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THEORETICAL BACKGROUND OF THE COORDINATE MEASUREMENT UNCERTAINTY ESTIMATION

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Abstract – The paper presents the assumptions and theoretical background of estimation the coordinate measurement uncertainty. The uncertainty for each characteristic is estimated separately. Mathematical model for each characteristic uses the minimal number of points of the measured workpiece. Each characteristic is defined by a formula presented in a form of function of coordinates' differences' of those points. The applied algorithms use the type B method according to the rules of GUM.

Keywords: coordinate measurement uncertainty, task-related uncertainty, type B method, geometrical product specification

1. INTRODUCTION

Commonly known methods for evaluation of uncertainty of coordinate measurements are:

- use of calibrated workpieces [1],
- use of simulation [2].

Both are quite expensive in use. First requires calibrated workpiece and may be used in case of serial production and in measuring laboratories performing calibrations gauges/standards of simple geometry, e.g. plug/ring gauges. Practical use of the second method is possible with the software VCMM developed by PTB and working with CMM software Quindos and Calypso. For wider use, especially in industry, a cheaper and easier solution is necessary.

The paper presents the assumptions and theoretical background of estimation the coordinate measurement uncertainty by the methodology developed by the authors. The uncertainty for each characteristic is estimated separately in accordance with the approach known as “task-specific” or “task related”. Mathematical model for each characteristic uses the minimal number of specially selected characteristic points on the surface of the measured workpiece. Each characteristic of workpiece (dimension, distance or geometrical deviation) is defined by a formula presented in a form of function of coordinates' differences' of those points. The applied algorithms use uncertainty budget in which the component uncertainties are evaluated by the type B method according to the rules of GUM.

2. ASSUMPTIONS

Assumptions for the evaluation of uncertainty of coordinate measurements are as follows. In coordinate measurements it is necessary to evaluate the uncertainty separately for each geometrical characteristic. Characteristic is understood as well defined dimension, distance or geometrical deviation (deviation of form, orientation, location or run-out) [3-5]. Each characteristic is presented in appropriate model by the use of minimal number of characteristic points of the measured workpiece. The points can be surface points, points of axis or centre point of sphere. The points can be toleranced features' points and/or datum features' points. The models of particular characteristics use different number (typically 3–8) of characteristic points. The points should represent the applied probing strategy and be distributed according to the good measurement practice. The models of particular characteristics are mainly derived from the formulae for point–point, point–straight line and point–plane distance [6, 7] but also from other dependencies or geometrical constructions. Each characteristic is defined by a formula presented in a form where it is a function of coordinates' differences' of points of the workpiece [8-10]. Measurement uncertainty is calculated as combined uncertainty (uncertainty of indirect measurement). Weights of uncertainties of particular coordinates' differences are calculated as partial derivatives.

Since the geometrical errors of a CMM are the most important source of measurement errors, the model should use commonly accepted kinematic model of a CMM and the associated model of error propagation [11-13]. The uncertainty budget includes at least the influences of geometrical errors of CMM and probing system errors.

3. MODEL OF THE MEASURED CHARACTERISTIC

The models of coordinate measurement uncertainty estimation are presented on the examples:

- distance between two spheres as point-point distance,
- perpendicularity deviation of axis to plane as point-straight line distance,
- parallelism of axes (in the plane normal to the common plane) as point-plane distance,
- perpendicularity of planes as complex model
- diameter of the circle as other model.

For evaluation of uncertainty of indirect measurement $u_c(y)$ it is necessary to know the measurement model, i.e. a function expressing the measurand y by means of measurands x_i, $i = 1 \ldots N$, which are used to calculate the value of y and uncertainties $u(x_i)$ corresponding to measurands x_i:

$$y = f(x_1, x_2, \ldots, x_N) \tag{1}$$

$$u_c(y) = \sqrt{\sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i)} \tag{2}$$

3.1 Models based on point-point distance

The simplest measurement model based on the point-point distance is the measurement model of the distance between two spheres. It is calculated as follows (Fig. 1):

$$l(A, B) = \sqrt{x_{BA}^2 + y_{BA}^2 + z_{BA}^2} \tag{3}$$

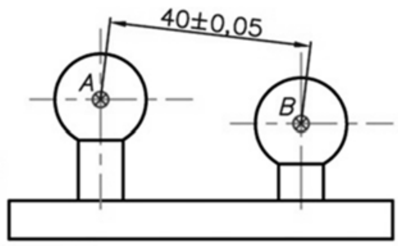

Fig. 1. Distance between two spheres

In all the formulae a simplified notation of the difference of coordinates of two points is used – for example: $x_{BA} = x_B - x_A$.

