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DEVELOPMENT OF CAA CORRECTION MATRIX FOR COORDINATE MEASUREMENT ARMS

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Abstract – Coordinate Measuring Arms (CMA) are devices that differ in many respects from conventional coordinate measuring machines (CMM). They have from 5 to 7 rotating pairs, in which the encoders are placed. These devices are manual, mobile, redundant, and at least 3 axes have a full rotation. Therefore, in this article, the method of development of CAA correction matrix for the CMA is described. What is a completely different issue of the development of CAA correction matrix for the conventional CMM.

Keywords: Coordinate Measuring Arm, Coordinate Measuring technique, CAA correction matrix

1. BASIC INFORMATION

CAA correction matrices are used to programmed correction of geometric errors of coordinate measuring machines. All of these errors affect the indication error. For the measuring machines the indication error e is calculated as the ratio of vector of 21 units of geometric errors k and matrix M showing the impact of each of these errors on the units x, y, z of machine indication error. Until recently, the development of this matrix was the domain of only the producers of the systems. Currently, the matrices shall be made available to users. This allows users to develop their own error correction, and thus improve the accuracy of measurement systems. Development of the quantities of the each errors using coordinate measuring systems allows to build a virtual measuring machines [1,2,3].

There is no doubt that the different structure of the CMA, forces the user to a different approach to the issues related to the development of geometric errors of these devices and also CAA model. Correction matrix for CMA will be based on simple kinematic task. In conventional CMM, geometric errors, derived from sliding pairs, can be described using the 21 units such as:

- 3 position deviations of each axis (xtx, yty, ztz) also denoted as (xpx, ypy, zpz)
- 6 straightness deviations of machine axis (xty, xtz, ytx, ytz, ztx, zty);
- 3 rotation deviations of axis (xrx, yry, zrz);
- 6 mutual twisting axis deviations (xry, xrz, yrx, yrz, zrx, zry);
- 3 squareness deviations of CMM axis (xwy, xwz, ywz).

For CMA in each rotating pair these errors will be as follows (Fig 1):

- ztz characterizing the shift error along the rotation axis of encoder;
- ξ_i characterized the shift "zero" of encoders;
- xtx characterized the shift along the each units of CMA;
- zrx characterized torsion of axis z with the axis x.

To determine the values of errors that occur in rotating pairs of CMA, provide a kinematic diagram of the CMA, and then describe it in accordance with the Denavit-Hartenberg notation (DH). From the CMA software we can get data such as: a) the encoder readings, b) directing of the stylus, c) coordinates of the measuring tip. Using a simple kinematic task we can build the equations with the many uncertainties that correspond to the geometric parameters of the CMA. Reading the indicated encoders and the coordinates of points and referral stylus from with the entire measuring space we are able to determine the actual geometric parameters of CMA[3,4,5,6].

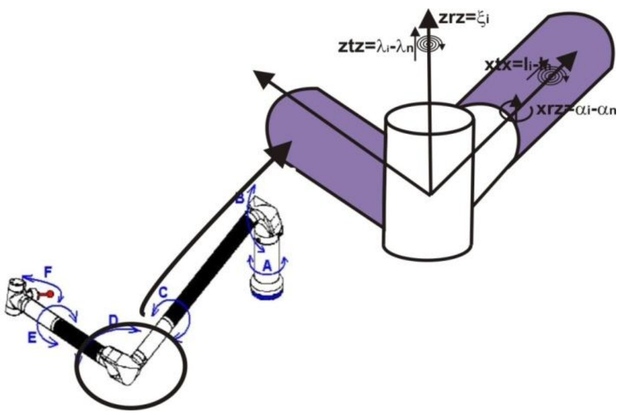

Fig.1. characteristic errors of rotation pairs in CMA
where
λ_n- nominal length (founded by designer),
λ_i - length read from the individual kinematic pairs,
l_n - nominal length,
l_i - length read from the individual kinematic pairs,
ξ_i - offset of zero of encoder of each configuration parameters
α_n - nominal length,
α_i - Value of twisting axis in each pairs

2. CMA KINEMATISC

The basis for CMA kinematics analysis is the description in accordance with the Denavit-Harterberg notation. This notation binds local coordinate system associated with each joint, and then determines the sequence of transformations of neighboring coordinate systems and leads to the calculation of the kinematics of the device as the submission of these transformations.

