

RESEARCH ON FREQUENCY RESPONSE OF PRESSURE TRANSDUCER

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Abstract—*The paper presents a kind of idea of determining uncertainty of frequency response based on dynamic calibration principles. The paper gives an example of pressure transducer which uncertainty of frequency response is determined by experimental data of multi-time dynamic calibration. It is crucial for widening operational frequency band-width of transducer concerned in the process of inverse filtering or correcting dynamic characteristics in practical applications.*

Keywords—*uncertainty; pressure transducer; frequency response*

1. INTRODUCTION

The concept of uncertainty can be traced back to uncertainty relationship presented by Heisenberg in 1927[1]. But the concept was not gradually determined and used until Recommendation INC-1 was presented by BIPM in 1980[2]. The workgroup of international uncertainty consists of ISO, IEC, OIML and CIPM in 1986. Guide to the Expression on Uncertainty in Measurement was published by ISO in 1992[3]. The criterion of Uncertainty in Measurement was established by China Metrology Institute in 1996. These data are only concerning uncertainty of static calibration and static measurement [4]. New requirements have been brought forward by the improvement of new scientific technology, which continuously accelerates the measurement technology to higher level, for example, the analysis of uncertainty has been brought forward from static to dynamic. However, there is no unified and systematic theory regarding dynamic uncertainty evaluation of measurement system at present. Professor Huang presented a kind of concept of dynamic uncertainty regarding operational frequency band-width of transducers and measurement systems in order to choose appropriate dynamic measurement systems according to frequency spectrum of measured signal[4][5]. Dr. Xie conducted an investigation in dynamic uncertainty of reliability predication based on LCR measurement system [6]. Dynamic characteristics of transducer or measurement system includes frequency response, impulse response, transfer function and so on. These dynamic performances can't be often described well and truly in practical applications. But, the quality of these dynamic performances, such as their repeatability or uncertainty, is crucial in the process of inverse filtering or correcting dynamic characteristics [11]. The paper presents a concept regarding uncertainty of frequency response of pressure transducer so that dynamic uncertainty can be estimated in the process of widening band-width of transducer or measurement system. This is an important problem in the case of correcting output signal resulting from frequency response of transducer in transient signal measurement.

2. PRINCIPLES OF DYNAMIC CALIBRATION AND DETERMINATION OF FREQUENCY RESPONSE

Based on theory of linear time invariant system, the relationship of input signal and output signal of a system can be described by Fredholm integral equations:

$$\int_{T_1}^{T_2} h(t, \tau)x(\tau)d\tau = A_h x = y(t) \quad t_1 < t < t_2$$

In the process of dynamic calibration, excitation signal(input signal) should cover wider frequency spectrum than band-width of transducer or measurement system. Typical methods of dynamic calibration are as follows.

Calibration by use of Quasi- δ Pressure Signal

If excitation signal and output signal are as follow respectively:

$$x(t) = \delta(t)$$

$$y(t) = h(t)$$

Then frequency response of calibrated pressure transducer can be calculated by following expression.

$$H(j\omega) = FFT[h(t)]$$

Because durative time of δ signal is zero, it is ideal and can't be generated by a practical system. So, generator of quasi- δ pressure signal is available. Its spectrum is flat from zero frequency to certain frequency. So frequency response of calibrated pressure transducer can be obtained by following formulae. Reference [7] researches the equipment of generator of quasi- δ pressure signal.

$$X(j\omega) = FFT[x(t)]$$

$$Y(j\omega) = FFT[y(t)]$$

$$H(j\omega) = \frac{|Y(j\omega)|}{|X(j\omega)|} = \frac{|Y(j\omega)|}{|X(j0)|}$$

Calibration by use of Sine Wave Pressure Signal

If excitation signal is as follow:

$$x(t) = \sin(\omega_0 t)$$

Then expression of output signal is as follow:

$$y(t) = A_0 \sin(\omega_0 t + \varphi_0)$$

Similarly, if excitation signal is as follow:

$$x(t) = \sin(\omega_n t)$$

Then expression of output signal is as follow:

$$y(t) = A_n \sin(\omega_n t + \varphi_n)$$

So frequency response of calibrated pressure transducer can be obtained by putting on sine wave pressure signals with different frequencies. For example, amplitude response $|H(j\omega)|$ consists of A_0, A_1, \dots, A_n . Phase response $\angle H(j\omega)$ consists of $\varphi_0, \varphi_1, \dots, \varphi_n$. Reference [8] researches the equipment of generator of sine wave pressure signal.

