CALIBRATED CAD MODEL OF FREEFORM STANDARD

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Abstract - The quality and relevance of reference standard in CAD-based metrology is significantly influenced by shape correspondence of the standard and its CAD representation. Based on geometrical-mathematical approach, a new freeform reference standard has been developed and manufactured so that its CAD model can be easily modified according to the values measured on physical standard. A two-stage calibration method for calibrated CAD model of the standard development is described in this paper.

Keywords: coordinate measurement, freeform standard, CAD model, calibrated CAD model, B-spline.

1. INTRODUCTION

Reliable evaluation of CAD-based freeform measurements in precision engineering leads to development of reference standards with sufficient precision and specific properties. Calibration standards of regular shapes are well developed [1] while the traceability and quality control in freeform manufacturing are issues due to lack of traceable verification standards [2]. To ensure a high level of information capability of CAD-based freeform measurements, the shape correspondence between the standard and its CAD model has to be as precise as possible. It follows that it is necessary to develop not only calibrated physical standard but also its calibrated CAD model. The idea of calibrated CAD model of freeform standard is mentioned in [3], [4], [5] and [6] where a freeform measurement is simulated with measurement of surfaces on regular objects (with calibrated CAD models) arranged in a manner that represents the actual shape as closely as possible.

Based on geometrical-mathematical approach, a new freeform reference standard Hyperbolic paraboloid for testing freeform measurement capability of coordinate measuring machines and machine tools [7] has been developed in CMI (Czech Metrology Institute) and manufactured in cooperation with CTU (Czech Technical University) in Prague. The development of freeform standard consists in purposeful application of B-spline representation and effective usage of their geometrical-mathematical properties. Thus, the CAD model of the standard can be easily modified according to the values measured on physical standard. In this paper, a two-stage calibration method developed to obtain calibrated CAD model of the standard is described.

The paper is organized as follows. B-spline representation and mathematical description of freeform standard is briefly described in section 2. Section 3 introduces calibration procedure developed to obtain calibrated CAD model of the standard. In section 4, all the obtained results are summarized.

2. B-spline representation

An analytic representation of uniform clamped B-spline surface of degree p in the u direction and degree q in the v direction is a bivariate vector piecewise polynomial function given by [8]

$$\mathbf{S}(u,v)=\sum_{i=0}^{m}\sum_{j=0}^{n}\mathbf{P}_{i,j}N_{i,p}(u)N_{j,q}(v) \qquad (1)$$

where $(\mathbf{P}_{i,j})$ are control points, $(N_{i,p}(u))$ and $(N_{j,p}(v))$ are univariate B-spline basis functions

$$\begin{aligned} N_{k,0}(t) &= \begin{cases} 1 & \text{if } t\in[t_k,t_{k+1}) \\ 0 & \text{otherwise}\end{cases} \\ N_{k,l}(t) &= \tfrac{t-t_k}{t_{k-l}-t_k}N_{k,l-1}(t)+\tfrac{t_{k+l+1}-t}{t_{k+l+1}-t_{k+1}}N_{k+1,l-1}(t), \\ & k=i,j;\ t=u,v;\ l=p,q, \end{aligned} \qquad (2)$$

defined on the knot vectors

$$\begin{aligned} U &=(\underbrace{0,\ldots,0}_{p+1},u_{p+1}\ldots,u_{r-p-1},\underbrace{1,\ldots,1}_{p+1}), \\ V &=(\underbrace{0,\ldots,0}_{q+1},v_{q+1},\ldots,v_{s-q-1},\underbrace{1,\ldots,1}_{q+1}, \end{aligned} \qquad (3)$$

where $r=m+p+1$ and $s=n+q+1$.

If $m,n=1$, the resulting B-spline surface

$$\begin{aligned} \mathbf{S}(u,v)=&(1-u)(1-v)\mathbf{P}_{0,0}+(1-u)v\mathbf{P}_{0,1}+ \\ &+u(1-v)\mathbf{P}_{1,0}+uv\mathbf{P}_{1,1},\ (u,v)\in[0,1]^2 \end{aligned} \qquad (4)$$

is created by one surface patch known as hyperbolic paraboloid. Hyperbolic paraboloid has very useful geometrical property independent on position of the four control points in three-dimensional space – both sets of parametric curves are straight lines as follows from substituting constant value $u\in[0,1]$ or $v\in[0,1]$ in (4). In fig. 1 left an example of such a surface is drawn together with two inner parametric straight lines in both directions.

