

APPLICATION OF THE KALMAN FILTERING FOR THE ESTIMATION OF CALIBRATION UNCERTAINTY

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Abstract – The purpose of this paper is to show as the optimal linear filtering of measurements during calibration of the instrument can simultaneously consider the factors that affect the accuracy of estimates. The Kalman filter or linear quadratic estimation allows taking into account the previous changes of the device characteristics, the correlation between the points in the measurement range and working space.

Keywords: test equipment, measurement instrument, calibration uncertainty, Kalman filtering

1. INTRODUCTION

Uncertainty of the calibration of measurement sets and test equipment is the competence indicator of testing and calibration laboratories. Estimation of the components of the calibration uncertainty is one of the most actual problems of modern applied metrology. Most of calibration procedures offer the evaluation of measurement uncertainty for some separate points in the range of measuring instruments and working space of test equipment without taking into account the statistical relationship between them. Not also taken into account the statistical nature of characteristics drift of the calibrated device. These circumstances have a significant impact on evaluation of the uncertainty of the calibration results depending on intervals between calibrations, measurement accuracy and information about statistical characteristics. We also should consider another factor. There is often a possibility for implementation checking the accuracy of the measurements between the periodic calibrations according to approved calibration methods. Examples of such checking are "zero test" then a comparison of the measuring instrument in case of redundancy of measurements or the measurement of parameters of the stable sample. Such estimates of accuracy can significantly reduce the components of the uncertainty associated with unknown drift characteristics. If the accuracy metrics are considered as the observed dynamic process then the above opportunities can be implemented. Evaluations of the accuracy metrics at some fixed time are the initial and periodic calibration, selective regular or irregular simplified verifications the accuracy of the device. The best assessment for certain limitations is provided by the Kalman filter or linear quadratic estimation. Kalman filter is now widely used for processing measurement results

of dynamic systems in electronics, communications equipment and other technologies [1].

2. CONSIDERATION OF MODELS

Authors consider the process of the device applying as two random sequences: 1. changes of accuracy metrics; 2. the sequence of periodic calibrations for their assessment. If both of these sequences satisfy several conditions then the Kalman filtering can ensure the effective evaluation of the accuracy of the calibration considering the previous changes of the device characteristics, the correlation between the points in the measurement range. The periodic estimating of the characteristics of measuring and testing devices is performed during the calibration of devices at discrete time t_k , $k=0, 1, \dots, N$. When the intervals $\Delta t = t_{k+1} - t_k$ are small enough to represent the change of equipment characteristics in the form of a linear relationship then the state model of the equipment can be characterized as a vectorial differential equation of the first order. The authors offer the sequence of characteristics device as following:

$$U_{k+1} = \Phi_{k+1/k} U_k + B_{k+1/k} W_k \quad (1)$$

where U_{k+1} , U_k are the vectors dimension n of the measured quantity values deviations from the true quantity values at time moments t_{k+1} , t_k , W_k is the sequence of random Gaussian vectors with zero mathematic expectations and covariance matrix R_{wk} , $\Phi_{k+1/k}$ and $B_{k+1/k}$ are the connection coefficient matrices between elements of the vector U_k and W_k , respectively.

During the device calibration we perform measurements, which are represented as the following equation

$$\Theta_k = A_k F_k U_k + V_k, \quad (2)$$

where Θ_k is the vector of measurement results, F_k is the projection matrix of the vector U_k in the measurement area Θ_k , V_k is the Gaussian random variable with covariance matrix R_{vk} , A_k is the matrix which diagonal elements take the values 0 or 1. The relevant elements of the matrix A_k are determined in accordance follows: $A_{ii} = 0$, if the value in the i -th point is not measured; $A_{ii} = 1$ - in the opposite case. The model of characteristics (1) and the model of its measurements (2) are the basis for obtaining of estimates of the accuracy. Accounting the dynamics of change in the characteristics of devices and their correlation is performed by an optimal linear filtering (the Kalman filtering). Then the evaluation of the deviation of the readings from the true

value is determined by the expression:

$$U_{k+1} = \Phi_{k+1/k} U_k + P_{k+1} [\Theta_{k+1} - A_{k+1} F_{k+1} (\Phi_{k+1/k} U_k)], \quad (3)$$

where P_{k+1} is the weighting matrix which is defined by the expression:

$$P_{k+1} = K_{k+1/k} (A_{k+1} F_{k+1})^T [(A_{k+1} F_{k+1}) K_{k+1/k} (A_{k+1} F_{k+1})^T + R_{vk}]^{-1}.$$

