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MEASUREMENT OF MICRO-SPHERE DIAMETER BASED ON WHISPERING GALLERY MODE

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Abstract – In this study it is proposed the new measurement method for the diameter of a microsphere, which utilizes the whispering gallery mode (WGM). WGMs are the light propagation mode, so it is necessary to determine the mode number to the obtained WGM wavelength for the accurate diameter measurement. Using wide wavelength bandwidth and polarization control, it was possible to identify the mode number of the detected WGMs, which was conformed by comparison with the theoretical values. Finally, the diameter measurement was implemented and the results were compared with other measurement instrument, revealing the performance of the proposed technique, repeatability was greater than 10 nm and resolution was less than 1 nm.

Keywords: Sphere, diameter, whispering gallery mode, resonance

1. BACKGROUND

In recent years, with miniaturization of products and parts, the 3-dimensional (3D) metrology has to be expanded to the micrometer scale [1,2]. Thus, instruments evaluating the 3D geometrical quantities of small parts such as dimension, size, tolerance have been developed [2]. The most promising instruments are micro-coordinate measuring machine (micro-CMM) [3], micro-X ray coherent tomography (micro-XCT) [4] and optical-CMM [5]. Micro-CMM is able to measure 3D geometry of small parts. However, small probing system for micro-CMMs has not yet matured. Downsizing a probe tip smaller than 50 µm and calibration of probe tip are challenging issues. Currently measurement uncertainly of the micro-CMMs is approximately 100 nm. Micro-XCT, which is highly studied recently, can image total 3D structure of the parts including the inside. Crucial drawback of the micro-XCT is the resolution that is still on the order to 100 nm. Development of micro-XCT is fundamental stage for metrology so that there are many issues such as measuring parts with multi materials, algorithm to reconstruct the product surface. Optical-CMM based on focus variation method is recently proposed for 3D metrology. Optics based microscopies are strong tools for surface metrology but many issue to be addressed for 3D metrology, for example, optical penetration depth and diffraction limit.

Calibration is necessary process for the dimensional metrology. A sphere is frequently used as a reference for calibration in the 3-dimensional metrology because of its isotropic shape. Assuring an accuracy of the reference sphere is responsible for measurement uncertainty of 3D metrology, that is, sphericity and diameter of the sphere need to be guaranteed. For micro-scale 3D metrology, size of the reference sphere is also micro-scale from several hundred micrometers to a few tens of micrometers, which has to be measured with accuracy of better than 10 nm.

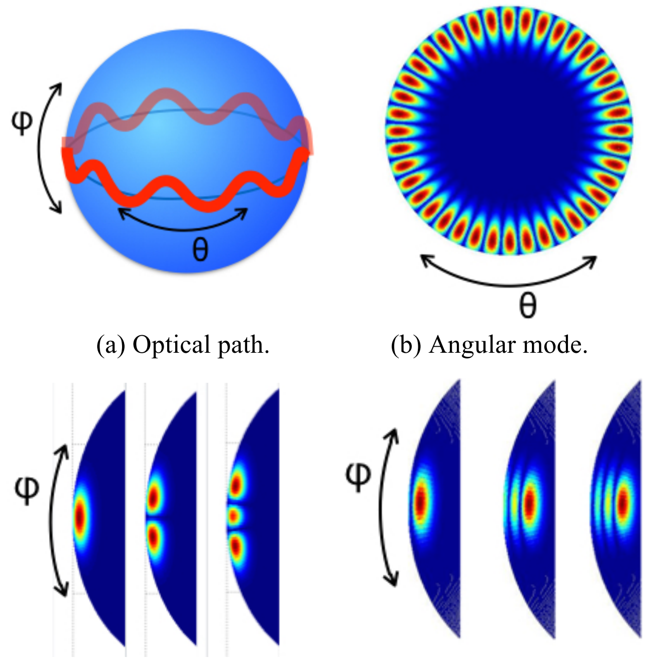
A macro-scale large sphere can be measured by means of interferometric technique with high accuracy of a few nanometer because the surface of the sphere can be treat as flat surfaces. For the interferometric method, however, it is difficult to determine the diameter of the micro-scale sphere due to the curvature of sphere surface, although still sphericity can be evaluated. Therefore, nowadays, measurement of a diameter of the micro-scale sphere is challenging issue. Therefore, we proposed the new measurement principle of a diameter for the micro-scale sphere to achieve 10 nm of the measurement accuracy.