Moreover, it is possible to create a piecewise bilinear ($p = q = 1$) surface represented by (1) of the same shape but with higher number of control points located at intersections of parametric straight lines of the original hyperbolic paraboloid. Individual patches of such a surface are C^0 continuously connected. An example of piecewise bilinear surface is drawn in fig. 1 right. Here, the surface is given by 13×13 control points, i.e. consists of 144 patches. The shapes of both surfaces depicted in fig. 1 are precisely the same even though their vector equations are different.

A piecewise bilinear B-spline surface consisting of more than one patch enables local modifications of the shape by changing position of control points. The higher number of control points, the finer local modifications of the shape can be applied. This remarkable property of bilinear B-spline surface has been used in process of freeform standard as well as its calibrated CAD model development.

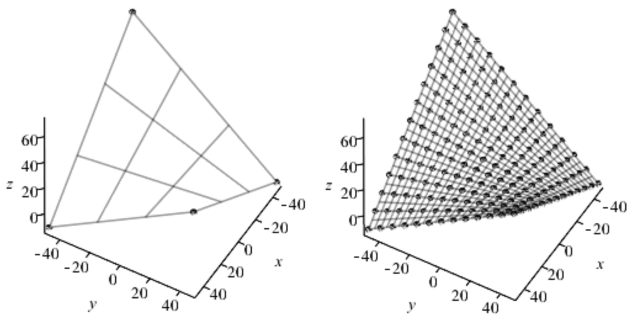


Fig. 1. Bilinear B-spline surface of the same shape given by 4 and 169 control points (all dimensions in mm).

3. Calibration of CAD model

The Hyperbolic paraboloid standard (120 mm $\times$ 120 mm $\times$ 67 mm) consists of step-squared base intended for clamping the standard on coordinate measuring machine, see fig. 2 left. The centre of the upper squared base lies at origin of coordinate system. Three precise reference spheres with radius equal to 8 mm are glued into the three of four spherical holes on the standard. The surface of hyperbolic paraboloid is trimmed by cylinder of revolution with axis identical with z-axis of coordinate system and radius equal to 40 mm. The common boundary between the upper squared base and the cylinder is filled with radius 4 mm, i.e. the transition surface is created by a part of torus. The standard has been manufactured by 3-axis milling on CNC milling machine US20 by high speed cutting from steel EN X10CrNi18-9.

During the whole process of calibration, see fig. 2 right, the manufactured standard was repeatedly measured in the Laboratory of Fundamental Metrology of CMI by tactile probe on SIP CMM 5 machine with maximum permissible error $(0.8 + 1.3L)$ μm without any changes of position in clamping device. The constant temperature (20 ± 0.2)°C in the environment was kept. The measured results were immediately processed and new input data for the measurement were continuously generated.

After coordinate system of the standard determination (by means of the three fiducial spheres), the CAD model of freeform standard has been calibrated in two stages.

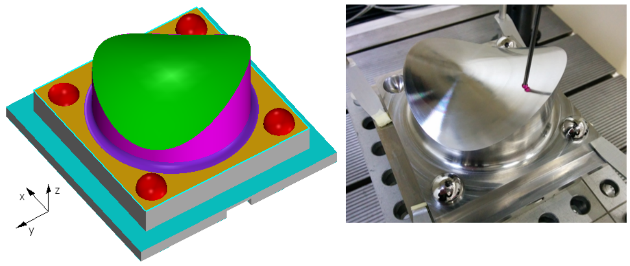


Fig. 2. CAD model (left) and calibration of standard (right)

3.1. Calibrated CAD model of the first stage

The procedure of calibrated CAD model of the first stage development consisted in the following steps.