The model (3) consists of 3 measurands: x_{AB}, y_{AB}, z_{AB} which leads to the following formula for the combined measurement uncertainty

$$u_c(l) = \sqrt{\left(\frac{\partial l}{\partial x_{BA}}\right)^2 u^2(x_{BA}) + \left(\frac{\partial l}{\partial y_{BA}}\right)^2 u^2(y_{BA}) + + \left(\frac{\partial l}{\partial z_{BA}}\right)^2 u^2(z_{BA})} \tag{4}$$

3.2 Models based on point-straight line distance

The distance of point *S* from straight line *p* given by a point *P* lying on the line and a unit vector *v* parallel to the line is calculated as:

$$l(S, p) = |(P - S) \times v| \tag{5}$$

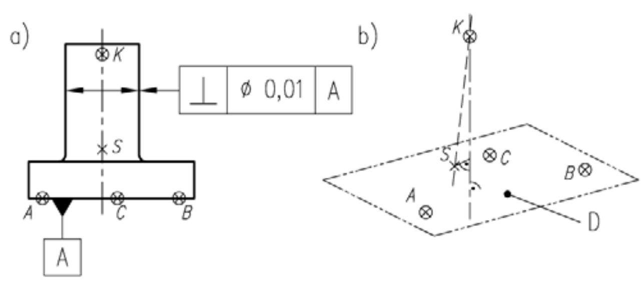

Fig. 2. Measurement of perpendicularity deviation of axis to plane: a) a design drawing with characteristic points, b) measurement model

The appropriate model for the perpendicularity deviation of axis to plane is shown on Fig. 2. The minimal number of points required to determine the deviation is 5. The datum plane is given by three points *A*, *B* and *C*. The axis, the direction of which is toleranced, is given by points *K* and *S*. The measuring task is to determine the distance *l* of the point *S* from the straight line perpendicular to the plane *ABC* and going through the point *K*. In the task, the straight line *p* from the formula (5) is perpendicular to the plane *ABC* and goes through point *K*. The unit vector *v* (normal vector of the plane *ABC*) can be calculated as:

$$v = (AB \times AC)/|AB \times AC| \tag{6}$$

Finally, the formula for perpendicularity deviation of an axis to a plane is:

$$l = \frac{a x_{KS} + b y_{KS} + c z_{KS}}{m} \tag{7}$$

$$a = y_{BA} z_{CA} - z_{BA} y_{CA}$$

$$b = z_{BA} x_{CA} - x_{BA} z_{CA}$$

$$c = x_{BA} y_{CA} - y_{BA} x_{CA}$$

$$m = \sqrt{a^2 + b^2 + c^2}$$

The model consists of 9 measurands x_{BA}, y_{BA}, z_{BA}, x_{CA}, y_{CA}, z_{CA}, x_{KS}, y_{KS}, z_{KS}.

3.3. Models based on point-plane distance

In the coordinate measuring technique the distance *l* of point *S* from the plane *p* given by any point *P* belonging to this plane and the unit normal vector *v* is calculated as follows:

$$l(S, p) = |(P - S) \cdot v| \tag{8}$$

The parallelism of axes (in the plane normal to the common plane) (Fig. 3) can be determined by the distance *l* between the point *S* of the toleranced axis and the plane defined by the two points (*A* and *B*) of the datum axis and one point (*C*) of the toleranced axis. The minimum number of points required to determine the deviation is 4. In this case, the point *S* nominally lies in the plane containing the points *A*, *B* and *C*.