2. MEASUREMENT PRINCIPLE BASED ON WHISPERING GALLERY MODE

2.1. Whispering gallery mode

As a measurement principle for the diameter of a microsphere, a new method using whispering gallery mode (WGM) has proposed [6]. WGMs are the light propagation mode inside the sphere. As shown in Figure 1 (a), the light propagates along the equatorial line of the sphere in WGMs. When an integer multiple of the wavelength of the propagating light is equivalent to the circumference of a sphere of the light, the light resonates, which is so-called WGM resonance. Based on the resonant wavelengths (WGM wavelength), it is possible to estimate the length of circumference, that is, the diameter of the sphere.

Geometrically the sphere is three-dimensional shape. Therefore, there are three kinds of mode as WGMs. First one is the angular mode (Figure 1 (b)), which means the number of the light waves along the traveling direction of the light. Other two modes related to extent of the electromagnetic field along the transverse to the traveling direction of the light. Depending on the direction of the extent, they are classified such as the azimuthal mode (Fig. 1 (c)) and the radial mode (Fig. 1 (d)). For the azimuthal and radial mode, the smallest spread of the electromagnetic field, which are the left side figure in Fig. 1(c) and (d), are here called the fundamental WGM (F-WGM). F-WGM has the highest mode number for the azimuthal mode and the lowest



(a) Optical path. (b) Angular mode.

(c) Azimuthal mode. (d) Radial mode.

Figure 1 Whispering gallery mode.

for the radial mode. By detecting the F-WGMs, it is possible to identify the mode number for these two modes.

In addition to the mode number, polarization of WGMs has to be concerned. Depending on the direction of the electromagnetic field, they are distinguished to the transverse electric (TE) or transverse magnetic (TM) WGM. For TE-(TM-) WGM, the electric (magnetic) field is parallel to the surface of a sphere.

To estimate the diameter of the sphere, firstly, WGM wavelengths are measured and, secondary it is necessary to determine the mode numbers and identify the polarization for the measured WGM wavelengths.

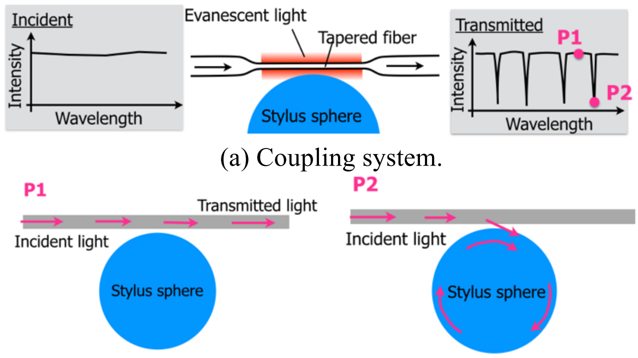
2.2. Measurement principle

In this measurement principle, the diameter of a sphere is estimated based on the analysis of the wavelengths of WGMs. The relationship between the WGM wavelength and the diameter of the sphere can be obtained by solving the following dispersion equation.

$$\frac{1}{j_l(\rho_1)}\frac{\partial[\rho_1 j_l(\rho_1)]}{\partial\rho_1}=\frac{c}{h_l^{(1)}(\rho_2)}\frac{\partial[\rho_2 h_l^{(1)}(\rho_2)]}{\partial\rho_2} \qquad (1)$$

Here, $j_l(\rho)$ is the spherical Bessel function, $h_l^{(1)}(\rho)$ is the spherical Hankel function, subscript l is the angular mode number, n is the refractive index (the subscript 1 and 2 means the sphere, surrounding air). $\rho_{1,2} = \pi d n_{1,2} / \lambda_0$ is the size parameter, λ_0 is the wavelength of light in vacuum, d is the diameter of the sphere. c is the polarization dependent factor that $\mu_2 n_1^2 / \mu_1 n_2^2$ for TE mode and μ_1 / μ_2 for TM mode. Wavelength dispersion of the refractive index is corrected by Sellmeier's equation.