2.1. Development of the kinematic scheme

To determine the kinematic errors of CMA, and thus to analyze the geometric accuracy of the arms, we must build a kinematic model of the device. This model must accurately reflect the behavior of the actual machine.

On the basis of the actual construction of the measuring arm and data from technical documentation the kinematic model was developed (Fig. 2), in the D-H coordinates [7,8].

On the basis of the CMA scheme the matrices of subsequent transformation of coordinate systems were determined.

$$A_i^{i-1} = \begin{bmatrix} \cos\theta_i & -\cos\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i & l_i \cos\theta_i \\ \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i \cos\theta_i & l_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & \lambda_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

where

α_i - the angles between the axes *z*,

θ_i - configuration coordinates which are the angles between the axes of x,

l_i - length measured along the axis *x*,

λ_i - length measured along the axis *z*.

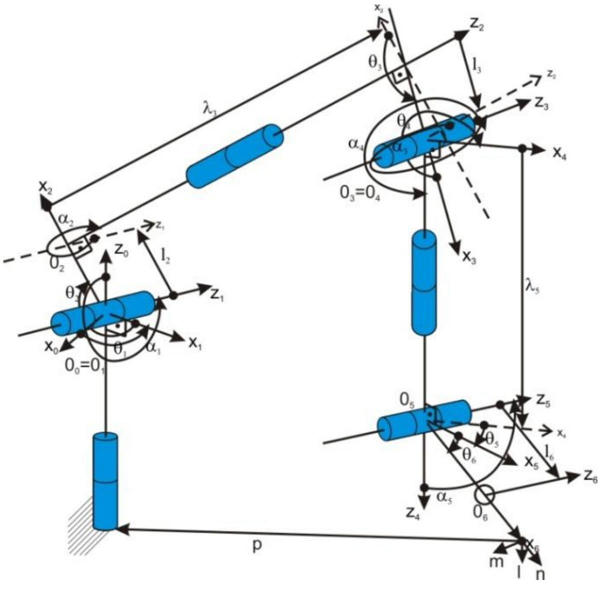

Fig. 2. Kinematic scheme of CMA

The matrix A contains information about the position and orientation of the individual units, using matrix algebra:

$$A_i^{i-1} = \begin{bmatrix} B & & & p \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

$$B_i = [l_i \quad m_i \quad n_i] = \begin{bmatrix} l_{ix} & m_{ix} & n_{ix} \\ l_{iy} & m_{iy} & n_{iy} \\ l_{iz} & m_{iz} & n_{iz} \end{bmatrix} \quad (3)$$

$$p_i = [p_{ix} \quad p_{iy} \quad p_{iz}]^T \quad (4)$$

where

B – orientation matrix of individual units,

p- position matrix of individual units,

p_x, p_y, p_z - location coordinates corresponding unit,

$l_{ix}, l_{iy}, l_{iz}, m_{ix}, m_{iy}, m_{iz}, n_{ix}, n_{iy}, n_{iz}$ - coordinates of the vector describing the rotation of the individual units.

2.2. Development of a simple and inverse kinematic task

The main job in the mechanical manipulation study is the so-called kinematics simple task. It is the static and geometric task, which is sometimes treated as a task of mapping of description of the device location from the configuration coordinates space to the description of the Cartesian coordinates space.

Position and orientation matrix of the *n* system relative to the system associated with the unit *i* can be written as the product of a sequence of transformations:

$$T_{n,i} = A_{i+1} A_{i+2} A_{i+3} \ldots A_n \quad (5)$$

where

A- matrix describing the location and orientation of the individual members,

T- transformation matrix of matrices A.