Calibration by use of Step Pressure Signal

If excitation signal $x(t)$ is step pressure, then output signal $y(t)$ of transducer calibrated can be recorded by experiment of step pressure generator. Reference [9] describes the equipment of generator of step pressure signal. The equipment consists of two elongated chambers, usually of constant cross section, separated by a burst diaphragm, see Figure 1. Initially the gas pressure is higher in one chamber than in the other. When the diaphragm ruptures the expansion of the high pressure gas into the low-pressure chamber generates a shock wave pressure which travels faster than the expanding gas. The rise time of the pressure is of the order of nanoseconds and is considered to be an ideal pressure step which includes wider frequency content.

The generated pressure step is calculable from gas dynamics provided that the pressure ratio, temperature, driven gas composition and shock wave velocity are accurately known. The assumptions used are that

- (1) Perfect gas laws apply
- (2) Flow is adiabatic
- (3) Isentropic relation between pressure and speed of sound
- (4) Specific heats are constant
- (5) Diaphragm burst is instantaneous
- (6) Viscous forces are negligible

The first three assumptions are somewhat more realistic than the last three ones. Transducers to be calibrated may be installed either in the side- or end-walls.

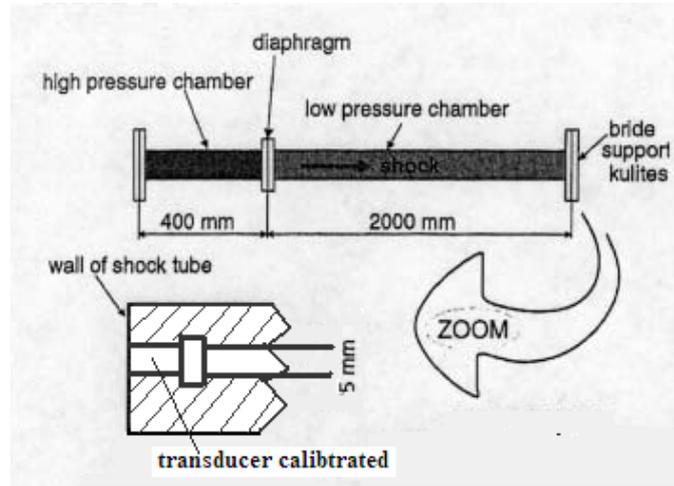


Fig.1 transducer mounted in the end-wall of shock tube

According to sequence $\{x(k), y(k)\} (k = 1, 2, \dots, N)$, linear difference formula with n order can be established

$$y(k) + \sum_{i=1}^n a_i y(k-i) = \sum_{i=0}^n b_i x(k-i) + e(k)$$

Where, $e(k)$ expresses sequence of noises.

$e(k)$ can be described as general formula as follow.

$$e(k) = \varepsilon(k) + \sum_{i=1}^n c_i \varepsilon(k-i) - \sum_{i=1}^n d_i e(k-i)$$

Thus, mathematic model above can be transformed into estimation with some parameters as follow.

$$y(k) = \boldsymbol{\varphi}_k^T \boldsymbol{\theta} + \varepsilon(k)$$

Where, $y(k)$ expresses output of transducer in disperse time k ;

$\boldsymbol{\varphi}_k^T$ expresses vector with M dimension of input sequence and output sequence;

$\boldsymbol{\theta}$ expresses estimated vector with M dimension;

$\varepsilon(k)$ expresses sequence of white noises.

