

OPTIMIZATION METHODS IN METROLOGY OF ELECTRICAL QUANTITIES

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Abstract—In electrical quantities metrology numerous examples of stochastic processes exist and a need for optimal solutions is posed. Here stochastic genetic algorithm optimization is used for solving two typical metrological problems: errors minimization in process of instrument's design and predicting metrological reference standards time-drift. The first case is optimal metrological design of a combined instrument transformer and the second case is analysis of resistance standard time-drift.

Keywords: optimization, genetic algorithm, electrical measurements, instrument transformer, resistance standard

1. INTRODUCTION

In metrology among other problems two of them represent general challenges:

1. Reduction of errors and uncertainties during the process of design of measurement instrumentation and devices;
2. Prediction of the time-behaviour of measurement instrumentation or standards, i.e. estimation of optimal recalibration interval.

In electrical measurements numerous examples of stochastic processes exist, like the electrical quantities standards time-behaviour, [1, 2].

In recent years application of stochastic optimization tools for design of electrical devices (including electrical instrumentation) is evident, [6-9]. Design applications generally aim to minimise manufacturing or material costs subject to performance criteria, which is an inverse problem requiring optimisation techniques, [3]. What mathematical physics denotes as inverse problems is the class of problems which are fundamental in measurement theory and practice, [4]. The main objective of such problems is to develop procedures for acquiring information on object and phenomena, accompanied by decreasing the distortion caused by measuring instruments, [4]. In [3] the implementation of Levenburg-Marquardt, sequential quadratic programming, Nelder-Mead, simulated annealing and practical swarm optimisation for metrology and design with continuous models is reported. In [6-9] the genetic algorithm (GA) as stochastic optimization method is applied for optimal design.

Calibration consists of two stages, [11]. In stage 1 a relation is established between values provided by measurement standards and corresponding instrument

response values. In stage 2 this relation is used to obtain measurement results from further response values. Polynomials of various degrees, determined by least squares, are extensively used as empirical calibration functions in metrology, [5]. In this contribution an original computer program with embedded genetic algorithm developed at the Ss. Cyril and Methodius University in Skopje, [6] is used for solving two metrological problems:

- a) Metrological optimal design of combined instrument transformer;
- b) Predictions of resistance standards time drift for determination of the optimal recalibration period, [12].

2. DESCRIPTION OF GENETIC ALGORITHM PROGRAM FOR METROLOGICAL OPTIMIZATION

On one side the concept of genetic algorithm is very simple as displayed in Figure 1, but on the other side it is a very powerful and robust computational tool. The GA implemented in the original computer program developed at the Ss. Cyril and Methodius University in Skopje, [6] starts with the initial *units (chromosomes)* stochastic generation. Each chromosome is composed of same number of *genes*. It is the initial *population (generation)*. The size of the population is between 20 and 100. An iteration in the GA is the cycle of estimation, selection and reproduction of the population. Each unit represents the eventual optimal solution. The possible solution, i.e. each unit is estimated according to defined criterion: the value of the *goal function* and each solution is assigned an *adequacy measure*. According to the comparison of all adequacy measures of the units, a decision is made which one of them will be allowed to form the next generation and with which probability in the step of *selection*. The selection strategy is the "virtual roulette wheel". All population units are potentially capable for *reproduction*. However, whether the unit will be selected depends on the value of the goal function (a sorted ranking list is formed). Finally, the virtual roulette wheel is formed according to the assigned number in the list. Conditionally, most of the "positions" in the wheel are assigned to the best units and the worst units have no "positions". The selected units from the "virtual roulette wheel" are copied into a new generation ready for generic operation *cross-over* and *mutation*. The *elitism* function is embedded, enabling automatic coping (*cloning*) of the best unit (according to the goal function value). Thus, the danger of losing the best unit during the cloning process is avoided.