In a perfectly spherical shape, azimuthal mode is degenerated to the F-WGM. However, the degeneracy is broken in the distorted sphere, appearing the low-order



(a) Coupling system.

(b) Decoupled. (c) Coupled.

Figure 2 Light coupling and detection system for whispering gallery mode.

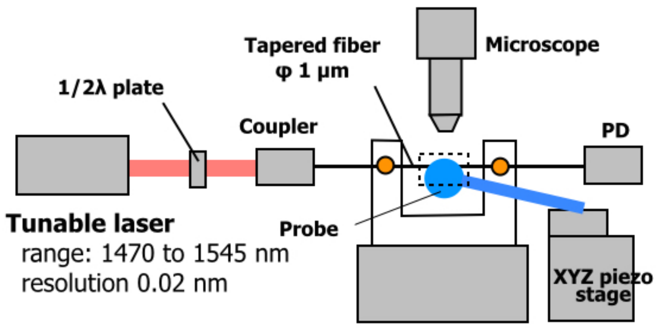


Figure 3 Experimental setup.

azimuthal mode [7]. Generally a sphere to be measured is not a true sphere, then the number of azimuthal modes can be found in measurements. Therefore, F-WGMs have to be distinguished from other modes. F-WGMs can be detected using the fitting algorithm, which is that the several WGM wavelengths obtained in experiments are compared with the theoretical WGM wavelengths.

3. MEASUREMENT SYSTEM

3.1. WGM coupling into the sphere

To excite the WGMs in a microsphere, it is necessary to couple the light from the outside of the sphere. A tapered optical fiber is used to introduce the light. The tapered optical fiber is an optical fiber that a partially thinned optical fiber by several μm in diameter. At the narrow portion, the light propagates in the fiber, being reflected totally between the fiber and the air. Therefore, the evanescent light appears to the outside of the optical fiber. The light enters into the sphere with evanescent coupling such as Fig. 2(a). In order to find the WGM wavelengths, the incident wavelengths are swept. When the wavelength of the incident light matches the WGM wavelength, the light is strongly coupled to and resonated in the sphere. Thus, at the WGM wavelength, the light transmitted to the other end of the optical fiber is significantly reduced. Therefore, the WGM wavelengths can be measured via the transmitted light.

3.2. Experimental setup

Figure 3 shows the schematic diagram of the experimental apparatus. The tapered fiber is fixed to the U-shaped holder. The tunable external cavity laser diode (wavelength resolution: 0.02 nm, tunable range: 1475 nm to 1545 nm) is used as the light source. After passing through the half-wave plate, the light is coupled into the optical fiber. At the other end of the fiber, the transmitted light intensity is measured with the photodiode (PD). The measured microsphere is fixed to the holder on the XYZ piezo-stage, controlling 3-dimensional position. Positional relationship between the optical fiber and the microsphere is observed with a microscope system installed above the fiber and sphere.

4. ANALYSIS OF WHISPERING GALLERY MODE

4.1. Determination of fundamental WGM

In the case measuring the WGMs in not truly spherical (distorted) sphere, various modes are appeared. For the diameter measurement, F-WGMs have to be explored to identify the mode number of the azimuthal and radial mode. Based on numerical analysis of the coupling coefficient, it was found that F-WGM is the most strongly coupled into the sphere rather than the other mode numbers [6]. Strong coupling leads deep peak of the transmitted light intensity.