So frequency response of calibrated pressure transducer can be obtained by estimation of linear difference formula with some parameters. The reference gives two kinds of methods of modeling and their differences [10].

3. UNCERTAINTY OF FREQUENCY RESPONSE

The concept of uncertainty of frequency response is defined as calculated statistic results of frequency response of calibrated pressure transducer based on experimental data of multi-time dynamic calibration. It includes two parts which are separately uncertainty of magnitude frequency response and uncertainty of phase frequency response. Their means and bias of magnitude frequency response and phase frequency response should be calculated in a certain probability level depending on experimental amounts. Given probability level is 95%, and then calibration experiments of twenty times are needed.

For example, Fig.2 shows nine experimental results of twenty time dynamic calibration of a transducer by using of step pressure excitation. Thus, mean value curve and its bias error of frequency response of the pressure transducer can be calculated and showed in Fig.3~Fig.6. The result explains that uncertainty of frequency response of the pressure transducer and its probability level to some extent.

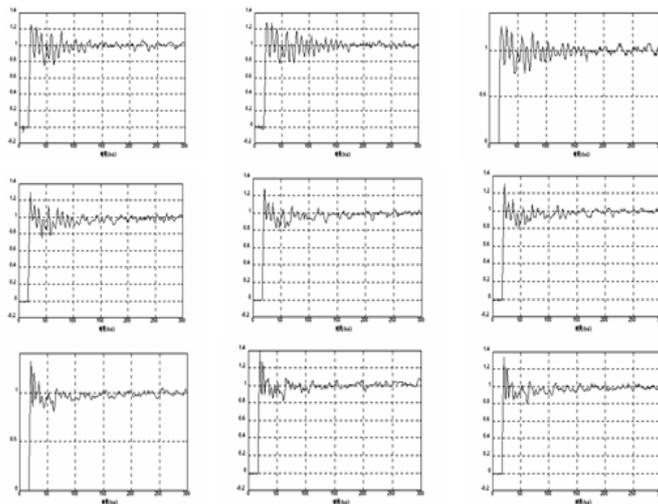


Fig.2 nine experimental results of dynamic calibration of a transducer

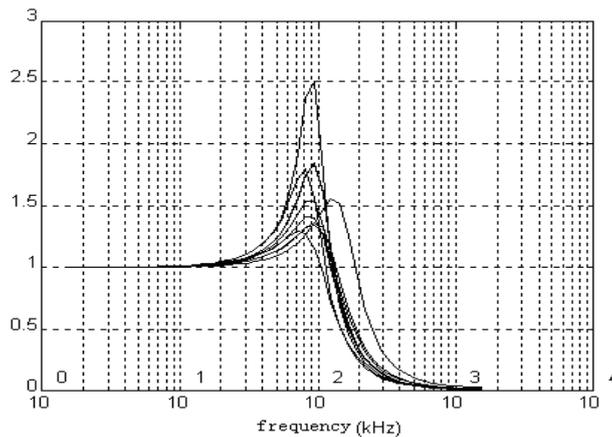


Fig.3 statistical results of unitary magnitude frequency response of a pressure transducer