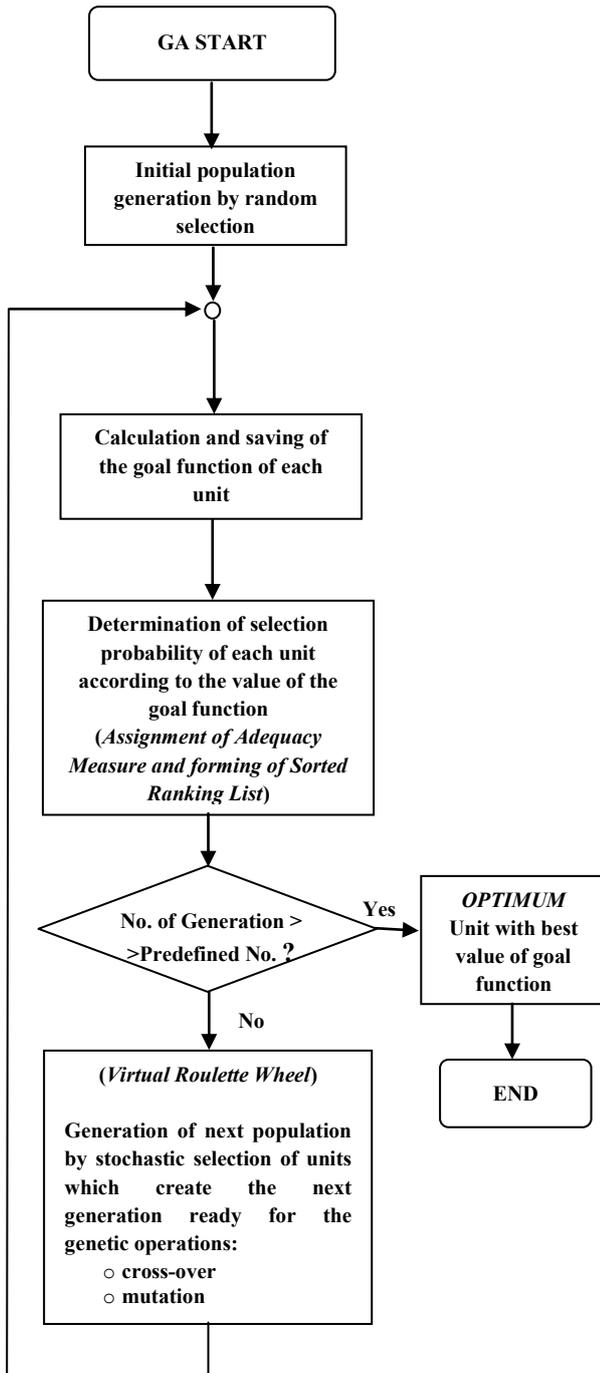


Fig. 1. Genetic algorithm for solving problems of stochastic nature in metrology of electrical quantities.

In the next step *reproduction* between pairs is performed, through simple coping of the units in the next generation. This is a process until the moment when *cross-over* and *mutation* happen according to previously defined probability. The selection of units exposed to the process of cross-over is random. They are grouped and the spot of division of the genes from the two parents is random again. Through the *cross-over* an exchange and recombination of the genetic material is achieved which forms a new chromosome with genes from the two parents. The *mutation* is a change of a unit in the chromosome. The mutation is a very important factor in the evolution, because it introduces

new genes in the process. This additionally increases the diversity of the units in the population. Pairs of chromosomes (parents) are exposed to the process of cross-over and the mutated units are returned in the population of the new generation. The cycle of estimation, selection and reproduction continues until predefined acceptable behaviour level is achieved. If the termination condition is the predefined generation number, then the best unit in the last generation is the optimum. The population units (chromosomes) in each next generation are more similar to each other with convergence towards specific part of the search space.

3. CASE STUDY NO. 1 - GA OPTIMAL DESIGN OF AN INSTRUMENT TRANSFORMER

The instrument transformers are of great metrological significance in the electrical power measurements, especially in the field of legal metrology and trade of electrical energy. The accurate metrological analysis of a 20 kV combined instrument transformer (CIT) (voltage transformation ratio $20000/\sqrt{3}\text{ V} : 100/\sqrt{3}\text{ V}$ and current transformation ratio $100\text{ A} : 5\text{ A}$) is possible only by numerical calculation of the magnetic field distribution in the 3D domain, [7]. The following four metrological parameters are relevant to the CIT:

- voltage measurement core (VMC) voltage error p_u ,
- VMC phase displacement error δ_u ,
- current measurement core (CMC) current error p_i ,
- CMC phase displacement error δ_i .