Coupling strength is also depending on the distance between the optical fiber and the sphere. If the optical fiber is approached nearly in contact with the sphere, the modes with low coupling coefficient are also appeared at peak signals, which also implies that the modes with high coupling coefficient can be found at relatively far distance. Thus, the F-WGM can be found the optical coupling with different distance between the fiber and the sphere. In experiment, a transmitted light signal was obtained while the optical fiber is gradually getting closer to the microspheres.

A glass (BK7) microsphere in a diameter of 90 μm is used for the specimen. The microsphere was approached to the optical fiber at the step of 100 nm. At each position, the wavelength was swept from 1500 to 1510 nm to detect the WGM wavelengths. Measured light spectrum was shown in Fig.4. Fig.4 (a) is the transmitted light intensity at initial position (0 nm) of the microsphere. When the microsphere is closing to the optical fiber, the peaks due to the WGM resonance were started to appear. As approaching the sphere to the fiber, the number of the peak to be found is increased. As a result of the contact of the sphere with the optical fiber at 600 nm (Fig.4 (d)), the numerous peaks can be found and transmittance are reduced because of the scattering. Appropriate adjustment of the distance of the sphere and the fiber can excite the only the mode with high coupling coefficient such as Fig.4(b). Comparison with the theoretical value, 1501.46 nm is expected to the wavelength of the F-WGM wavelength. Thus, appeared strong peaks adjusting the distance of the fiber and sphere are recognized to be F-WGM. By detecting the F-WGMs, the mode numbers can be identified for both the radial and azimuthal mode.

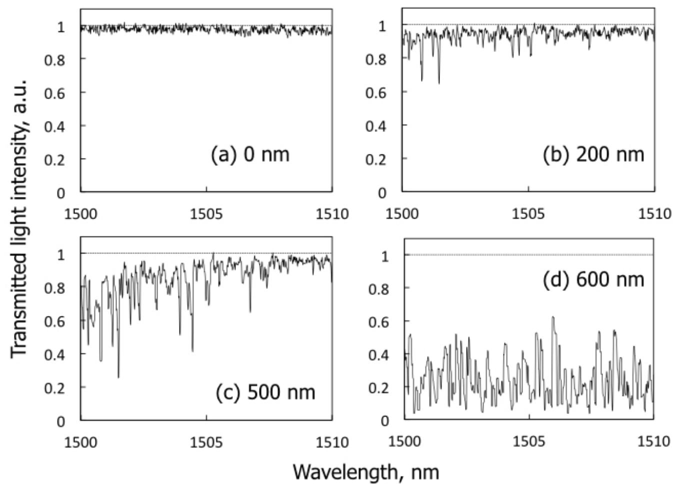


Fig.4: Transmitted light spectrum with different distance between the fiber to the microsphere.

4.2. Determination of angular mode number

The angular mode number for the WGM wavelength obtained in experiments is estimated by fitting a theoretical wavelength. The solid lines in Fig.5 shows the relationship between the diameter of the sphere and the F-WGM wavelength of SiO_2 glass sphere derived using equation (1). Small letter of L in the figure means the angle mode number. Suppose the diameter is assumed to be 80.5μm, the intersection of the black dashed and solid line is the F-WGM wavelength for each angular mode number. Considering the long wavelength range (upper side of the figure), the red dotted horizontal arrows show three of the F-WGM wavelength for a sphere with diameter of 80.5μm. As the three F-WGM wavelengths are obtained in experiments, there are several combinations to diameters (red dotted vertical line). Thus, it is hard to identify the angular mode numbers to the F-WGM wavelengths. Therefore, two wavelength ranges are used in.