In the above task, plane *p* is given by the three points *A*, *B* and *C*. As the point *P* you can adopt one of these points (*A*, *B* or *C*) and the normal unit vector *v* can be calculated according to the formula:

$$v = \frac{AB \times AC}{|AB \times AC|} \tag{9}$$

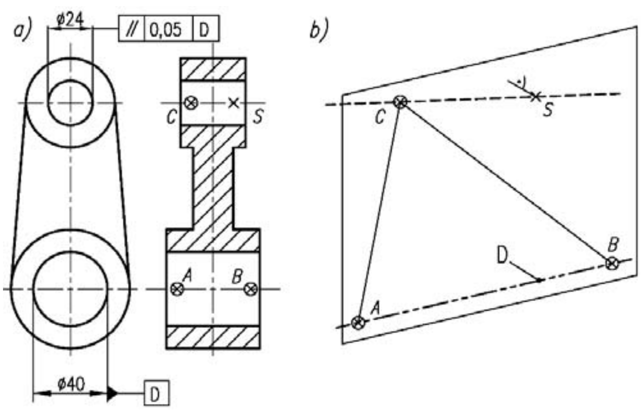

Fig. 3. Measurement of parallelism of axes in the plane normal to the common plane: a) a design drawing with characteristic points, b) measurement model

If we assume point A as the point P, we get the following model:

$$l = \frac{ax_{SA}+by_{SA}+cz_{SA}}{m} \tag{10}$$

$$a = y_{BA}z_{CA} - z_{BA}y_{CA}$$

$$b = z_{BA}x_{CA} - x_{BA}z_{CA}$$

$$c = x_{BA}y_{CA} - y_{BA}x_{CA}$$

$$m = \sqrt{a^2 + b^2 + c^2}$$

The model consists of 9 measurands x_{BA}, y_{BA}, z_{BA}, x_{CA}, y_{CA}, z_{CA}, x_{SA}, y_{SA}, z_{SA}.

3.4. Complex models

Models of some geometrical deviations are more complex. For example, the perpendicularity of two planes can be determined as the distance l between the point S and the plane perpendicular to the plane ABC and going through points K and L (Fig. 4). The minimum number of points required to determine the deviation is 6. The datum plane is represented by the points A, B and C. The plane the orientation of which is toleranced is represented by the points K, L and S.

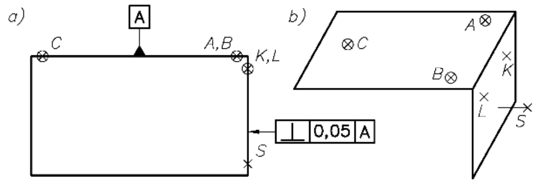

Fig. 4. Measurement of perpendicularity of the planes: a) a design drawing with characteristic points, b) measurement model

In this case the formula (8) for point-plane distance can be used. In this task, the plane p is perpendicular to the plane ABC and going through points K and L. The normal unit vector v of the plane can be calculated as:

$$v = ((AB \times AC) \times KL)/|(AB \times AC) \times KL| \tag{11}$$

As the point P in the formula (8) the point K or the point L can be assumed. Finally, this gives

$$l = |e + f + g| \tag{12}$$

where

$$e = \frac{x_{SK}(cy_{LK} - bz_{LK})}{t}$$

$$f = \frac{y_{SK}(az_{LK} - cx_{LK})}{t}$$

$$g = \frac{z_{SK}(bx_{LK} - ay_{LK})}{t}$$

$$t = \sqrt{(cy_{LK} - bz_{LK})^2 + (az_{LK} - cx_{LK})^2 + (bx_{LK} - ay_{LK})^2}$$

3.5. Models based on other constructions

To determine the diameter of a circle 3 points of this circle are required (A, B and C).

