

THE MEASUREMENT OF TREATMENT EFFECTS IN MATHEMATICAL ABILITIES: COMPARING TRAININGS IN PENCIL-AND-PAPER AND COMPUTERISED FORMATS

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Abstract – The aim of this study was to verify whether five-year-old children (attending the last year of kindergarten) could have different enhancements to their abilities to accomplish two homologous trainings on numerical intelligence: pencil-and-paper format and computerized format. We have evaluated the effects of two types of training, observing that both training are useful in relation to numerical abilities.

Keywords: Measurement, enhancement, trainings, format, mathematical abilities

1. INTRODUCTION

Since early childhood pre-schoolers show basic mathematical skills that will be crucial to learn mathematics in primary school, such as counting, magnitude comparison, conservation, enumeration, numeracy-related logic and computation skills [e.g., 1]. Specifically numeracy refers to many aspects, such as the study of numbers, arithmetic, problem solving and measurement [e.g., 2].

The idea that the early numerical competencies of kindergarteners consistently predict their upcoming arithmetic abilities in primary school has gained increasing attention in latest years [e.g., 3, 4].

The processing of spatial representation of number magnitude does have specific relevance for subsequent development of numerical intelligence [5].

Dehaene et al. [6] suggested that in early infancy number magnitude can be successfully learned using a left-to-right oriented mental number line on which numbers are spatially presented in an analogical format. This number line is supposed to be retrieved spontaneously by pre-schoolers every time that a number is presented and it is assumed to be developed quickly even before the attendance of primary schools [7, 8].

Therefore, it is crucial that the accuracy of learning the positional value of numbers using the mental number line improves as a function of education, practice and age [3].

It has been showed that the accuracy of children's number line representations impacts on their achievement in mathematics [3]. For instance, Booth and Siegler [3] highlighted that pre-schoolers showing a correct number line representation carried out better classical mathematical tasks (e.g., sum) and performed better even some arithmetic problems that are usually presented in primary school.

Overall, these findings suggest the existence of a strong relationship between the quality of the mental number line representation and future mathematical achievements [2]. Accordingly, trainings of mathematical abilities focusing on the development of number line mental representation have been included in successful psychoeducational interventions for kindergarteners [9, 10, 5].

There is evidence [e.g., 11, 12] that mathematical achievements at school are predicted by early numeracy skills, that is, children with low numerical knowledge at kindergarten learn slowly mathematics later on [13]. This would explain why Aubrey, Dahl, & Godfrey [14] stress the importance of implementing specific early interventions throughout primary school years to enhance the above mentioned mathematical skills in children at risk for or with evident problems in mathematical learning.

Overall, although there is not agreement about the definition of mathematics difficulties, according to Mazzocco, Feigenson and Halberda [15] they refer to individuals showing a performance between 11th and 25th percentiles on mathematics tests.

Furthermore, Fuchs and Fuchs [16] claim that low performers or low achievers in mathematics tasks (i.e., performance below 25th percentile average mathematics skills expected for the considered age) are children at risk for severe mathematics problems.

Since there is great variability in pre-schoolers' mathematics skills, during the last decades a series of trainings has been developed in order to enhance numerical knowledge in early childhood, that is, to promote early numeracy learning especially in low-performing learners [17].

These psychoeducational interventions might focus on some specific components of numeracy, emphasizing the development of particular aspects [e.g., 2]. Indeed, an effective intervention needs to embrace a series of aspects, as the number sense. It refers to the ability to quickly understand, estimate and manipulate numerical quantity [18]. This is an essential component of mathematical cognition and it is the essential aspect empowered in numerous pre-school psychoeducational interventions, mainly based on the enhancement of the capacity to build internal representations of mathematical concepts [19].

This might be enhanced by using planned concrete-representational instructions or specific activities requiring the manipulation of block visual images and illustrations of numerical sets [2]. These instructional aspects of psychoeducational trainings for kindergarteners are particularly successful to learn the positional value of numbers [20], and to enhance problem solving abilities [21, 2].