In 3D optical resonator, optical mode density becomes larger with the shorter optical wavelength range. Then the F-WGM becomes dense at shorter wavelengths, and free

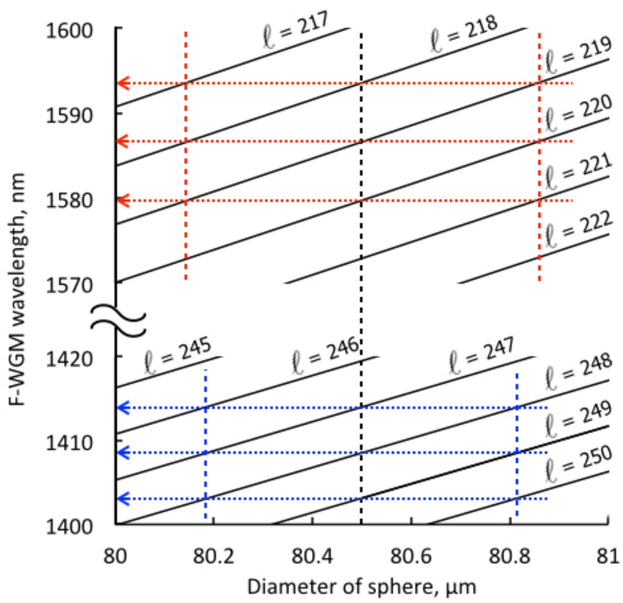


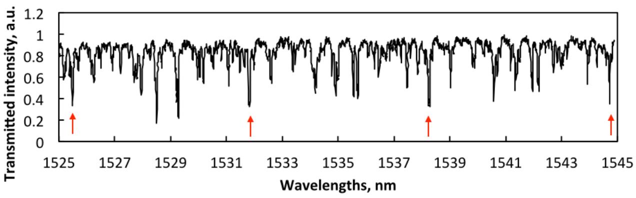
Fig.5: Relation of diameter of sphere and F-WGM wavelength.

spectral range (FSR) is narrowed. Thus, the candidate diameters corresponding to the obtained angle mode numbers are different for two separated wavelength ranges. Consequently only the true diameter is coincident. Thus, the angular mode number can be determined by comparing the two wavelength ranges.

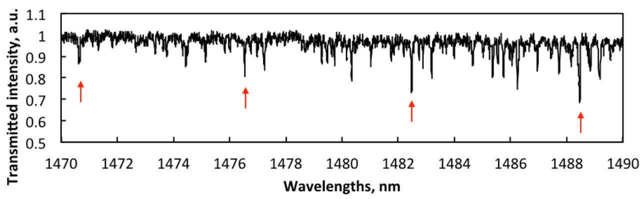
This method was experimentally confirmed. A glass sphere (BK7) of a diameter of about 80 μm is used. Sweeping range of the incident wavelength is 20 nm for each that 1470 to 1490 nm and 1525 to 1545 nm. After adjusting the distance of the microspheres and the optical fiber, the transmitted light spectrum was measured. The results are shown in Fig. 6. The red arrows indicate the F-WGM wavelengths, which are summarized in Table 1, 2 as Measured F-WGM wavelength. The candidate diameter in Table 1, 2 was estimated with allocated the angular mode numbers. The candidate diameters of both wavelength ranges are quite agreed well at 75.867 nm although the differences get larger for different set of angular mode number. Thus, the diameter and the angular mode numbers are estimated by using two wavelength ranges.

4.3. Determination of polarization of WGMs

The polarization of the excited WGMs depends on the polarization in the optical fiber. Therefore, the polarization of the WGMs can be controlled by the incident polarization to the optical fiber. The incident angle of polarization of the



(a) Longer wavelength.



(b) Shorter wavelength.

Fig.6: Transmitted light spectrum with different wavelength bandwidth.

Table 1: Measured result for longer wavelength.

