XXI IMEKO World Congress "Measurement in Research and Industry"
August 30 – September 4, 2015, Prague, Czech Republic

EXTENSION OF THE CLASSICAL MEASUREMENT METHOD BASED ON LENGTH ACCUMULATION IN SERIES

LI Bin

School of Mechanical Engineering, Southeast University, Nanjing, China, lbin_ali@aliyun.com

Abstract – No upper limit exists for the classical measurement method based on length accumulation in series. By extending this method, such a characteristic can be used to solve many difficult problems such as measuring the diameter of a shaft, measuring the race diameter of a bearing, and measuring the pitch diameter of a thread screw, all with arbitrarily large sizes. If the calibration precision of the measuring units (accumulative objects) is sufficiently high, and different measuring units are accumulated in measurement, high precision can be obtained with this method.

Keywords: length, accumulation, measurement, extension

1. INTRODUCTION

The classical measurement method based on length accumulation in series may be one of the oldest and simplest measuring methods; for example, the measurement of a distance by stepping has been used since ancient times and requires only that one goes on foot and counts. In addition, the method has an important characteristic: there is no upper limit of measurement. By extending this method, it was found that this characteristic can be used to solve many difficult large-scale measurement problems such as measuring the diameter of a shaft [1], measuring the race diameter of a bearing [2], and measuring the pitch diameter of a thread screw [3], all with arbitrarily large sizes. In addition, this extended method also has the advantages of easy operation, simplicity, portability, low cost, and, even more importantly, high precision under certain conditions.

In sections 2 and 3, the mathematical model and some applications of the method are given, respectively. The accumulative error analysis, which is the issue of most concern with regard to all kinds of accumulative methods, is performed in section 4. Finally, in section 5, the conclusions of this paper are presented.

2. MATHEMATICAL MODEL

Here, we present the mathematical model of the extended classical measurement method based on length accumulation in series. The model is a quotation from the following fundamental equation of the generalized measurement methods based on length accumulation proposed in [4]:

$$\sum_{j=0}^{n-1} \boldsymbol{P}_j \boldsymbol{P}_{j+1}(l) = \boldsymbol{0}\,, \tag{1}$$

where l is the quantity to be measured, n is the total number of accumulative objects, P_j is the accumulative point or central accumulative point with the serial number of j, and $\boldsymbol{P}_j\boldsymbol{P}_{j+1}$ is the vector from P_j to P_{j+1}.

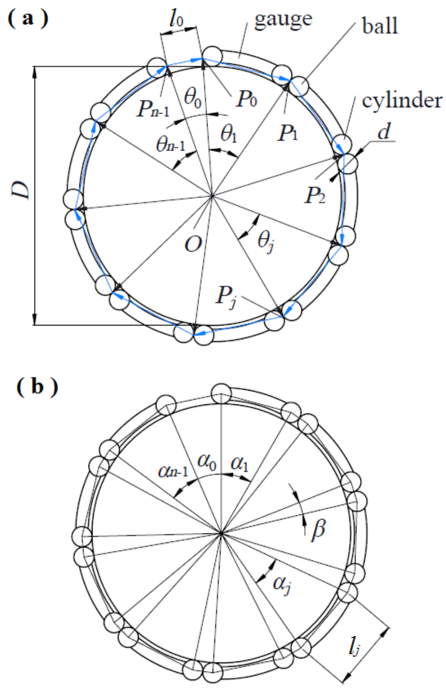

Fig.1. Measurement of the diameter of a large shaft with accumulative gauges: (a) configuration of the accumulated gauges together with accumulative points P_j and vectors $\boldsymbol{P}_j\boldsymbol{P}_{j+1}$ and (b) equivalent diagram of (a) to illustrate the method of deriving the equation to solve the diameter.

Without loss of generality, the diameter measurement of a large shaft can be taken as an example to illustrate the meaning of (1) [1].

As shown in Fig.1 (a), for every gauge, there is a ball and a cylinder, each with a diameter d, located separately at the two ends of the gauge. As the accumulative objects, the gauges are accumulated in series around the circumference of the large shaft and fixed with magnets. The contacting point P_j (j = 0, 1,…, n − 1) between two accumulated objects including the measuring tool used to measure the gap l_0 between the first and last gauges (not shown in Fig.1) is just the accumulative point mentioned above.