The aspect of situated cognition is also identified as a further crucial feature of worthwhile psychoeducational interventions for early mathematics learning [2]. Specifically, situated cognition is based on the planning of tasks enhancing a specific cognitive process, which are strictly related to everyday life activities by using well-known materials related to one's own experiences. Overall, this type of psychoeducational interventions is very useful from kindergarten [22] to secondary school [23], reinforcing the motivation of learners, too [2].

Another relevant aspect of psycho educational trainings is related to the development of whole-class interventions, which seem very useful from early infancy [2], especially in children risking the development of mathematics difficulties [e.g., 10, 24, 25].

Although the psychoeducational interventions proposed to the classroom are effective, it is relevant to highlight that psychoeducational interventions applied individually might be more effective for individuals with severe numeracy difficulties [26, 21].

According to Clarke et al. [27] and Witzel, Mink, and Riccomini [28] the implementation of specific interventions, mainly based on the use of explicit and simple instructions, on the employment of visual materials (e.g., cubes, number lines), on demonstrations about how to solve a problem, on breaking complex tasks into simpler units, seem to favour the empowerment of numeracy skills in early childhood, especially in kindergarten children at risk for severe mathematics problems.

In this regards, Slavin [29] suggests that in order to be effective, interventions to enhance early numeracy should be carried out for at least 12 weeks. The accomplishment of this type of trainings could support the effectiveness of treatment.

Specifically, some authors stated that there are benefits using computer assisted trainings in comparison with the classical pencil-and-paper approach [30].

Precisely, the point of strength of a training in computerized format is the interactivity with which materials are presented by an avatar (e.g., an animated puppet that drives the child in performing a cognitive task), an effect that is evident both when psychoeducational programmes empower cognitive processes and the metacognitive one [31].

Indeed, the use of the computer could provide immediate feedbacks without the involvement of teachers by using the afore-mentioned avatars, an aspect that is important for the positive effects on child's motivation [32].

Other advantages in using a pc training are represented by the use of different cognitive channels, specifically through the simultaneous presentation of some stimuli as images, text, animations, sounds and voices [33].

Nonetheless, further studies [e.g., 34, 35, 36] suggest that the pencil-and-paper-and psychoeducational programmes can be useful tools to enhance cognitive functions in childhood.

In summary, using a pc and pencil-and-paper trainings has some advantages, which could be further enhanced when the same specific training is settled and presented in two formats.

2. AIM AND METHOD

This work aimed at exploring the development of numeracy skills in pre-schoolers in relation to the application of two parallel kinds of trainings in mathematical abilities: one in pencil-and-paper format and one in computerised format. We would evaluate the efficacy of the training and, specifically, if the format of presentation affects differently the children's achievement. In agreement with literature, we can hypothesized:

1) the participants that carried out a computer training show greater improvement in mathematical learning; indeed some authors state that computerised educational interventions in kindergarten are more effective than pencil-and-paper training [36];

2) the participants that carried out two modalities of training could have a similar enhancement because both psychoeducational programs are effective in the same way; the efficacy of both modalities count on the novelty of materials presented and on the general educational approach used [34].

Therefore, at pre-test and post-test, kids were collectively administrated a battery of tests assessing their abilities.

2.1. Participants, instruments and procedure

Fifty-eight 5-year-old children were recruited in several public schools sited in Sardinia (Italy). Participants (44.8% female) were divided into one control group (n= 24) and two experimental groups (Table 1).

These last ones followed two different formats of homologous mathematical trainings, respectively in pencil-and-paper (n=13) and computerized organizations (n=21).

In relation to gender, we observed that there were no differences among participants in relation to the sex variable ($\chi^2 = .557$, $df = 2$, $p = .757$).

The children belonged to the middle socioeconomic status. The research was conducted on the base of the ethical requests defined by the Italian Association for Psychology. We applied a non-probabilistic sampling.

In order to assess the abilities at pre and post-test, our participants compiled a battery of standardized tests (Raven's Coloured Progressive Matrices - CPM [37]; Numerical Intelligence Scale - BIN [38]), in order to have respectively a measure of fluid intelligence and to assess their numerical knowledge.