Inverse kinematics task of CMA consists in determining a set of values of angular or linear displacement (configuration coordinates) in the kinematic pairs that provide specific position and orientation of the working unit (in the case of measuring arms stylus).

Developing of inverse kinematics task of CMA we base on the equation (5) and calculate the values of the variables $\theta_1, \theta_2, \theta_{3,\ldots,} \theta_n$ and then consider the solvability task due to the:

- workspace;
- multiple solutions [9,10,11].

3. ALGORITHM OF DEVELOPMENT OF GEOMETRICAL PARAMETERS

Determination of the geometric accuracy of the CMA, is a multistep and task as well as the most time-consuming and the most difficult part of the entire implementation. However, most of the tasks we do once, and the remaining ones we repeat after the device calibration or after it is completely dismantled and re-assembled.

3.1. Determination of the mathematical model

The first step is to determine, in a symbolic way, the simple kinematics task. This task has saved information about the location and orientation of the stylus. Then we build a system of equations which comprises 28 unknowns describing the parameters needed to characterize the individual units of the kinematic diagram, recorded in accordance with the D-H notation. Algorithm of development of geometrical parameters is shown in fig. 3.

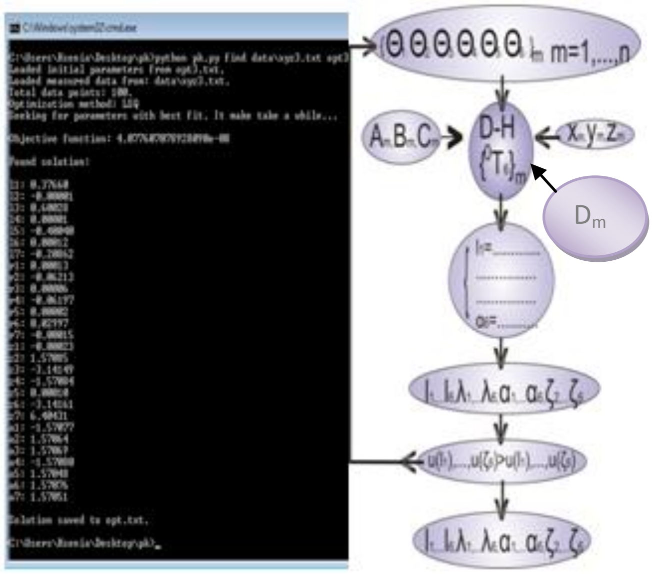

Fig. 3 The algorithm for calculating the geometric accuracy of the CMA
where *(*Θ_m – readings of angular measurement systems, A_m, B_m, C_m - unit vectors of referral stylus, X_m, Y_m, Z_m - coordinates obtained from the manufacturer's CMA software, X_{ni}, Y_{ni}, Z_{ni}- coordinates obtained from the length artifact, lm – distance from zm-1--axis to zm-axis measured along the xm-axis, αm – the angle between the zm-1-axis and zm-axis measured around the xm-axis, λm – distance from xm-1-axis to the xm-axis measured along the zm-1-axis). D_m- Length read from the artifakt.

3.1. Measurement strategy

Fig.4. The measurement strategy with the designed artifact adapted to the measuring tip.

To obtain the values of individual, geometrical units of the device, we measure the artifact allowing for unambiguous reading of length. The measurement is carried out across the whole measurement space.
From the manufacturer's software, we can read the configuration coordinates θ_i (readings from the encoder), A, B, C (unit vector describing the referral stylus), as well as cartesian coordinates x, y, z. Then we read the nominal data from the length artifact x_n, y_n, z_n. After obtaining the data from the actual measuring device and from the artifact, we substitute the data to the system of equations built on the formula (6):

$$D_1 = \sqrt{\left(X_{1S} - X_{2S}\right)^2 + \left(Y_{1S} - Y_{2S}\right)^2 + \left(Z_{1S} - Z_{2S}\right)^2} \qquad (6)$$

