random function parameters $E[X(t_i)]$ estimation. Also was estimated expanded uncertainty for measurement results in measurement points and those were used for the random function parameters $D[X(t_i)]$ estimation. Correlation moment $K_x(t_1, t_2)$ was presented as autocorrelation coefficient r_a.

Measurement model (2) was ground for the uncertainty estimation. Estimation results were, dial gauge uncertainty was u_{DG}=1,0 µm, uncertainty from the reading u_{READ}=0,6 µm, uncertainty from the measurement method (measurement force, velocity of the movement, distance between measurement points and other minor) u_{MET} = from 0,5 µm, which was by smaller h due to variation of measurement results around the measurement points up to 0,3 µm where h was bigger, uncertainty from workpiece (mainly surface roughness incl probe parameters) u_{OBJ}=0,2 µm, uncertainty from the datum line (mainly regression analyse deviations) u_{DL}= from 0,6 µm on the both ends of measurement line up to 0,2 µm on the midpoint of the line, uncertainty from the movement accuracy through datum line (movement velocity, distance between measurement points) during measurements u_{MOV}= from 0,5 µm for the bigger velocity and smaller distance up to 0,2 µm for the opposite case.

Uncertainty from environment was taken as minor.

All presented values have coverage factor k=1.

Summary results of the measurements straightness of one workpiece are given in Table 1. Based on those were found the random function parameters $E[X(t_i)]$ and $D[X(t_i)]$ for the straightness deviation measurement.

Table 1 Straightness deviations measurement summary results for random function for one velocity value

Measurement point No	Average straightness deviation Δ_{STR} [µm] from datum line and expanded uncertainty U, (k=2), [µm]		
	h=5 mm	h=3 mm	h=2 mm
1	1; U=2,8	2; U=2,7	-2; U=2,6
2	1; U=2,8	0 ; U=2,7	0; U=2,6
3	2; U=2,7	-2; U=2,6	-2, U=2,5
4	1; U=2,7	-2; U=2,6	-2, U=2,5
5	2; U=2,7	4; U=2,6	4; U=2,5
6	-8; U=2,6	5; U=2,5	5; U=2,4
7	-9; U=2,7	-8; U=2,6	-8; U=2,5
8	-10; U=2,7	-1; U=2,6	-1; U=2,5
9	-20; U=2,7	-8; U=2,6	-8; U=2,5
10	5; U=2,8	-5; U=2,7	-5; U=2,6
11	15; U=2,8	10; U=2,7	10; U=2,6

Autocorrelation coefficient r_a was estimated using (3) for various combinations of h_i and v_i. Summary results of the autocorrelation calculation are given in Table 2. Critical values of autocorrelation by Anderson are on a confidence level α=0,05. Calculated values of autocorrelation coefficient r_a assured the assumptions that minor autocorrelation effect exists by static measurements and essential effect by rapid measurements. Of cause, in initial stage there was small quantity of results.

Table 2 Summary results for the autocorrelation coefficient

Autocorrelation coefficient	Autocorrelation coefficient values depending on distance		
	h=6 mm	h=3 mm	h=2 mm
r_a; v_1=1,0 cm/s	0,40	0,84	0,85
r_a; v_2= 0,5 cm/s	0,45	0,78	0,80
$r_{acritical}$ (α=0,05)	0,35	0,30	0,30

On Fig 4 are presented measurement results as graph which can be used for random function mathematical model estimation. Shown is only measurement procedure where distance between points was h=2 mm and velocity was v_2≈ 0,5 cm/s and autocorrelation coefficient was r_a=0,85.

Axes x presents workpiece distance parameter L, where straightness deviation was measured with measurements points as random function argument t. Axes y presents straightness deviation Δ_{STR} measurement results in measurement points and its uncertainty. Continuous line presents measurement results function $X(t)= E[X(t_i)]$. Dashed lines present function $D_x(t)$ as the expanded uncertainty of the measurement results.

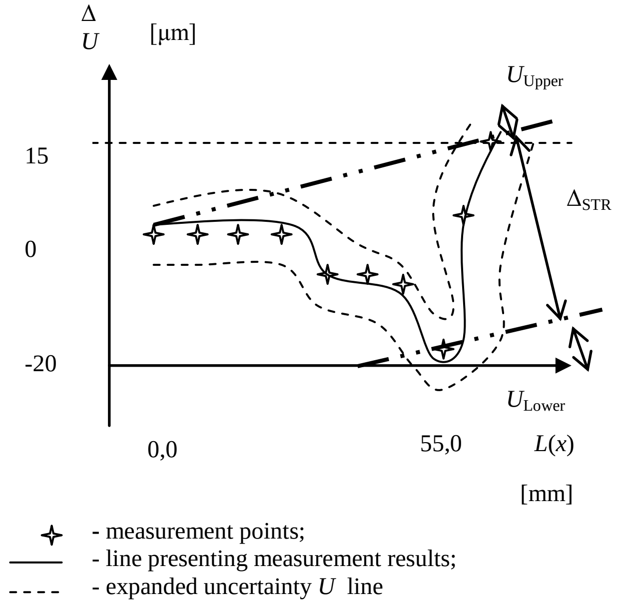

Fig. 4.Random function graph for the measurement of Δ_{STR}

On a similar way can be found graphs for other measurement procedure parameters. Combined those together can be drawen up the measurement procedures picture for wide values of factors. Using this, can be found corrections if measurement results which were performed rapidly.

4. CONCLUSIONS

As result of this work can be given next:

– measurement and uncertainty estimation model for measurements of the geometrical deviations is usefull to give as random function;

– random functions for measurement presents more information and allows further simulation of the measurement process and founding corrections to measurement result..

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