1. **Mathematical model** – in the first step of calibrated CAD model development, the hyperbolic paraboloid given by four control points (4) was chosen as a suitable mathematical model of freeform surface. In particularly, the following coordinates of control points were used (all dimensions in mm)

$$\mathbf{P}_{0,0} = (-48, -48, 73),\ \mathbf{P}_{0,1} = (-48, 48, -11),$$
$$\mathbf{P}_{1,0} = (48, -48, -11),\ \mathbf{P}_{1,1} = (48, 48, 49),$$

   see fig. 1 left. Thus, vector equation (4) has the form

$$\mathbf{S}(u, v) = \begin{pmatrix} 96u - 48 \\ 96v - 48 \\ 144uv - 84u - 84v + 73 \end{pmatrix}. \quad (5)$$

2. **Theoretical CAD model** – due to the fact that CAD systems are based on NURBS (non-uniform B-spline) representation and all surfaces used to design the freeform standard can be expressed in NURBS representation, it was possible to create a precise CAD representation of mathematical model, see fig. 2 left.

3. **Theoretical piecewise bilinear surface** – a mesh of 625 control points ($\mathbf{P}^{625}$) as function values of surface (5) uniformly distributed with respect to the domain of parametrization was generated

$$(\mathbf{P}^{625}) : \left(\mathbf{P}_{i,j} = \mathbf{S}(\tfrac{i}{24}, \tfrac{j}{24}) = (x_{\mathbf{P}i,j}, y_{\mathbf{P}i,j}, z_{\mathbf{P}i,j})\right)_{i,j=0}^{24}$$

   see the mesh of points in fig. 3 left.

4. **Input data for the primary measurement** – a subset ($\mathbf{P}^{301}$) of 301 control points belonging to the area of physical standard

$$(\mathbf{P}^{301}) : \left(\sqrt{x^2_{\mathbf{P}i,j} + y^2_{\mathbf{P}i,j}} \le 40\right)_{i,j=0}^{24}$$

was selected from the set $(\mathbf{P}^{625})$ as input data for the primary measurement, see the subset in fig. 3 right.

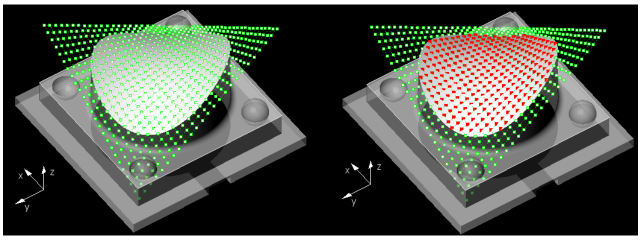


Fig. 3. Mesh $(\mathbf{P}^{625})$ (left) and subset of points belonging to the area of physical standard $(\mathbf{P}^{301})$ (right)

5. **Primary measurement** – the physical standard was measured at points $(\mathbf{P}^{301})$ with theoretical CAD model $\mathbf{S}(u, v)$ as the reference surface. The set $(\mathbf{M}^{301})$ of measured points was obtained and the following range of form error was determined

$$FE_{PS} = [-10.3, 8.7]\ \mu\text{m}.$$

The distribution of measured points as well as the colour map of measured deviations is shown in tab. 1, measurement 1.1.

Note: Form errors FE in this text are distinguished by subscript given by abbreviation of input data set following by abbreviation of reference CAD model, e.g. FE_{PS} in this case. All colour maps in this text are drawn in the range $[-5, 5]$ μm together with the histogram of the corresponding measured values.

6. **Modification of original theoretical CAD model** – the set of control points $(\mathbf{P}^{625})$ was modified by replacement of the subset $(\mathbf{P}^{301})$ on subset $(\mathbf{M}^{301})$. Thus, the modified set of control points $(\mathbf{R}^{625})$ was obtained.

7. **Possible calibrated CAD models of the first stage** – the three following interpretations of calibrated CAD model of the first stage were investigated: bilinear B-spline surface $\mathbf{L}(u, v)$, biquadratic B-spline surface $\mathbf{Q}(u, v)$ and bicubic B-spline surface $\mathbf{C}(u, v)$ given by the set of control points $(\mathbf{R}^{625})$, degrees $p, q = 1$, $p, q = 2$, $p, q = 3$ and knot vectors

$$\begin{aligned} U, V &= (0, 0, \tfrac{1}{24}, \tfrac{2}{24}, \ldots, \tfrac{23}{24}, 1, 1), \\ U, V &= (0, 0, 0, \tfrac{1}{24}, \tfrac{2}{24}, \ldots, \tfrac{23}{24}, 1, 1, 1), \\ U, V &= (0, 0, 0, 0, \tfrac{1}{24}, \tfrac{2}{24}, \ldots, \tfrac{23}{24}, 1, 1, 1, 1) \end{aligned}$$

in the given order.