where
D1 - distance of two points on the length artifact,
*X*1s - formula on the X coordinate of point 1, on the artifact determined from the simple tasks,
*X*2s - formula on the X coordinate of point 2, on the artifact determined from the simple tasks,
*Y*1s - formula on the Y coordinate of point 1, on the artifact determined from the simple tasks,
*Y*2s - formula on the Y coordinate of point 2, on the artifact determined from the simple tasks,
*Z*1s - formula on the Z coordinate of point 1, on the artifact determined from the simple tasks,
*Z*2s - formula on the Z coordinate of point 2, on the artifact determined from the simple tasks,
As, for the calculation we have 28 variables, we have to develop a minimum of 28 equations, and each of the equations will be built in an analogous way.

3.3. Presentation of the results

As a result of a series of calibration measurements and calculations listed in the programming language, and based on the measurement of the length, we get 28 variables that describe the errors of parameters of the CMA (Fig. 3). Parameters xtx_i, ztz_i, xrx_i and ξ_i are characterized as an unchangeable errors for given configuration of the measuring device. The encoder displacements or its mounting errors are characterized by ξ_i errors. In a futher stage of works another parameters corresponding to errors in whole measuring volume will be obtained. They will be different accordingly to the measuring volume part. One additional parameter to every encoder.

Errors characterized the shift along the each units of CMA:

$$xtx_i\ [mm] = \begin{pmatrix} 0,0629 \\ 0,2482 \\ -0,5457 \\ -0,0877 \\ -0,3595 \\ -0,1295 \\ -0,7545 \end{pmatrix}$$

Errors characterizing the shift error along the rotation axis of encoder:

$$ztz_i\ [mm] = \begin{pmatrix} -0,1295 \\ 0,0468 \\ -0,0349 \\ 0,0155 \\ 0,0062 \\ -0,0063 \\ 0,0063 \end{pmatrix}$$

Errors characterized torsion of axis z with the axis x:

$$zrx_i\,[°]=\begin{pmatrix}0,0124\\0,0141\\0,0040\\0,0037\\0,0066\\0,0064\\0,0021\end{pmatrix}$$

Errors characterized the shift "zero" of encoders:

$$\xi_i[°]=\begin{pmatrix}0,0137\\0,0137\\-0,0097\\-0,0111\\0,0010\\-0,0031\\0,5894\end{pmatrix}$$

All of these errors affect the indication error e. For CMA the CAA matrix will be:

$$e=k*M \quad (7)$$

where

$$e=[e_x, e_y, e_z], \quad (8)$$

$$k=[xtx_i, ztz_i, xrx_i, \xi_i], \quad (9)$$

$$M=[x_{zp}, y_{zp}, z_{zp}]^T \quad (10)$$

and

xtx_i, ztz_i, xrx_i, ξ_i- units of errors occurring in individual rotation pairs

x_{zp}, y_{zp}, z_{zp} –coordinates from simple task

e - indication erros, k - units of geometric errors, M - matrix of interactions of each of these errors on the units x, y, z of machine indication error.

4. CONCLUSIONS

Determination of the correction matrix for the CMA is a different issue than for conventional CMM. The operator must take into account completely different approach to the problem because CMA does not have in its design rotating pairs only sliding ones and because they are redundant devices [12,13,14,15,16].

The study will help to build a virtual machine and to carry out the correction of the measuring device which is CMA. The algorithm determining the parameters is designed so that it allows to use it for all devices with open kinematic chain.

In the next stage of works authors are going to construct a special standard for mapping the errors in the whole CMA measuring volume.

Research on the CMA are conducted within the research project “Accuracy assessment system of on-line coordinate measurements implemented using redundant devices”.

REFERENCES

[1] E. Ratajczyk, *Coordinate Measuring Technique, Współrzędnościowa technika pomiarowa*, Warsaw University of Technology, Poland, 2008.