Table 1. Descriptive statistics in the sample.

Into each group	CONTROL GROUP	PENCIL-AND-PAPER	COMPUTERISED
N	24	13	21
% OF FEMALES	41.7	53.8	42.9
AGE MEAN (IN MONTHS)	64.33	64.23	64.62
AGE STANDARD DEVIATION	3.30	3.29	3.81
LOWEST AGE	58	59	59
HIGHEST AGE	70	70	71

Specifically, the BIN [38] assesses four areas of numerical abilities: the lexical subscale (i.e., measures the ability to read and write Arabic numbers as well as the skill to connect the number-word to the exact symbol); the semantic subscale (i.e., appraises the ability to link numerical sizes, dots and Arabic digits); the pre-syntactical subscale (i.e., considers the ability to associate numbers to their representation and to order some sizes); the counting subscale (i.e., measures the ability to declaim the number-words system forward and backward, as well as the knowledge of the order of Arabic digits).

Subsequently, pupils were presented the numerical training “Sviluppare l'intelligenza numerica I” developed by Lucangeli et al. [39, 40] for 10 weekly sessions lasting each 30’ in pencil-and-paper and computerized modalities.

The trainings presented in this research involve several activities following the same theoretical organisation, settled to improve numerical abilities in pre-schoolers. The tasks were established to endow a slow, progressive and amusing approach to numerical knowledge in pupils, in order to get them ready for primary school. The psychoeducational intervention enhances various processes: it enhances the use of the Arabian code until 10, the knowledge of numbers, the automation of the sequence of numbers, the evaluation of the weight of numbers, the assessment of space, size and objects and the concept of ordinality.

Precisely, the computerized modality of training presentation is characterised by a series of activities which are realised in the playful context of North Pole; there is an interactive tutor that gives the guidelines and the feedbacks about the accuracy of the performance and encourages the pupil in case of mistake. The level of difficulty of these tasks might be adjusted in order to tailor them to the child’s competence.

Likewise, the pencil-and-paper modality of training was aimed at improving numerical knowledge, following the same criteria and postulates. The program embraces different parts and learning units, defined by specific new elements and growing difficulty, structured in a sequence of printable worksheets for learning activities. The pencil-and-paper training was directed by trained researchers.

Both computer-assisted and pencil-and-paper trainings include some exercises stimulating metacognitive awareness

and regulation of the cognitive processes implied in the tasks presented.

In order to verify our hypotheses and compare the effects of trainings, we applied a Mixed Design ANCOVA in relation to the pre-test and post-test assessments in overall mathematical abilities. Afterwards, in order to deeply appreciate the modification of these abilities in experimental groups, we applied a Multivariate Linear Regression using the total score in numerical abilities at post-test as a criterion.

3. RESULTS

Firstly the descriptive statistics were assessed in relation to all measures collected. Then the linear correlations were evaluated between all pairs of variables (at pre-test and post-test) by the application of Pearson’s *r* Coefficient. Before verify our hypotheses, we evaluated the potential existences of differences at pre-test among pupils embedded randomly in different experimental groups; these differences are not statistically significant.

Formerly, to assess the specific role of trainings in the variation of performance from pre-test to post test, a Mixed design Analysis of Covariance was computed. The repeated measures were constituted by the overall score in numerical abilities at pre-test and post-test [38], the between factor was the “training group” (control, pencil-and-paper, computer assisted), the covariates were the age at pre-test and the score in the fluid intelligence at pre-test (CPM) [37]. This statistic was suitable in order to compare the total performance for all groups in two moments, controlling the effects of potentially influencing factors.

The multivariate tests were significant; specifically we observed a significant effect of ‘repeated measures’ ($F(1;53) = 5.307$, $MSE = 5853.557$, $p=.025$, $\eta^2 = .091$) and ‘repeated measures * training group’ ($F(2;53) = 4.381$, $MSE = 4832.380$, $p=.017$, $\eta^2 = .142$) (see Table 2).

The covariates (age and CPM) did not have a significant effect on performance.