The CIT must comply with the standard IEC 61869-4, [10]. It comprises two transformer systems in complex electromagnetic coupling:

- a current measurement system with a regime close to a short circuit in the secondary winding and
- a voltage measurement system with a regime of almost unloaded transformer, i.e. open-circuit regime.

Both measurement systems are in a common housing with strong electromagnetic influence which significantly contributes to the metrological properties of the whole device. The geometry of the CIT is displayed in Figure 2. The goal during the optimal design process of the CIT is the minimisation of its metrological parameters p_u , δ_u , p_i and δ_i , as well as the VMC voltage error $p_{u0,25}$ at 0,25 of VMC rated load, according to the specifications of the IEC standard [10], as discussed and with initial results given in [7, 8]. Also, the difference of the absolute values of the VMC voltage error at rated load and at 0,25 of the rated load should be as minimal as possible. The minimization requirements of all the metrological parameters could be contradictory. So, the process of metrological optimal design has to be performed by posing a single objective function comprising them all. This is an example of a multi objective optimisation problem. The optimization design of the 20 kV CIT, the case study in this paper, is made through variation of eleven input constructional and geometrical variables of the CIT. The program GA maximizes the objective function, therefore the reciprocities of the errors are considered.

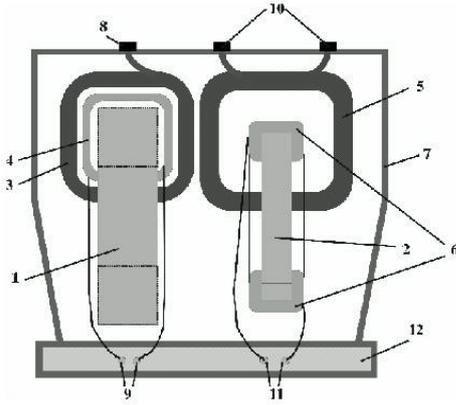


Fig. 2. Geometry of instrument transformer

1. VMC iron core
2. CMC iron core
- 3., 4. VMC windings
- 5., 6. CMC windings

The objective function can be defined with a *conservative approach* from metrological point of view - the certain measurement errors expressed through the absolute values are the metrological objective function:

$$f_{opt1} = \frac{1}{1 + |p_u + p_{u0,25}|} + \frac{1}{1 + |p_u|} + \frac{1}{1 + |p_{u0,25}|} + \frac{1}{1 + |p_i|} \quad (1)$$

as combination of the VMC voltage error p_u at rated load, VMC voltage error $p_{u0,25}$ at 0,25 of the rated load and CMC current error p_i at rated load. The phase displacement errors are constraints during the optimization process in order to satisfy the achieved accuracy class of the both measurement cores. The CIT is a highly non-linear electromagnetic system, with mutual electromagnetic influence of both cores which determines the metrological performance of the device. The GA optimal design is coupled to the results of the FEM-3D analysis of the magnetic field distribution in the CIT as in [7, 8]. Some of initial results of the optimal design process with the goal function defined as in (1) are given in [7, 8].

According to the Guide for Expressing the Uncertainty in Measurements (GUM) [11], a *statistical approach* can be applied:

$$f_{opt2} = \frac{1}{1 + \sqrt{(p_u + p_{u0,25})^2 + p_u^2 + p_{u0,25}^2 + p_i^2}} \quad (2)$$

In the objective functions (1) and (2) the phase displacement error δ_u and δ_i are constraints (a control criterion of the maximal allowed values of the phase displacement errors for certain CIT accuracy classes in the GA algorithm CIT mathematical model is embedded, as in [10]). The CIT optimal design starts with the parameters derived by classical analytical design of the CIT based on the T-equivalent circuit as a CIT model given in [7, 8] by coupling with FEM-3D modelling and calculation of the main electromagnetic characteristics. The chromosome comprises 11 genes (input variables-geometrical, constructional and electrical parameters). The mapping range of the eleven GA input variables is defined according to the previously derived results by FEM-3D and the analytical transformer design, some most

characteristic are given in Table 1. The optimal solution is derived by the following genetic parameters: cross-over probability 0,65, mutation probability 0,03, population size 16, maximal number of generations 30000.