[2] J. Sładek, “Errors identification and measurement accuracy assessment of coordinate measuring machine (CMM), *Advance in technology of Machines and Equipment*, vol. 23, n^o. 4, pp. 113-136, 1999.

[3] J. Sładek, K. Ostrowska, A. Gąska „Virtual Portable Arm” „Wirtualne Współrzędnościowe Ramię Pomiarowe (WWRP)” *Measurement Automation and Monitoring, Pomiary Automatyka Kontrola*, vol. 56, n^o. 1, pp. 75-77, 2010.

[4] J. Sładek, K. Ostrowska, K. Gacek, M. Bryndza, „*Designation of operator impact on errors of measurements realized by coordinate measuring arm*”, Advances in Coordinate Metrology, pp. 130-137, University of Bielsko-Biała, Bielsko-Biała 2010.

[5] A. Gąska, J. Sładek, K. Ostrowska, R. Kupiec, M. Krawczyk, W. Harmatys, P. Gąska, M. Gruza, M., D. Owczarek, R. Knapik, A. Kmita “Analysis of changes in coordinate measuring machines accuracy made by different nodes density in geometrical errors correction matrix“ Measurement, vol. 68, pp. 155–163, May 2015.

[6] Z. Kotulski, W. Szczepiński *Errors calculation for engineers, Rachunek błędów dla inżynierów*, Wyd. Naukowo-Techniczne, Warsaw, 2004.

[7] Craig John J. *Introduction to Robotics, Mechanics and Control, Wprowadzenie do robotyki, Mechanika i sterowanie*, Wydawnictwo Naukowo – Techniczne, Warsaw, 1995.

[8] K. Ostrowska *The method of assessing the accuracy of measurements carried out with the use of coordinate measuring arm, Metoda oceny dokładności pomiarów realizowanych przy zastosowaniu Współrzędnościowych Ramion Pomiarowych*, PhD Thesis, Cracow University of Technology, Cracow, 2009.

[9] J. Sładek *Modeling and assessment of the accuracy of machines and coordinate measurements, Modelowanie i ocena dokładności maszyn i pomiarów współrzędnościowych*, Cracow University of Technology, Cracow, 2001.

[10] E. Ratajczyk *Coordinate measuring technique. Measuring machines and robots, Współrzędnościowa technika pomiarowa. Maszyny i roboty pomiarowe*, Warsaw University of Technology, Warsaw, 1994.

[11] K. Ostrowska, „Methods for assessing the accuracy of hybrid measuring machines and the use of them“, „Metody oceny dokładności hybrydowych maszyn pomiarowych i ich zastosowanie”, *7th International Conference Automation in Production Planning and Manufacturing*, pp.112-122, Zilina, Slovakia, 2006.

[12] J. Sładek, K. Ostrowska, K. Gacek „Kinematic metrological model of the coordinate measuring arm (MCMA)”, *XIX IMEKO World Congress*, pp. 1987-1992, Lisbon, Portugal, September 2009.

[13] J. Santolaria, J.J. Aguilar, J.A. Yague, J. Pastor “Kinematic parameter estimation technique for calibration and repeatability improvement of articulated arm coordinate measuring machines”, *Precision Engineering*, vol. 32, n^o. 4, pp. 251-268, October 2008.

[14] J. Sładek, K. Ostrowska, A. Gąska „Modeling and identification of errors of coordinate measuring arms with the use of a metrological model” *Measurement*, vol. 46, n^o. 1, pp. 667–679, January 2013.

[15] K. Ostrowska, D. Szewczyk, J. Sładek, “Determination of operator’s impact on the measurement done using coordinate technique” *Advances in Science and Technology Research Journal*, vol. 7, n^o. 20, pp. 16–21, Dec. 2013.

[16] K. Ostrowska, A. Gąska, J. Sładek, “Determining the uncertainty of measurement with the use of a Virtual Coordinate Measuring Arm” *International Journal of Advanced Manufacturing Technology*, vol. 71, no. 1-4, pp. 529-537, March 2014.