The application of Bonferroni's Post Hoc Comparisons showed precisely the differences between the training groups at post-test. In particular we detected that the score in mathematical ability is higher in the experimental groups than in the control group; nevertheless, there are not differences among the final performances consequent to the application of two types of homologous trainings (pencil-and-paper format and computerised).

Table 2. Mixed design Analysis of Covariance.

SOURCE	DF (b; w)	MEAN SQUARE	F	p	η^2
PRE – POST	1; 53	5853.55	5.30	.02	.09
PRE – POST * AGE	1; 53	3514.16	3.18	.07	.05
PRE – POST * CPM	1; 53	30.30	.02	.86	.00
PRE – POST * TRAINING	2; 53	4832.38	4.38	.01	.14

The results are illustrated in Figure 1. Then, in order to further investigate the relation among variables in two experimental groups we applied a Multivariate Linear Regression (Backward Method) only in this sub-sample (n=34).

We used as a criterion the overall numeric performance at post-test and as predictors variables the age at pre-test, the score at pre-test in CPM evaluation, the subscales at pre-test in Numerical Intelligence Scale (BIN) (lexical, semantic, pre-syntactical, counting), the format of training (pencil-and-paper format and computerised).

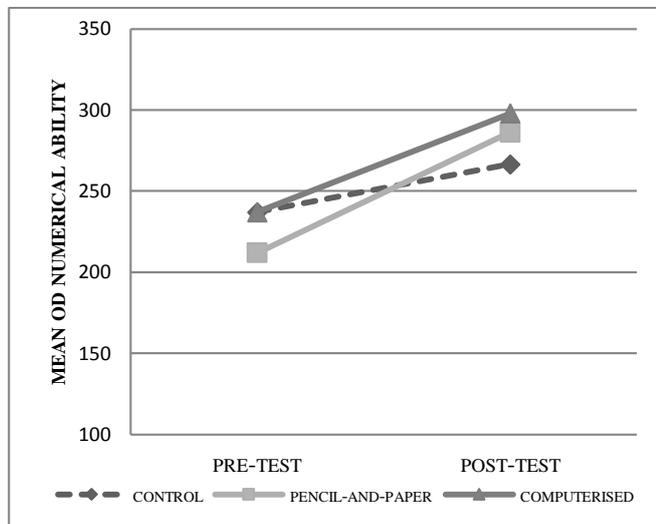


Fig. 1. Mean of numerical abilities in experimental groups at pre-test and post-test assessments.

Table 3 sums up these results, showing the values of R Squared, Standard Error of Estimation, Beta Coefficient, t value and their significance.

This analysis highlighted the significant effects of previous individual abilities (specifically lexical and pre-syntactical), confirming that the age and the CPM scores were not significant. It is crucial that there was not the effect of the specific type of training group realized.

4. DISCUSSION AND CONCLUSIONS

This work intended to study the effects of trainings in five-year-old children on numerical abilities, using different modalities (i.e., pencil-and-paper and computerised one). Evaluating the efficacy, we could observe that the modality of task presentation did not affect children achievements. Indeed compared with the control group that only carried out curricular activities, at post-test similar achievements were showed both by those children who performed tasks in the computer-assisted modality and by those participants who performed pencil-and-paper psychoeducational programmes.

These outcomes support our second hypothesis, in agreement with authors [e.g. 34] affirming that the aspects ensuring the efficacy of trainings are typically the sequencing of activities, the segmentation of material, the control of task difficulty, the application in small interactive groups, the directed questioning/responding and the addressed instructions.