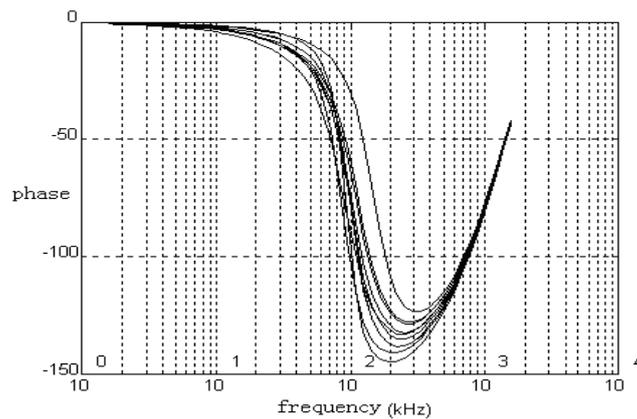


Fig.4 statistical results of phase frequency response of a pressure transducer

Taking amplitude frequency response for example, operational frequency band-width of the transducer is 18 kHz (amplitude error is within $\pm 5\%$), or 27 kHz (amplitude error is within $\pm 10\%$) according to result of Fig.5. Its repeatability of amplitude frequency response is $\pm 5\%$ (within 35 kHz), or $\pm 10\%$ (within 45 kHz). Thus, operational frequency band-width of the transducer can be widened by using of correction method of inverse filter.

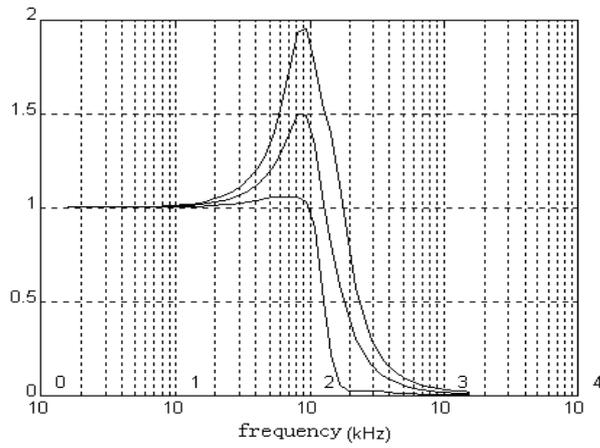


Fig.5 mean value curve and its bias error of unitary magnitude frequency response of a pressure transducer

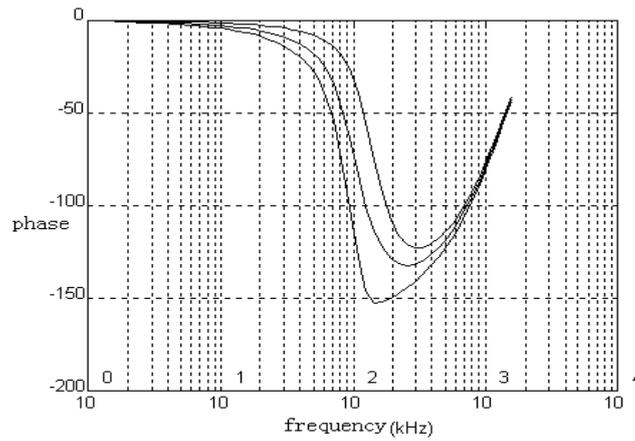


Fig.6 mean value curve and its bias error of phase frequency response of a pressure transducer

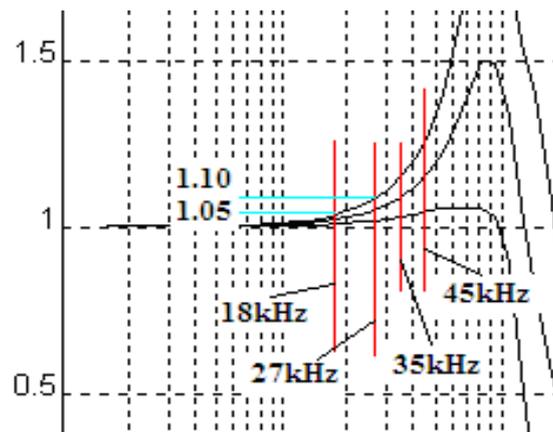


Fig.7 details of zoom in Fig.5

4. CONCLUSIONS

Dynamic characteristics of measurement systems is a key part for dynamic or instantaneous process measurement. Research on frequency response of system and its uncertainty expression depends on the level and development of dynamic calibration and technique. It has not uniform criterion until now. The paper has had a try on frequency response of transducer and given an experimental result. It is useful to select transducers in practical applications. At the same time, It is helpful that band-width of transducers is widened and corrected error of dynamic characteristics of measurement systems can be reduced or controlled within certain range.

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