Table 1. Input variables for CIT optimal design (mapping range)

Input variable	Minimum	Maximum	Initial value (analytical)
VMC primary winding number of turns	23584	24000	24000
VMC primary winding current density [A/mm ²]	1,5	3,0	2,04
VMC secondary winding current density [A/mm ²]	2,0	3,0	2,61
VMC magnetic core outside length [mm]	183	193	185
CMC secondary winding number of turns	115	125	120
CMC primary winding current density [A/mm ²]	1,0	1,6	1,36
CMC secondary winding current density [A/mm ²]	2,0	3,0	2,55
CMC magnetic core outside length [mm]	136	162	142

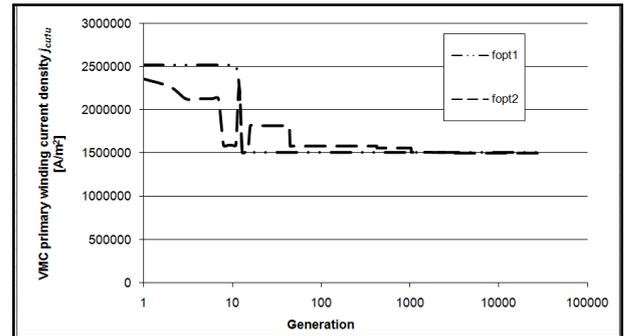


Fig. 3. Genetic algorithm changes through the generations of the VMC primary winding current density.

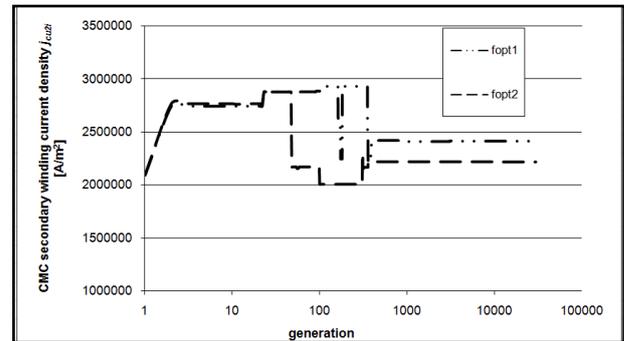


Fig. 4. Genetic algorithm changes through the generations of the CMC secondary winding current density.

In Figures 3-5, the changes throughout the 30000 generations in the GA optimisation process of the current densities in the CIT windings, as input variables (genes) are

comparatively displayed for the both objective functions: conservative approach f_{opt1} and statistical approach f_{opt2} .

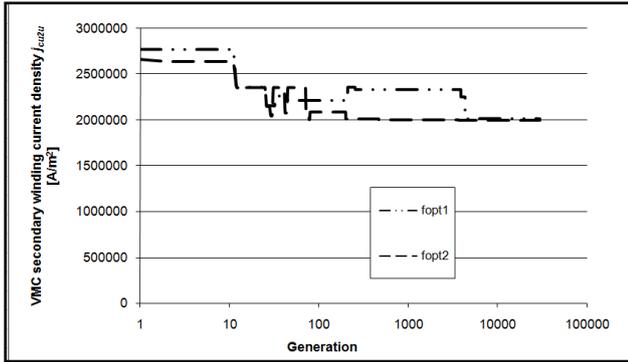


Fig. 5. Genetic algorithm changes through the generations of the VMC secondary winding current density.

The main input variables in the calculation of the CIT metrological parameters are the equivalent T-circuit parameters, i.e. the windings resistances and the leakage reactances. In Figures 6-12, changes throughout the 30000 generations in the GA optimisation process of the CIT equivalent T-circuit parameters are comparatively displayed for the both objective functions: conservative approach f_{opt1} and statistical approach f_{opt2} .

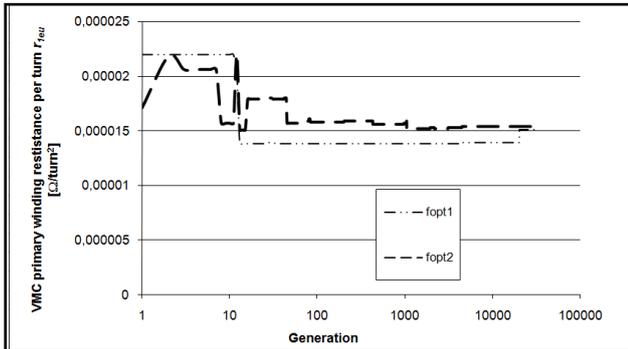


Fig. 6. Genetic algorithm changes through the generations of the VMC primary winding resistance per turn.