Table 3. Results of Multivariate Linear Regression at the first and last steps (Backward method). Criterion variable: Overall Numerical abilities at post test

Step	R ² (S. E.)	VARIABLES	Beta	t	p
1	.478 (44.02)	AGE	-.061	-.463	.647
		PRE TEST CPM	.056	.420	.678
		PRE TEST LEXICAL	.315	1.883	.071
		PRE TEST SEMANTIC	.083	.526	.603
		PRE TEST COUNTING	.267	1.367	.183
		PRE TEST PRE-SYNTACTICAL	.293	1.807	.082
		MODALITY OF TRAINING	-.031	-.222	.826
2	.496 (43.24)	AGE	-.063	-.484	.632
		PRE TEST CPM	.062	.479	.636
		PRE TEST LEXICAL	.320	1.968	.059
		PRE TEST SEMANTIC	.087	.565	.576
		PRE TEST COUNTING	.251	1.408	.171
		PRE TEST PRE-SYNTACTICAL	.296	1.861	.074
3	.510 (42.64)	AGE	-.056	-.441	.662
		PRE TEST LEXICAL	.326	2.036	.051
		PRE TEST SEMANTIC	.092	.609	.548
		PRE TEST COUNTING	.239	1.375	.180
4	.523 (42.05)	PRE TEST LEXICAL	.319	2.032	.051
		PRE TEST SEMANTIC	.089	.600	.553
		PRE TEST COUNTING	.237	1.380	.178
		PRE TEST PRE-SYNTACTICAL	.300	1.990	.056
5	.534 (41.59)	PRE TEST LEXICAL	.324	2.088	.045
		PRE TEST COUNTING	.265	1.620	.116
		PRE TEST PRE-SYNTACTICAL	.329	2.325	.027
6	.510 (42.67)	PRE TEST LEXICAL	.455	3.347	.002
		PRE TEST PRE-SYNTACTICAL	.409	3.012	.005

Furthermore, in agreement with Penna and Stara [41] these results suggest that the role of the modality in presenting the tasks to be carried out can be marginal, whereas what is crucial is the content of the activity and the use of an immediate feedback. Indeed, the efficacy of two formats of training could be related to the fact that both provide feedbacks on results and on metacognitive processes, crucial in learning [42]. Moreover, in both modalities, psychoeducational interventions were accompanied by another relevant aspect, which could be identified in the presence of a new teacher, specialized in the promotion of these activities [29]. Besides, the novelty of the material used to train children can have motivated them in performing the task [34].

Our outcomes confirmed as both types of trainings seem to give sustenance to the emergence of a specific cognitive function [35, 41, 43] that support the consolidation of basic numerical processes (lexical, syntactical, counting and semantic). Another crucial aspect of both trainings was the similar structured program of activities, presented to pupils with a strict, continuative and defined calendar, lasting ten weeks [44].

Moreover, the accomplishment of trainings in the small groups could, on the other hand, support the achievement of learning aims, overcoming together the difficulties experienced by each individual [29]. Indeed also Wilson and Räsänen [2] observed as the training at kindergarten accomplished in small groups are very effective.

Aiming at further investigating the specific processes implied in the outcome at post-test, we applied a regression in order to identify lexical and pre-syntactical processes as specific predictors. Specifically the lexical processes referred to the number name and to their symbolic representation. To enhance these aspects, during the training we presented apposite doggerels and ditties, supporting this type of learning [45]. In relation to the empowerment of pre-syntactical processes, we focused on further comprehension of relations among different orders of magnitude [46]. These aspects were crucial in acquiring the concepts of inclusion and hierarchy, in the attribution of a name at the number or of the positional value of the number [45].

Then, during the training kids were presented specific tasks distinguishing among a unit and a compound of units. Another point of strength was that all these activities were applied by the presentation of specific images and activities, fitted on the kids interests [35].

Finally, in order to optimise the effectiveness of psychoeducational programs, the choice of a specific modality to propose the training had to be connected synchronously (i.e., at the same time) and diachronically (i.e., in a sequential way) to the general educational program [40, 41, 44]. Indeed, it is important to select the training to empower cognitive abilities paying attention to both the modality of stimuli presentation (i.e., computer assisted or pencil-and-paper) and the type of contents more appropriate to the children characteristics (e.g., expertise, familiarity with computer) [39, 40, 47, 48, 49].

In summary, from an educational perspective, an intensified stimulation on numerical knowledge by using new technologies as well as classical psycho-educational programs like those proposed in the current study can represent useful tools that can be easily administered by teachers to promote numerical development.

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