Unlike the bilinear B-spline surface which passes through all control points, B-spline surfaces of higher degrees approximates the control mesh. This property is visible from figs. 4 and 5, where bilinear B-spline surface $\mathbf{L}(u, v)$ and bicubic B-spline surface $\mathbf{C}(u, v)$ are drawn in the space (u, v, d); d is the distance between the theoretical and real control point in μm. Thus, i is possible to consider biquadratic and bicubic B-spline surface as calibrated CAD models of the first stage covering the measurement uncertainty.

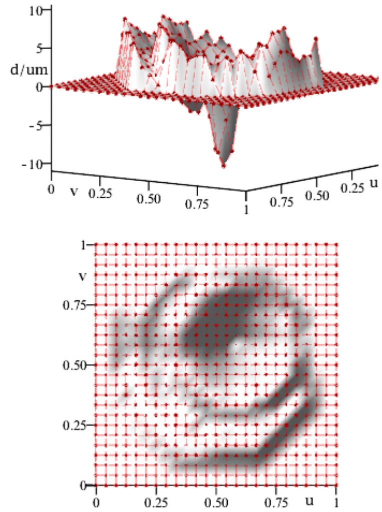


Fig. 4. Bilinear B-spline surface drawn in the space (u, v, d)

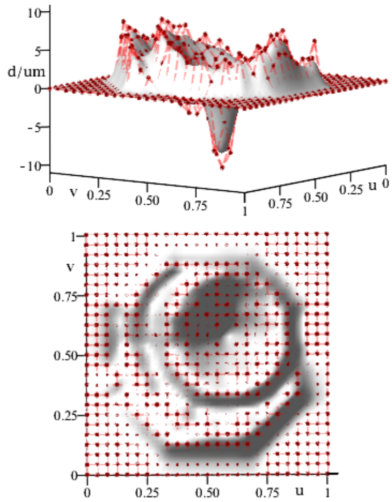


Fig. 5. Bicubic B-spline surface drawn in the space (u, v, d)

8. **The best calibrated CAD model of the first stage selection** – the physical standard was measured at points $(\mathbf{P}^{301})$ and $(\mathbf{M}^{301})$ with bilinear B-spline surface $\mathbf{L}(u, v)$ as reference CAD model, see tab. 1, measurement 1.2 and 1.3. Based on the obtained ranges of form errors

$$\begin{aligned} FE_{PL} &= [-1.0, 2.4]\ \mu\text{m}, \\ FE_{ML} &= [-0.8, 1.9]\ \mu\text{m}, \end{aligned}$$

the set $(\mathbf{M}^{301})$ was selected as input data for the measurement with biquadratic and bicubic B-spline surface as reference CAD model, see tab. 1, measurement 1.4 and 1.5. Here, the following ranges of form errors were obtained

$$\begin{aligned} FE_{MQ} &= [-0.8, 1.6]\,\mu\text{m}, \\ FE_{MC} &= [-0.8, 1.7]\,\mu\text{m}. \end{aligned}$$

Ranges of FE_{ML}, FE_{MQ} and FE_{MC} were comparable. It was the consequence of using the set $(\mathbf{M}^{301})$ as input data for the measurement.

To select the best calibrated CAD model of the first stage, it was necessary to generate a test set of points independent on control mesh $(\mathbf{R}^{625})$ of the surfaces representing reference CAD models. Thus, the test set $(\mathbf{T}^{225})$ of 255 points distributed in uniform (x, y) grid was generated and used as input data with bilinear, biquadratic and bicubic B-spline surface as reference CAD model, see tab. 1, measurement 1.4, 1.5 and 1.6. Then, the following range of form errors were obtained

$$\begin{aligned} FE_{TL} &= [-7.6, 27.7]\,\mu\text{m}, \\ FE_{TQ} &= [-2.5, 25.5]\,\mu\text{m}, \\ FE_{TC} &= [-2.0, 21.1]\,\mu\text{m}. \end{aligned}$$

From the colour maps representing results of measurements 1.4, 1.5 and 1.6 it follows that the biggest deviations of the measured data set from the reference surface are found along the circular boundary of freeform surface. It is caused by physical basis of the measured data used for modification of theoretical model. From B-spline theory point of view, the boundary conditions have very important influence on the shape along the boundaries. To avoid the unsuitable distortion, an extended B-spline surface as the first approach is commonly used and trimmed in appropriate dimensions to obtain the final model with required properties in actual area. However, if the measurement of physical object is a part of the process, it is impossible to extend measurement range beyond the physical dimensions. In this case, it is more useful to trim the reference surface and finish the process of calibrated CAD model development.