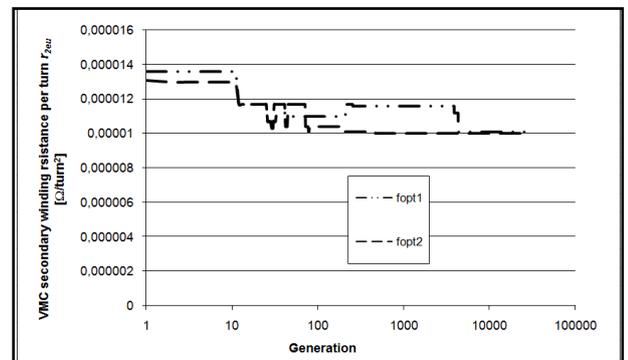


Fig. 7. Genetic algorithm changes through the generations of the VMC secondary winding resistance per turn.

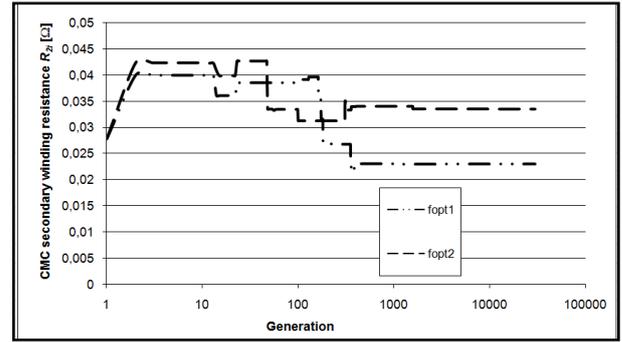


Fig. 8. Genetic algorithm changes through the generations of the CMC secondary winding resistance per turn.

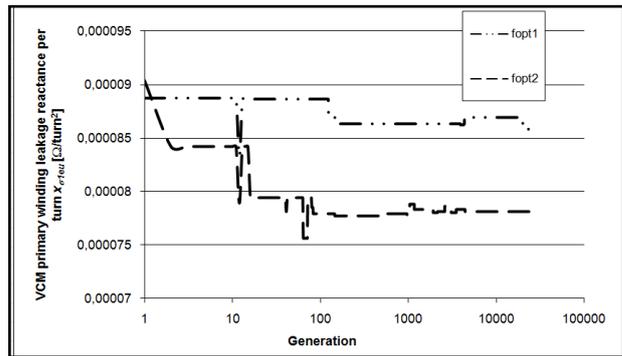


Fig. 9. Genetic algorithm changes through the generations of the VCM primary winding leakage reactance per turn.

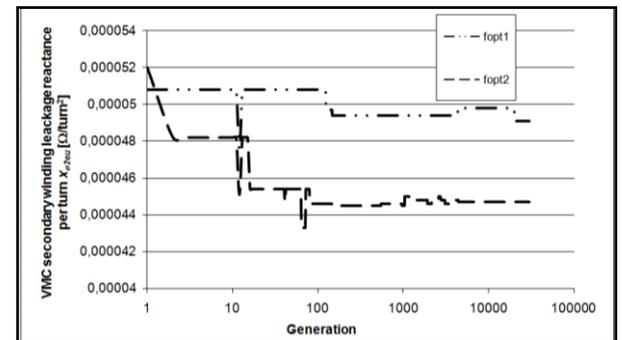


Fig. 10. Genetic algorithm changes through the generations of the VMC secondary winding leakage reactance per turn.

