

## ALTERNATIVE MODELS AND NUMERICAL SIMULATIONS OF RAREFIED GAS FLOW IN VACUUM SYSTEMS

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**Abstract** – An analytical and a numerical studies of rarefied gas effects on the flow through a vacuum pipe at low operating pressure are performed by the use of both continuum and kinetic approaches. The pipe operates in a high-Knudsen-, low-Reynolds- numbers regime where the viscous effects dominate the flow. Since only the isothermal flow is focused, the slip-flow at the wall is the main anomaly. A comparison of the calculation results with those from the available measurements shows good agreement.

**Keywords:** vacuum, slip-flow, leak, rarefied gas

### 1. INTRODUCTION

The recent advances in the vacuum techniques and the micro-machinery have led to the creation of various systems with a very high vacuum (extremely low operating pressure), which could be in a very small size. Due to these conditions, the gas is rarefied which makes the flow behaviours differ from those near the ambient pressure and/or in a normal size system. At an extremely low operating pressure, the numbers of gas molecules will be dramatically decreased causing the gas to be rarefied. The situation is magnified if the size of the system is reduced. Consequently, due to the small number of gas molecules, the flow behaviours will be different from general gas, where the number of gas molecules is large enough to consider the gas as a continuum medium. The continuum medium assumption is not valid for the aforementioned cases if the flow behaves as slip, transition, or free molecular flow. Therefore, tube characteristic length,  $L_C$  and the tube pressure,  $p$ , are the two main factors of Knudsen number,  $Kn$  that characterise the flow regime through a sonic nozzle. The Knudsen number is defined as:

$$Kn = \frac{\lambda}{L_C} \quad (1)$$

where the mean free path,  $\lambda$  is a function of the pressure,  $p$ , the viscosity,  $\mu$  and the most probable molecular velocity,  $\bar{v}$ . There are various models to describe the molecular mean free path [1]. However, to avoid the confusion, the mean free path is estimated using Maxwellian theory.

As well as the Knudsen number, the rarefaction parameter,  $\delta$  is another quantity that is also used to describe the flow regime and is defined as [2]:

$$\delta = \frac{\sqrt{\pi} L_C}{2 \lambda} = \frac{\sqrt{\pi}}{2} \frac{1}{Kn} \quad (2)$$

From the above equations, the Knudsen number as a function of the pressure of gas flowing through ISO-standard tube, the imperial size tube and the leak artefacts [3], is plotted in Figure 1.

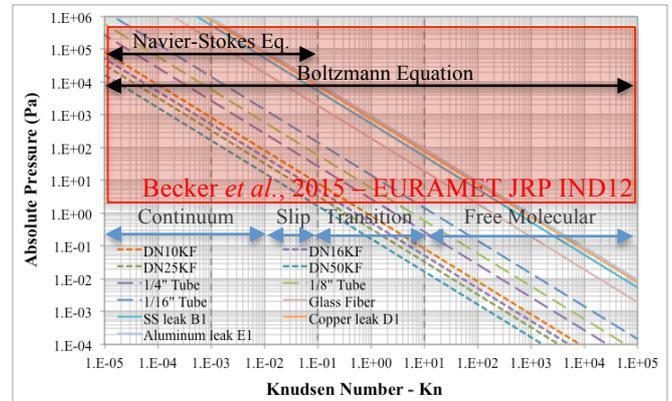


Fig. 1. Pressure versus  $Kn$  for usual gas flow through the tubes and the leak artefacts [3].

When  $Kn$  is very small, there are enough molecules for the gas to be considered as in a continuum regime. The slip-flow starts to appear at values of  $Kn$  greater than 0.001 and become dominant at around 0.01, where the slip-flow regime begins. The flow through leak artefacts that Becker *et al.* [3] investigated enters this regime when the operating pressure is around ambient pressure except glass fibre. As the gas becomes more and more rarefied, its flow is characterised as being in the transition and free molecular regimes, when  $Kn$  reaches 0.1 and 10 respectively. The leak artefacts (B1, D1 and E1) enter these two regimes around  $10^4$  Pa and  $10^2$  Pa. Due to the larger size, the flows in vacuum tubes enter either slip-flow, transition or free molecular flow regimes in a much lower pressure than those of leak artefacts. Normally, the vacuum systems operate in either three regimes. Therefore, the flow models to predict the rarefied gas play an essential role in the design of vacuum systems and their components.