Based on ranges of obtained form errors, the bicubic B-spline surface was selected as the calibrated CAD model of the first stage. Then, this calibrated CAD model of the first stage was used as an original surface for the calibrated CAD model of the second stage development.

3.2. Calibrated CAD model of the second stage

The procedure of calibrated CAD model of the second stage development consisted in the following steps.

1. **Input data for the secondary measurement** – the set $(\mathbf{B}^{9409})$ of 9 409 points uniformly distributed (with respect to the domain of parametrization) on the bicubic B-spline surface $\mathbf{C}(u, v)$

   $(\mathbf{B}^{9409})$:
   $$\left(\mathbf{B}_{i,j} = \mathbf{C}(\tfrac{i}{96}, \tfrac{j}{96}) = (x_{\mathbf{B}i,j}, y_{\mathbf{B}i,j}, z_{\mathbf{B}i,j})\right)_{i,j=0}^{96}$$

   was generated. Then, a subset $(\mathbf{B}^{5002})$ of 5 002 points belonging to the area of the physical standard

   $$(\mathbf{B}^{5002}) : \left(\sqrt{x^2_{\mathbf{B}i,j} + y^2_{\mathbf{B}i,j}} \leq 40\right)_{i,j=0}^{96}$$

   was selected as input data for the secondary measurement.

2. **Secondary measurement** – the physical standard was measured at points $(\mathbf{B}^{5002})$ with bicubic B-spline surface $\mathbf{C}(u, v)$ as reference CAD model. The set $(\mathbf{M}^{5002})$ of measured points was obtained and the following form error was determined

   $$FE_{BC} = [-6.6, 38.3]\,\mu\text{m},$$

   see tab. 2, measurement 2.1. Such a wide range is caused by the above mentioned influence of boundary of physical freeform surface.

3. **Modification of the calibrated CAD model of the first stage** – the set of control points $(\mathbf{B}^{9409})$ was modified by replacement of the subset $(\mathbf{B}^{5002})$ on subset $(\mathbf{M}^{5002})$. After that, the modified set of control points $(\mathbf{R}^{9409})$ was obtained.

4. **Calibrated CAD model of the second stage** – based on previous experience, the bilinear B-spline surface $\mathbf{K}(u, v) = (x(u, v), y(u, v), z(u, v))$ given by the set of control points $(\mathbf{R}^{9409})$, degrees $p = q = 1$ and knot vectors

   $$U, V = (0, 0, \tfrac{1}{96}, \tfrac{2}{96}, \ldots, \tfrac{95}{96}, 1, 1)$$

   as the calibrated CAD model of the second stage was chosen. Its mathematical expression is given by (1) and by the following condition

   $$\sqrt{[x(u, v)]^2 + [y(u, v)]^2} \leq 37. \qquad (6)$$

   Considering condition (6), the surface $\mathbf{K}(u, v)$ is trimmed by cylinder of revolution with axis of revolution identical with z-axis and radius equal to 37 mm (i.e. 3 mm boundary zone is removed) and the above mentioned unsuitable influence of boundary of physical freeform surface is eliminated.