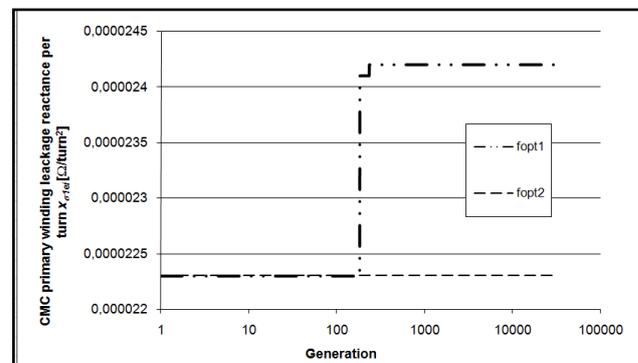


Fig. 11. Genetic algorithm changes through the generations of the CMC primary winding leakage reactance per turn.

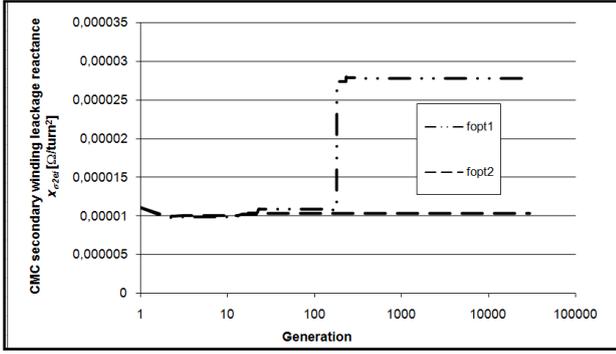


Fig. 12. Genetic algorithm changes through the generations of the CMC secondary winding leakage reactance per turn.

Table 2. Comparison of the CIT metrological, electromagnetic and constructive parameters derived by goal function with conservative and statistical approach

GA derived CIT parameter	Conservative approach	Statistical approach
p_u [%]	-0,795	-0,625
$p_{u0,25}$ [%]	0,795	0,855
p_i [%]	0,0	10^{-7}
δ_u [min]	-69,43	-61,62
δ_i [min]	-10,45	2,30
B_{mu} [T]	0,424	0,336
B_{mi} [T]	0,539	0,165
m_{Fe} [kg]	35,18	46,58
m_{Cu} [kg]	12,02	13,57

In Table 2 some of the optimal results (metrological parameters, geometrical, constructional and electromagnetic) derived by the both goal functions (1)-conservative approach and (2)-statistical approach are compared.

4. CASE STUDY NO. 2 - GA FOR PREDICTION OF RESISTANCE STANDARDS TIME DRIFT

The estimation of the optimal moment for calibration of the standards is a challenge for most of the metrological laboratories, [12]. The decisions are made based on the empirical experience of the laboratory staff, the behaviour history and the conditions of the standard, [12]. The trend lines are usually extrapolated to predict the time-behaviour of the standards by using the classical least squares method, as in [1]. However, these models do not take into account the random nature of the calibration results and uncertainties.

In this contribution a new model of time-drift trend line of resistance standard derived by using the Lagrange polynomial (LP), coupled with genetic algorithm optimization is presented. The time points of the performed calibrations and the gained values of standard resistance are input data. The final model of the trend line using five input experimental values can be reduced to polynomial of fourth order:

$$R(t)=a_0t^4+ a_1t^3+ a_2t^2+ a_3t+ a_4 \quad (3)$$

where t is the time, R is the resistance and $a_i, i=0, \dots, 4$ are the LP coefficients, depending on the values of the resistance at five calibration moments. The LP model is derived on the basis of the experimental data of calibrations of 1 Ω standard in time intervals from 1996 to 2005 year as in Table 3. The goal function of the optimization process by genetic algorithm (GA) is the minimum difference between the theoretical resistance value, derived by the LP, and the experimental value of the resistance at certain time moment (20. 04. 2007). The original GA program maximizes the goal function and the following goal function f_{opt} is given in the optimization process:

$$f_{opt} = \frac{1}{1 + |R_6 - R_{6exp}|} \quad (4)$$

where $R_6=R_{2007}$ is the resistance of the standard derived by the LP, determined by GA, at the time moment 20. 04. 2007 and R_{6exp} is the resistance of the standard derived by experimental calibration on 20. 04. 2007.