Measured F-WGM wavelength	Angular mode number				
1525.49 nm	245	246	247	248	249
1531.84 nm	244	245	246	247	248
1538.24 nm	243	244	245	246	247
1544.73 nm	242	243	244	245	246
Candidate diameter	75.224 μm	75.546 μm	75.866 μm	76.187 μm	76.508 μm

Table 2: Measured result for shorter wavelength

Measured F-WGM wavelength	Angular mode number				
1470.65 nm	258	259	260	261	262
1476.53 nm	257	258	259	260	261
1482.48 nm	256	257	258	259	260
1488.46 nm	255	256	257	258	259
Candidate diameter	75.249 μm	75.558 μm	75.867 μm	76.176 μm	76.485 μm

incident laser (linear polarization) was rotated from 0 to 180° for each 30°. The microsphere used is a glass sphere (SiO_2) with a diameter of about 70 μm. Sweeping range of the incident wavelength was from 1480 to 1500 nm. After adjusting the microspheres and the optical fiber length, and was measured the transmitted light spectrum. The most clear peaks were found when the incident polarization angle of 0°and 90°, which shown in Fig.7. Theoretically, at the same angular mode number, TE-WGM is appeared at around 5 nm longer wavelength than the TM-WGM when the diameter of the sphere is considered 70 μm. Therefore, upper data in Fig.7 was considered as the TE-WGM, and lower was TM-WGM, and then as a result of calculating the diameters, the results were well agreed at 67.651μm. Therefore, different polarization of WGMs were excited by rotating the polarization angle of the incident laser. Thus, controlling the incident polarization, TE- or TM-WGM are excitable selectively.

5. DIAMETER MEASUREMENT

In order to verify the performance of the proposed method, the measurement result was compared with the result by coherent scanning interferometer (CSI). The measured sphere was made by melting the thinned optical fiber, that is, a material of the sphere was quartz glass and has a diameter of around 65 μm by optical microscope.

First, the diameter was measured by the proposed method. Direction of the polarization of the incident laser was changed with a step of 30° from 0° to 90° to determine the polarization mode of F-WGM. Sweeping range of the wavelength of the incident laser was 1470 to 1500 nm and 1515 to 1545 nm. Detected wavelengths of the F-WGMs are summarized in Table 3. Comparing these wavelengths with the theoretical values, the angular mode number was estimated as shown also in Table 3. Finally the estimated diameter was 65.244 μm with maximum variation of 10 nm. For comparison, the diameter was measured with the CSI. The distance between the surface of the substrate and the summit of the sphere was measured as the diameter. Measurement was implemented 3 times. The average was 66.41 μm with 205 nm differences at maximum. Repeatability of the WGM based method was much better than the CSI, although the average diameter was different around 0.8 μm. The CSI requires calibration of the piezo-stage. Uncertainty analysis is indispensable to reveal that difference of the measurement result.

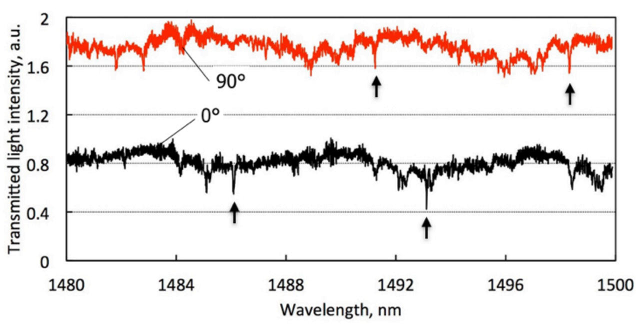


Fig.7: Transmitted light spectrum with different polarization directions of input light.

Table 3: Detected wavelength of F-WGM.

Measured F-WGM wavelength, nm	Estimated angular mode number	Estimated diameter, μm
1474.34	191	65.247
1481.65	190	65.247
1489.15	189	65.247
1496.74	188	65.248
1504.30	187	65.244
1512.01	186	65.243
1519.80	185	65.242
1527.66	184	65.240
1535.57	183	65.238
Estimated diameter		65.244 μm
Standard diviation		4 nm
Maximum variation		10 nm

6. CONCLUSIONS

In this study, new diameter measurement technique using WGMs was proposed for a microsphere. A light resonates in the measured sphere. By measuring resonant wavelengths, the diameter can be measured. In this paper, estimation methods of mode numbers of WGM and polarizations of WGM have developed. Using this technique, the diameter of the micro-scale sphere could be measured with variation of 10 nm.

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