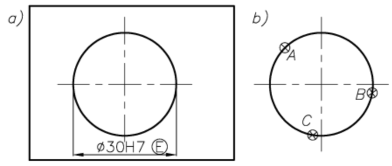

Fig. 5. Measurement of diameter of a circle: a) a design drawing with characteristic points, b) measurement model

The radius of the circle including points A, B and C can be evaluated with the formula for radius of a circle circumscribed on a triangle

$$R = \frac{abc}{4S} \tag{13}$$

where

$$a = \sqrt{x_{CB}^2 + y_{CB}^2 + z_{CB}^2}$$

$$b = \sqrt{x_{CA}^2 + y_{CA}^2 + z_{CA}^2}$$

$$c = \sqrt{x_{BA}^2 + y_{BA}^2 + z_{BA}^2}$$

$$S = \sqrt{p(p-a)(p-c)(p-c)}$$

$$p = \frac{1}{2}(a + b + c)$$

The model consist of 9 measurands x_{BA}, y_{BA}, z_{BA}, x_{CA}, y_{CA}, z_{CA}, x_{CB}, y_{CB}, z_{CB}.

4. MODELS OF CMM ERRORS

The influence of the CMM geometrical errors on the CMM error of indication in a point A for conventional CMMs can be described as:

$$\begin{bmatrix} ex(A) \\ ey(A) \\ ez(A) \end{bmatrix} = \begin{bmatrix} xpx(A) \\ xty(A) \\ xtz(A) \end{bmatrix} + \begin{bmatrix} ypx(A) \\ yty(A) \\ ytz(A) \end{bmatrix} + \begin{bmatrix} zpx(A) \\ zty(A) \\ zpz(A) \end{bmatrix} + $$

$$+M_1 \begin{bmatrix} xrx(A) \\ xry(A) \\ xrz(A) \end{bmatrix} + M_2 \begin{bmatrix} yrx(A) \\ yry(A) \\ yrz(A) \end{bmatrix} + M_3 \begin{bmatrix} zrx(A) \\ zry(A) \\ zrz(A) \end{bmatrix} \tag{14}$$

$$+M_4 \begin{bmatrix} ywz(A) \\ xwz(A) \\ xwy(A) \end{bmatrix}$$

where M_1, …,M_4 are matrices of weights depending on the design type of the CMM, its dimensions, the stylus used and coordinates of point A.

The measurement error of coordinates' differences of points A and B is equal to difference of the indication errors of the CMM in points B and A:

$$\begin{bmatrix} ex(AB) \\ ey(AB) \\ ez(AB) \end{bmatrix} = \begin{bmatrix} ex(B) \\ ey(B) \\ ez(B) \end{bmatrix} - \begin{bmatrix} ex(A) \\ ey(A) \\ ez(A) \end{bmatrix} \tag{15}$$

The influence of the geometrical errors of the articulated measuring arms (ACMM) error of indication in a point A

CMMs can be described by means of Denavit and Hartenberg (D–H) convention [14] (Fig. 6).

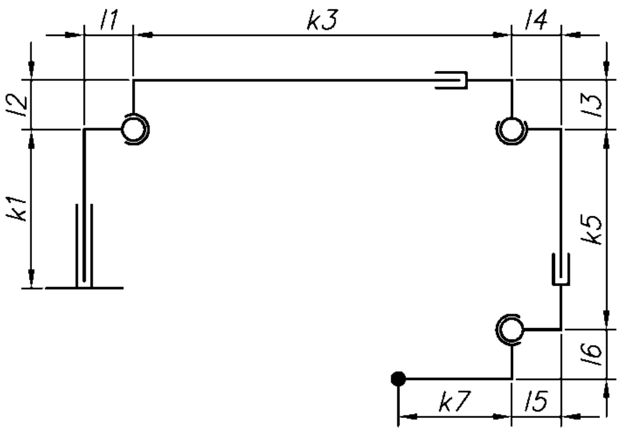

Fig. 6. The kinematic model of the measuring arm

The coordinates of stylus tip centre point of the ACMM (x, y, z) can be obtained as the elements [1, 4], [2, 4] and [3, 4] of the matrix resulting from product of six or seven transformation matrices of the form

$$\begin{bmatrix} \cos(\theta_i) & -sin(\theta_i)\cos(\alpha_i) & sin(\theta_i)\sin(\alpha_i) & l_i\cos(\theta_i) \\ sin(\theta_i) & cos(\theta_i)\cos(\alpha_i) & -cos(\theta_i)\sin(\alpha_i) & l_i sin(\theta_i) \\ 0 & \sin(\alpha_i) & \cos(\alpha_i) & k_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (16)$$

where: k_i – the distance on the rotation axis (length of particular joint), θ_i – rotation about the joint axis, l_i – the length of each common normal (joint offset), α_i – the angle between two successive joint axes (joint twist).