The general slip-flow equations given in this paper are the simple tool to describe rarefied gas flow through any size of the tube from microchannel leak artefacts to ISO-standard tube. For more complex passage, the slip-flow could be predicted by the aid of commercial CFD software like ANSYS FLUENT. The implement of slip-flow in such software is shown in this paper.

## 2. SLIP-FLOW

The slip-flow regime is a slightly rarefied one, which could occur in the gas flows through a small passage like the leak artefacts and/or in a low-pressure condition like the vacuum systems, as shown in Fig. 1. It typically corresponds to the Knudsen number ranging between 0.01 and 0.1, easily reached for the flow either through a micrometer scale leak artefact under ambient pressure or through an ISO-standard tube in rough vacuum.

The Knudsen layer plays a fundamental role in the slip-flow regime. This thin layer, one or two molecular mean free paths in thickness, is a region of local non-equilibrium, observed in any gas flow near a surface. In non-rarefied flow, the Knudsen layer is too thin to have any significant influence, where as in the slip-flow regime, it needs to be considered [4].

Although the Navier-Stokes equations are not valid in the Knudsen layer, due to non linear stress/strain-rate behaviour within it [5], their use with an appropriate boundary velocity slip and temperature jump conditions can provide an accurate prediction of the mass flow rate [6]. The slip-flow condition was originally proposed by Maxwell and has since been developed up to the second order. Several models have been proposed, mostly in similar form but differing slightly in the coefficients used. If the isothermal flow is assumed, the general second order slip-flow model is derived as:

$$u_{slip} = \frac{2-\alpha}{\alpha} \lambda \left[ \frac{\partial u}{\partial n} - \frac{3}{2} \frac{\mu}{\rho T} \frac{\partial^2 T}{\partial s \partial n} \right]_{wall} + \frac{3}{4} \left[ \frac{\mu}{\rho T} \frac{\partial T}{\partial s} \right]_{wall} \quad (3)$$

where  $s$  and  $n$  denote the tangential and the normal directions to the wall. The tangential momentum accommodation coefficient,  $\alpha$  is unity for perfect diffuse reflection and is zero for purely specular reflection of molecules with the wall. If the isothermal flow is assumed, the general second order slip-flow model is derived as:

$$u_{slip} = u_s - u_w = \frac{2-\alpha}{\alpha} A_\alpha \lambda \left. \frac{\partial u}{\partial n} \right|_{wall} - A_\beta \lambda^2 \left. \frac{\partial^2 u}{\partial n^2} \right|_{wall} \quad (4)$$

where  $u_{slip}$  is the slip velocity,  $u_s$  is the flow velocity at the wall, and  $u_w$  is the velocity of the wall, with its normal direction noted as  $n$ . The mean free path of the molecules is  $\lambda$  and  $\alpha$  is the tangential momentum accommodation coefficient, which are equal to unity for perfectly diffuse molecular reflection and equal to zero for purely specular reflection.  $A_\alpha$  and  $A_\beta$  are the first and second order dimensionless coefficients, respectively. In Maxwell's model,  $A_\alpha$  equals to unity, which overestimates the velocity at the wall, but leads to a good prediction of gas velocity outside the Knudsen layer. Example values of  $A_\alpha$  and  $A_\beta$  proposed in the literature are given in Table 1.