5. **Calibrated CAD model of the second stage verification** – two test sets of points $(\mathbf{T}^{225})$ and $(\mathbf{D}^{4644})$ uniformly distributed in (x, y) grid and independent on control mesh $(\mathbf{R}^{9409})$ was generated and measured with the calibrated CAD model of the second stage $\mathbf{K}(u, v)$ as the reference surface. The obtained form errors

   $$\begin{aligned} FE_{TK} &= [-1.2, 1.4]\,\mu\text{m}, \\ FE_{DK} &= [-1.5, 1.4]\,\mu\text{m} \end{aligned}$$

Table 1. Calibrated model of the first stage – colour maps of deviations of measured points from the reference CAD models

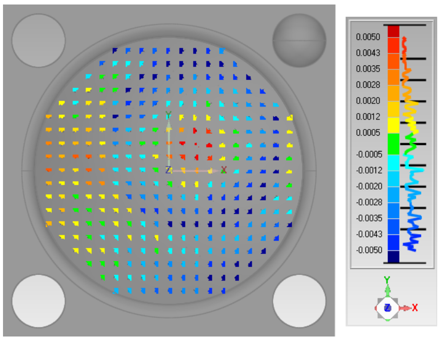
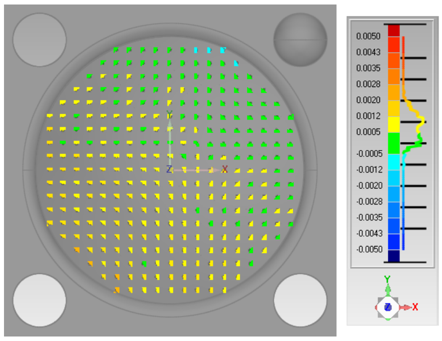
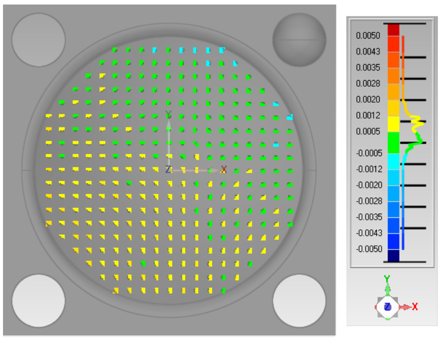
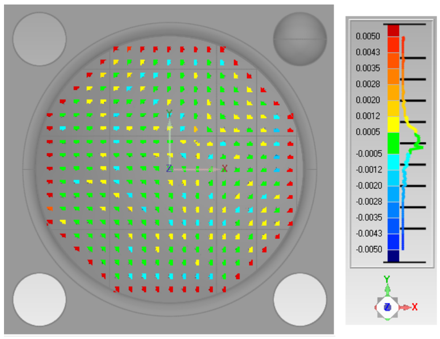
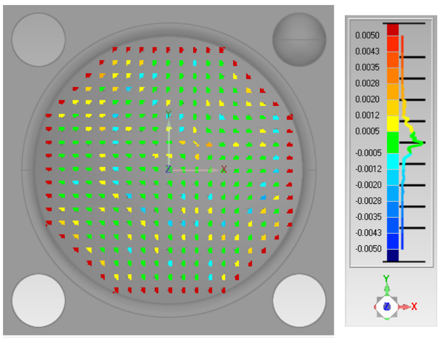
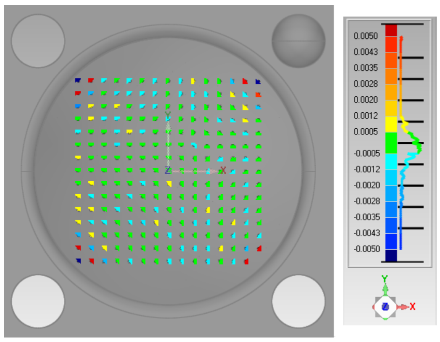
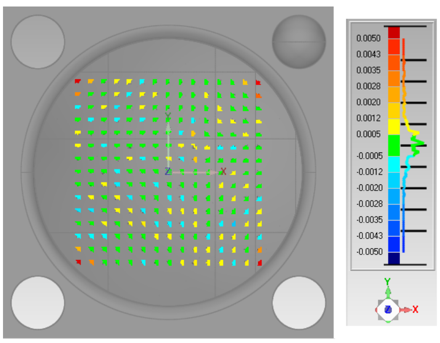
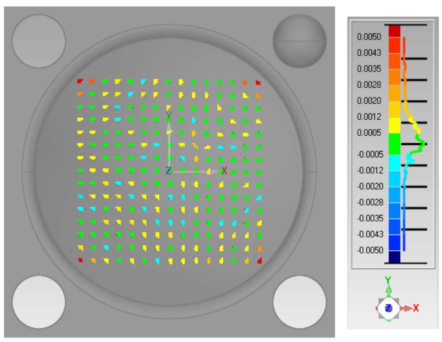
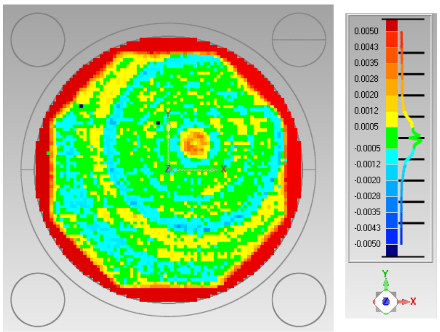
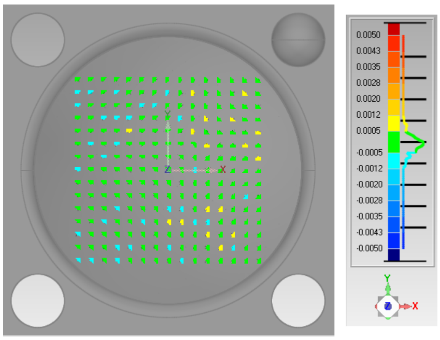
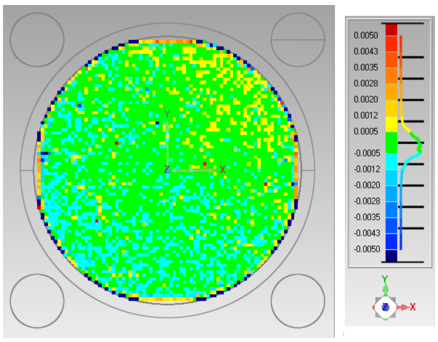
Measu-rement	Colour map	Measu-rement	Colour map
1.1	Input data: $\mathbf{P}^{301}$, reference CAD model: $\mathbf{S}(u, v)$ 	1.2	Input data: $\mathbf{P}^{301}$, reference CAD model: $\mathbf{L}(u, v)$ 
1.3	Input data: $\mathbf{M}^{301}$, reference CAD model: $\mathbf{L}(u, v)$ 	1.4	Input data: $\mathbf{M}^{301}$, reference CAD model: $\mathbf{Q}(u, v)$ 
1.5	Input data: $\mathbf{M}^{301}$, reference CAD model: $\mathbf{C}(u, v)$ 	1.6	Input data: $\mathbf{T}^{225}$, reference CAD model: $\mathbf{L}(u, v)$ 
1.7	Input data: $\mathbf{T}^{225}$, reference CAD: $\mathbf{Q}(u, v)$ 	1.8	Input data: $\mathbf{T}^{225}$, reference CAD: $\mathbf{C}(u, v)$ 