Inputting nominal values in the formula one gets the nominal coordinates of the stylus tip, if the input values include errors –the tip coordinates are effected by the errors.

5. CONCLUSIONS

The uncertainty for each characteristic (dimension, geometrical deviation) must be estimated separately. The presented examples clearly show that weights for particular component uncertainties calculated as partial derivatives of the assumed models are different for each case. The weights depend not only on the type of characteristic arising from the measurement model but also on the probing strategy, and especially distribution of the extreme probing points on the surface.

In the presented methodology, the mathematical model for each characteristic is defined by a formula presented in a form of function of coordinates' differences' of a few points. This way the problem of correlation is eliminated from the uncertainty analysis.

The uncertainty of coordinates' differences of the pair of points can be estimated on the base of difference of the component indication errors of the CMM in those points.

REFERENCES

[1] ISO 15530-3:2011 Geometrical product specifications (GPS). Coordinate measuring machines (CMM): Technique for determining the uncertainty of measurement. Part 3: Use of calibrated workpieces or measurement standards

[2] ISO/TS 15530-4:2008: Geometrical Product Specifications (GPS). Coordinate measuring machines (CMM): Technique for determining the uncertainty of measurement. Part 4: Evaluating task-specific measurement uncertainty using simulation

[3] ISO 14405-1:2010 Geometrical product specifications (GPS) - Dimensional tolerancing - Part 1: Linear sizes.

[4] ISO 1101:2012 Geometrical Product Specifications and Verification. Geometrical Tolerancing. Tolerancing of Form, Orientation, Location and Run-out.

[5] Z. Humienny, P. Turek, "Animated visualization of the maximum material requirement", *Measurement*, vol. 45, pp. 2283–2287, 2012.

[6] W. Jakubiec, "Analytical estimation of uncertainty of coordinate measurements of geometric deviations. Models based on distance between point and straight line", *Advances in Manufacturing Science and Technology*, 33 (2), pp. 45–54, 2009.

[7] W. Jakubiec, W. Płowucha, "Analytical evaluation of the uncertainty of coordinate measurements of geometrical deviations. Models based on the distance between point and plane". *Advances in Manufacturing Science and Technology,* vol. 37 (3), pp. 5–16, 2013

[8] W. Jakubiec, "Adequacy and generality conditions in estimation of uncertainty in measurements of geometrical quantities", *Advances in Manufacturing Science and Technology*, 31 (4), pp. 55–66, 2007.

[9] W. Płowucha, W. Jakubiec, "Theory and practice of uncertainty evaluation of coordinate measurements". *Key Engineering Materials*, Vol. 613, pp. 344–353, 2014

[10] W. Jakubiec, W. Płowucha, M. Starczak, "Analytical estimation of coordinate measurement uncertainty", *Measurement,* 45, pp. 2299–2308, 2012.

[11] E. Trapet, E. Savio, L. De Chiffre, "New advances in traceability of CMMs for almost the entire range of industrial dimensional metrology needs", *Annals of CIRP,* 53 (1), pp. 433–438, 2004.

[12] H. Schwenke, B.R. Siebert, F. Waeldele, "Assesment of uncertainties dimensional metrology by Monte Carlo simulation. Proposal of a modular and visual software", *Annals of the CIRP*, 49, 2000.

[13] J. Sładek, A. Gąska, "Evaluation of coordinate measurement uncertainty with use of virtual machine model based on Monte Carlo method", *Measurement*, 45, pp. 1564–1575, 2012.

[14] K. Ostrowska, A. Gąska, J. Sładek, "Determining the uncertainty of measurement with the use of a Virtual Coordinate Measuring Arm", *International Journal of Advanced Manufacturing Technology*, 71, pp. 529–537, 2013.