As shown in Fig.1 (a), from (1), one can obtain:

$$\sum_{j=0}^{n-1} \theta_j = 2\pi . \quad (2)$$

As shown in Fig.1 (b), (2) is equivalent to

$$\sum_{j=0}^{n-1} \alpha_j + (n-2)\beta = 2\pi , \quad (3)$$

where:

$$\alpha_0 = 2\sin^{-1}\frac{l_0 + d}{D + d}, \quad (4)$$

$$\alpha_j = 2\sin^{-1}\frac{l_j}{D + d}, \qquad j = 1,2,\ldots,n-1 \quad (5)$$

$$\beta = 2\sin^{-1}\frac{d}{D + d}, \quad (6)$$

where l_j is the central distance between the ball and cylinder of the gauge that the serial numbers of the two accumulative points located at it are $j-1$ and j, respectively.

The diameter to be measured (D) then can be solved from (3).

Fig.2. Example of the diameter measurement of a large shaft with accumulative gauges in situ.

Fig.2 shows the photograph of an example of the diameter measurement of a large shaft with accumulative gauges in situ.

3. CASES

Some examples of applying the method are given in this section.

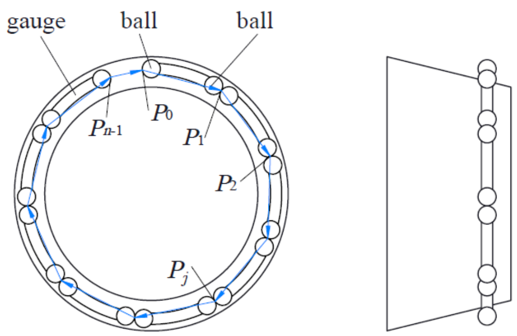

Fig.3. Measurement of the end-face diameter of a large cone with accumulative gauges: configuration of the accumulated gauges together with accumulative points P_j and vectors $\boldsymbol{P_jP_{j+1}}$.

Fig.3 shows the case of the end-face diameter measurement of a large cone [5].

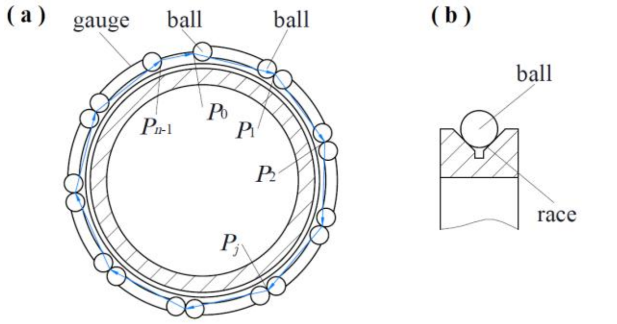

Fig.4. Measurement of the race diameter of a large bearing with accumulative gauges: (a) configuration of the accumulated gauges together with accumulative points P_j and vectors $\boldsymbol{P_jP_{j+1}}$ and (b) configuration of a ball contacting with the race of the bearing.

Fig.4 shows the case of the race diameter measurement of a large bearing.

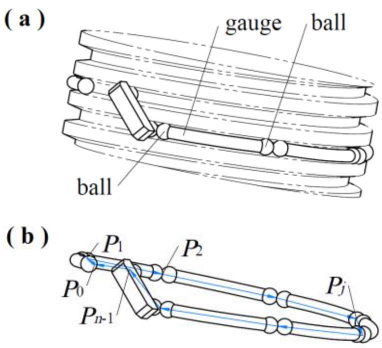

Fig.5. Measurement of the pitch diameter of a large thread screw with accumulative gauges: (a) configuration of the accumulated gauges and (b) accumulative points P_j and vectors $\boldsymbol{P_jP_{j+1}}$.

Fig.5 shows the case of the pitch diameter measurement of a large thread screw.

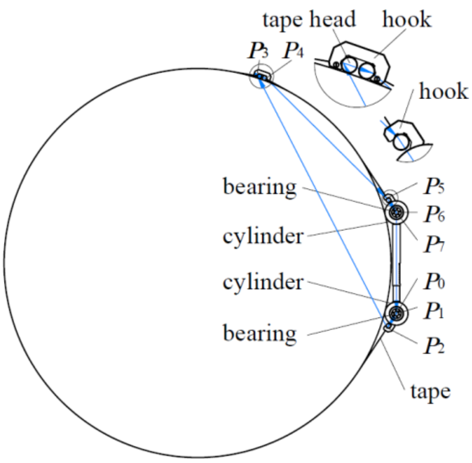

Fig.6. Measurement of the diameter of a large shaft with accumulative tapes: configuration of the accumulated tapes and other accumulative objects together with accumulative points P_j and vectors $\boldsymbol{P_jP_{j+1}}$.

Fig.6 shows another case of the diameter measurement of a large shaft [6].