Table 2. Calibrated model of the second stage – colour maps of deviations of measured points from the reference CAD models

Measu-rement	Colour map
2.1	Input data: $\mathbf{B}^{5002}$, reference CAD model: $\mathbf{C}(u,v)$ 
2.2	Input data: $\mathbf{T}^{225}$, reference CAD model: $\mathbf{K}(u,v)$ 
2.3	Input data: $\mathbf{D}^{4644}$, reference CAD model: $\mathbf{K}(u,v)$ 

were comparable with measurement uncertainty and repeatability of SIP CMM 5 machine. Thus, the calibrated CAD model of the second stage $\mathbf{K}(u,v)$ was confirmed.

4. CONCLUSIONS

The paper reports on individual steps of calibration process which has been developed to obtain calibrated CAD model of freeform standard. Due to the geometrical-mathematical properties of the standard, the calibration process is based on two-stages modification of CAD model according to the coordinates of surface points measured on the real physical standard. The original theoretical shape of the standard as well as the calibrated one are described by B-spline representation. The final values of form error obtained by CAD-based measurement of the physical standard with respect to the developed calibrated CAD model are comparable with measurement uncertainty and repeatability of coordinate measuring machine on which the measurement has been performed. Thus, the suggested method is fully confirmed.

If a more precise correspondence of real standard and its CAD model is required, it is possible to extend the method and consider calibrated CAD model of the third stage. However, in this case the machine time is necessary to take into consideration. For example, the measurement of 5 002 data points described in this paper took approximately 14 hours.

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