The previously discussed slip-flow models could be implemented into continuum-assumptions Navier-Stokes Equations to predict fully developed gas flow through a vacuum piping system. Therefore, the reduced flow rate for slip-flow through a tube is derived in terms of the rarefaction parameter as:

$$G_p = \frac{\delta}{4} + \frac{2-\alpha}{\alpha} A_\alpha \frac{\sqrt{\pi}}{2} - A_\beta \frac{\pi}{4\delta} \quad (5)$$

A flow parameter that is of common interest is the mass flow rate through the passage. This can be calculated from the reduced flow rate as follows [7][8]:

$$\dot{m}_{lb} = \frac{\pi R^3}{\bar{v}} G_p^{lb} \frac{dp}{dz} \quad (6)$$

where  $R$  is the radius of the tube.

Table 1. First and Second order slip coefficient.

Author, Year	$A_\alpha$	$A_\beta$
Maxwell, 1879	1.0000	0.0000
Cercignani, 1964	1.1466	0.9756
Deissler, 1964	1.0000	1.1250
Hadjiconstantinou, 2003	1.1466	0.6470

## 3. TRANSITION AND FREE-MOLECULAR FLOWS

The slip-flow models are limited to use within the slip-flow regime, whereas within the transition and the free-molecular flow regimes, the kinetic gas theory is required. The solutions based on this theory are valid throughout the whole range of the Knudsen number from the free molecular, through the transition, and up to the slip and hydrodynamic regimes. In this paper, the BGK (one kinetic gas model) is chosen and the linearized BGK model is solved numerically by DVM (Discrete Velocity Method) to determine a reduced flow rate of the fully developed flow through a tube. Therefore, the mass flow rate for each specific problem could be determined from the reduced flow rate using equation (6).

## 4. NUMERICAL SIMULATION OF SLIP-FLOW

ANSYS FLUENT, the well-known commercial CFD software, could be employed to simulate fully developed flow through a tube. FLUENT is capable to determine the flow from laminar to turbulent as well as viscous and inviscid flows. In addition, it also gives an additional option for the "Low Pressure Boundary Slip (LPBS)" condition. Since the flows of the investigated vacuum tubes and leak artefacts are in the low Reynolds number range, it is clear that the flow is laminar with high viscous effects. The LPBS option, which is based on Maxwell's model, is enable to determine the slip-flow effect at the nozzle walls. However, the LPBS method is limited to only first order model. To simulate slip-flow using higher order models, the "Moving Wall" method was proposed [4].

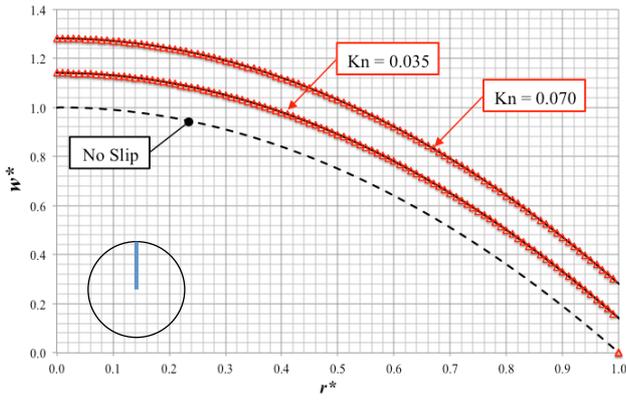


Fig. 2. Dimensionless velocity along the radius of tube; comparison between no-slip result (--), analytical result of Maxwell's first order slip-flow (-) and LPBS ( $\Delta$ ) for  $Kn = 0.035$  and  $0.07$ .

Fig. 2 shows the comparison between the analytical results of the Maxwell's first order slip-flow (-) and the numerical results obtained from the LPBS method ( $\Delta$ ) in terms of the dimensionless velocity along the radius of tube for  $Kn$  equals to  $0.035$  and  $0.07$ . It could be clearly seen that there are no differences between the results of both methods.