Table 3. The experimental GA input data for LP model

Time moment	R [Ω]	Uncert. of R [Ω]
26. 09. 1996	0,999992300	$\pm 5 \cdot 10^{-7}$
23. 10. 1998	0,999992450	$\pm 3 \cdot 10^{-8}$
28. 12. 2000	0,999992330	$\pm 6 \cdot 10^{-9}$
08. 04. 2003	0,999992306	$\pm 6 \cdot 10^{-9}$
09. 05. 2005	0,999992353	$\pm 1 \cdot 10^{-8}$
20. 04. 2007	0,999992349	$\pm 1 \cdot 10^{-8}$
14. 06. 2010	0,999992461	$\pm 1 \cdot 10^{-8}$

A verification of the gained model with known experimental data is done (for the time point 14. 06. 2010). The verification criteria k is:

$$k = \frac{|R_7 - R_{7exp}|}{|s_{R7}| + |s_{R7exp}|} \quad (5)$$

where $R_7=R_{2010}$ is the resistance of the standard derived by the LP, determined by GA, at the time moment 14. 06. 2010 and R_{7exp} is the resistance of the standard derived by experimental calibration on 14. 06. 2007, s_{R7} and s_{R7exp} are the respective uncertainties. The optimal solution is at following genetic parameters: cross-over probability 0,65, mutation probability 0,03, population size 10, maximal number of generations 10000.

Table 4. Input variables derived from the GA optimization process

R_{1996} [Ω]	R_{1998} [Ω]	R_{2000} [Ω]	R_{2003} [Ω]	R_{2005} [Ω]
0,99999245	0,99999242	0,9999923	0,9999923	0,99999235

The values of the five input variables derived from the process of genetic algorithm optimization are in Table 4.

The derived GA results after 10000 generations for the Lagrange polynomial coefficients are given in Table 5.

Table 5. The GA output data for LP model

LP coefficient	GA derived value
a_0 [Ω/month^4]	0,00000000000
a_1 [Ω/month^3]	0,00000000000
a_2 [Ω/month^2]	-0,00000000025
a_3 [Ω/month]	0,00000000378
a_4 [Ω]	0,999992438

The value of the 1 Ω standard resistance in 2007 derived by the LP coupled with the GA optimization process is $R_{2007} = 0,999992349 \Omega$. The verification of the method is done through comparison of the theoretical standard resistance in 2010, with experimentally derived value through calibration. The theoretical value of the standard resistance on 14. 06. 2010 gained by the LP trend line and through the GA process is 0,9999916844, with difference of $-7,77 \cdot 10^{-7} \Omega$, which is the same order of uncertainty as the input variable with highest uncertainty R_{1996} . The experimental input values of the 1 Ω resistance standard are compared in Figure 13 to the resistance values in different time points derived by the Lagrange polynomial coupled to the genetic algorithm optimization process. The uncertainties of the theoretical values are compared to the measurement uncertainties of the experimental data and overlapping of the both uncertainties interval exists. The overlapping is satisfactory for the point in 2010, which is a verification point of the methodology.

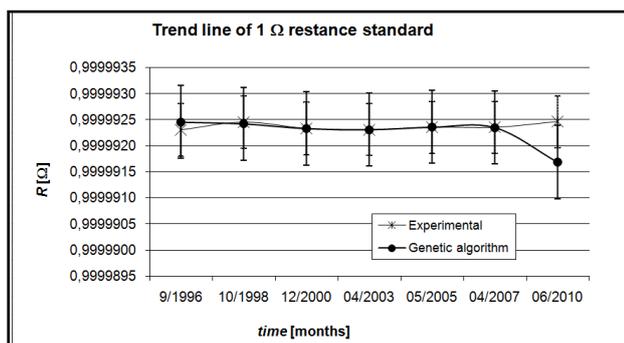


Fig. 13. Comparison of the GA derived LP time-trend line and the experimentally derived time-drift of 1 Ω resistance standard.

5. CONCLUSIONS

In the paper the stochastic optimization method-genetic algorithm is applied for gaining solution of two different, but challenging problems in metrology of electrical quantities:

1. Reduction of errors and uncertainties during the process of design of an instrument transformer;
2. Prediction of time-drift of a resistance standard.

One original GA software solution is used for two problems of diverse nature. The derived results are satisfactory and verified. It can be concluded that lots of options exists, i.e. the genetic algorithm further to be applied for gaining other optimal solutions in different metrology areas and problems.

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