There are still many other geometric parameters of large workpieces that can be measured using the developed method, such as the distance between two shoulders of a large shaft and the indexing error of nominally uniform holes on a large cylinder.

4. ACCUMULATIVE ERROR ANALYSIS

Accumulative error, which is usually the major factor affecting the precision of the method, contains two components: the sum of the contacting or aiming errors between accumulated objects; and the synthesis of the geometric errors of the accumulated objects themselves.

The first component of accumulative error is usually easy to control. For example, for the case shown in Fig.1, because the contact area between the ball and cylinder of two accumulated gauges is theoretically a point, the error caused by the contact can be restricted to such a small value that it can be ignored [1].

Without loss of generality, the second component of the accumulative error can be demonstrated using the measurement of a length with a ruler as an example.

There are two ways to accomplish the measurement. The first one is by accumulating the same one ruler; the measurement error ΔL and relative mean square deviation σ_L/L of the measured length L can be respectively obtained by (1) as

$$\Delta L = \sum_{j=0}^{n-1} \Delta l = n\Delta l \quad \text{and} \tag{6}$$

$$\frac{\sigma_L}{L} = \frac{\sigma_l}{l}\,, \tag{7}$$

where n is the accumulating number of the ruler, l is the nominal size of the ruler, and Δl and σ_l are its error and mean square deviation of calibration, respectively.

The second way to accomplish the measurement is by the accumulation of different rulers. Supposing that the calibration results of the rulers are irrelative, the measurement error ΔL and relative mean square deviation σ_L/L of the measured length L are then given respectively by

$$\Delta L = \sum_{j=0}^{n-1} \Delta l_j \quad \text{and} \tag{8}$$

$$\frac{\sigma_L}{L} = \frac{1}{\sqrt{n}}\frac{\sigma_l}{l}\,, \tag{9}$$

where n is the number of the accumulated rulers, $l_j=l$ ($j = 0, 1,\dots, n-1$) is the nominal size of the jth accumulated ruler, and Δl_j and σ_l are its error and mean square deviation of calibration, respectively.

Comparing (9) with (7), it can be concluded that the precision of the second way to accomplish the measurement is higher than that of the first due to the average effect caused by the accumulation of the rulers. In another words, when different rulers are accumulated for measurement as opposed to the same one ruler, the measurement precision is increased due to the average effect.

5. CONCLUSIONS

The classical measurement method based on length accumulation in series has no upper limit. By extending this method, this lack of an upper limit can be used to solve many difficult large-scale measurement problems.

In addition, the extended method has the advantages of easy operation, simplicity, portability, and low cost. If the calibration precision of the measuring units (accumulative objects) is sufficiently high, and different measuring units are used to take advantage of the average effect, high precision can be obtained with this method.

Note that the results obtained using this method (e.g., the diameter measurement of a large shaft) are usually averages; whether this aspect is unfavorable or not depends on the application itself. Another common worry about this method is that it might be tedious because it requires the same manual operation to be repeated. However, compared with most other methods for large-scale measurements, this concern is not always justified.

REFERENCES

[1] B. Li, “Polygon-based large diameter measurement with modular gauges”, *Measurement*, vol. 27, nº 1, pp. 1-11, Jan. 2000.

[2] B. Li, “High precision and non-upper limitation method of measuring race diameter of slewing bearing with two-sided race”, *Chinese Patent*, 200910035222.6, Sept. 15, 2009. （李彬, “高精度无上限双面滚道回转支承滚道直径测量方法”, *中国专利*, 200910035222.6, 2009.9.15）

[3] B. Li, “High precision and non-upper limitation apparatus of measuring pitch diameter of ultra large cylindrical thread and its operation method”, Chinese Patent, 201010581495.3, Dec. 10, 2010.（李彬, “高精度无上限特大圆柱螺纹中径测量装置及测量方法”, *中国专利*, 201010581495.3, 2010.12.10）

[4] B. Li, “Principle and fundamental equations of generalized measurement methods based on length and angular accumulation”, this proceeding.

[5] B. Li, “End-face diameter measurement of large cone based on a polygon by means of modular gauges”, *Journal of Astronautic Metrology and Measurement*, vol. 19, nº 6, pp. 17-20, Dec. 1999. （李彬, “基于多边形的积木式大圆锥端面直径测量”, *宇航计测技术*, vol. 19, nº 6, pp. 17-20, 1999.12）

[6] B. Li, “A non-full circle Π tape used for measuring large outside diameter”, Chinese Patent, 201210091673.3, March 30, 2012.（李彬, “一种用于大外径测量的非整圆Π尺”, *中国专利*, 201210091673.3, 2012.3.30）