## 5. COMPARISON

In the present work, the reduced flow rates are compared with the available experimental results [3]. From the flow conditions, the average reduced flow rate through a tube or a leak artefact could be calculated from the mass flow rate as:

$$G^* = \dot{m} \frac{\bar{v}}{\pi R^3} \frac{L}{p_{in} - p_{out}} \quad (6)$$

where  $R$  is the average radius of the tube. The quantity  $G^*$  corresponds to the average rarefaction parameter  $\delta_0 = (\delta_{in} + \delta_{out})/2$ , where  $\delta_{in}$  and  $\delta_{out}$  are determined from the pressures  $p_{in}$  and  $p_{out}$  as well as the radius at the inlet and the outlet respectively. The comparisons of rarefied gas flows through the leak artefacts are shown in Figure 3. The comparison range covers from the slip-flow until the free molecular flow regimes.

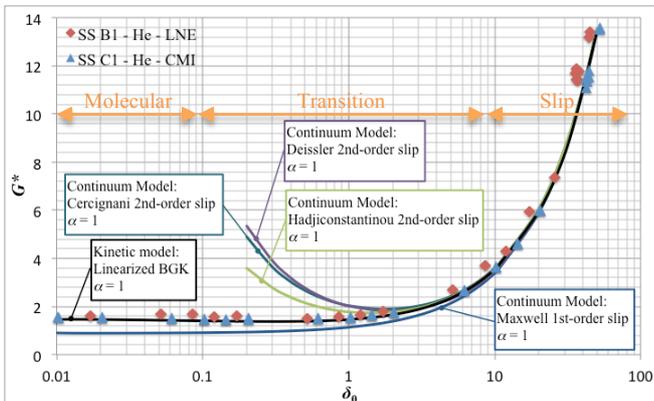


Fig. 3. Comparisons of slip-flow models with kinetic model and experiments in the form of reduced flow rate versus the rarefaction factor. Experimental data obtained by LNE, France ( $\blacklozenge$ ) and CMI, Czech Republic ( $\blacktriangle$ ) [3].

It could be seen that the linearized BGK model is in good agreement with the experimental results as expected. The slip-flow model could predict the rarefied gas flow until the early transition regime but totally fail from the mid transition regime until the free molecular regime. Therefore, the slip-flow models must be carefully employed. The first order Maxwell model is in excellent agreement with the BGK model from the continuum regime where the value of  $\delta$  is large until the early transition regime where  $\delta$  is around 5 to 6. The second order slip model could extend the validity of Navier-Stokes equations until almost the mid transition regime where the value of  $\delta$  is as low as 2.

## 5. CONCLUSIONS AND PERSPECTIVES

The simple general equation of fully developed slip-flow through a circular cross section has been introduced. The result of the equation in term of flow rate could be directly calculated from a rarefaction parameter. The rarefaction parameter, which is used to classify a type of flow, is determined from the operation conditions of flow. The first order slip-flow model proposed by Maxwell is valid until the early transition regime. However, the second order slip-flow models could extend the validity to almost the mid transition regime. Therefore, their results are in good agreement with those from the experiment until their limit. The rarefied gas flow is well predicted from the linearized BGK model for the entire regime as expected.

Moreover, for more complex flow passage where exact solution does not exist, the slip-flow could be simulated by commercial CFD software using the validated methods.

## ACKNOWLEDGMENTS

Several years ago, Prof. Stéphane Colin, Prof. Lucien Baldas, Prof. Sandrine Geoffroy, and their colleagues at the institut clément ader opened a door to the world of rarefied gas dynamics to the first author, giving him the opportunity to join their research. A few years later, the chance to meet Prof. Dimitris Valougeorgis and colleagues at the University of Thessaly brought him a key to solve kinetic theory of gases. He is forever grateful to them for their kind support in knowledge and mind.

The authors would like to express our appreciation to Asso. Prof. Asi Bunyajitradulya, Asst. Prof. Boonchai Lertnuwat and Asst. Prof. Niphon Wansophark from the Faculty of Engineering, Chulalongkorn University for their kind support in Ansys FLUENT's license to perform the calculation in the investigation.

This work is a part of the project, which is supported by the Coordinating Center for Thai Government Science and Technology Scholarship Students (CSTS), National Science and Technology Development Agency (NSTDA) to spin-off the development of a rarefied gas flow measuring device.

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