Similarly, we get the assessments of prediction of the covariance matrix and its filtration:

$$K_{\kappa+1/\kappa} = \Phi_{\kappa+1/\kappa} K_{\kappa} \Phi_{\kappa+1/\kappa}^T + B_{\kappa+1/\kappa} R_{wk} B_{\kappa+1/\kappa}^T, \quad (4)$$

$$K_{k+1} = K_{k+1/k} - P_{k+1} (A_{k+1} F_{k+1}) K_{k+1/k}.$$

The equations (1)-(4) are the recursive algorithm of the Kalman filtering for evaluation the accuracy metrics of devices and their covariance matrix according to the results of the calibrations. The information about the previous state of the device is presented in the weight matrix P_{k+1} . The diagonal elements of the covariance matrix should be considered as the sum of the point estimates of the variance that are associated with the statistical properties of the calibrated device characteristics. These properties are associated with the time variability of characteristics, with the correlation between the points in measuring range and also the characteristics of the measurement standards, which are used for the calibration. In accordance with the GUM these components of the measurement uncertainty are usually evaluated by the type A. Components of the uncertainty budget of the calibration, as measured by type B, can be taken into account in the standard way. Thus, we are able to obtain estimates of the total uncertainty of the device calibration.

3. THE ESTIMATION OF MEASUREMENT INSTRUMENT CALIBRATION UNCERTAINTY

The calibration of measuring instruments can be presented as evaluating the differences between the readings and the quantity values by standards at the number of points of the measurement range and under different measurement modes. A calibration may be expressed by calibration function, calibration curve, table, additive or multiplicative correction of the indication with associated measurement uncertainty. The measurement uncertainty includes components associated with corrections and the assigned quantity values of measurement standards, as well as the definitional uncertainty, which associated with the inadequacy of the suggested model of the measurand. Sources contributing to the uncertainty include, but are not necessarily limited to, the reference standards and reference materials used, methods and equipment used, environmental conditions, properties and condition of the item being tested or calibrated, and the operator. Individual properties of the calibrated device, such as the correlation characteristics, variability in time, are usually expressed in time intervals much larger than the interval between calibrations. Therefore, these long-term behavior is not commonly taken into account when estimating the calibration uncertainty.

The invariance of the models (1), (2) in relation to calibration methods allows the use of filtering for those cases where the laboratory use the simplified calibration methods, various inspections, checking of the device not by the standard methods. These evaluations are often carried irregularly, they use other calibration sets, don't cover the whole measuring range. The Kalman filtering allows to use

the results of such inspections and calibrations to simulate the lifetime process of the devices in the form of sequence of the evaluations of device characteristics and to make recommendations on measurement volume and calibration frequency.

Authors present the calibration at time t_k as estimations of the difference vector Θ_k between of the device and reference readings at several points of the measuring range. The calibrated instrument is considered as a dynamic system described by the vector U_k of the random deviations of the readings from the measured value. There are correlations between the values of the deviations of the readings at the points of measuring range and between deviations at consecutive times t_k . They can be used as a source of the additional information. In order for the Kalman filtering (1)-(4) provide the best estimate in terms of minimum variance of the vector U_k the following restrictions must be provided: 1.the random deviations vector sequence of the readings of the calibrated devices should be a Gauss-Markov sequence; 2.the random variations of accuracy features of the calibrated instruments and used standards are not correlated among themselves. The estimation of the covariance matrix of the calibration results takes in to account the device characteristics prehistory. That allows the use of a priori information about the properties of the devices. At the initial time t_0 the deviation U_0 of the readings are the mathematic expectation of the deviations during the initial calibration, as the initial covariance K_0 is a diagonal matrix of the variances of the deviations. The covariance matrix R_{v_0} of measurement accuracy at the calibration is taken from the calibration certificate of the used standard. The authors consider the relationships between the accuracy of the calibration results and the accuracy of the used standards, the duration of the interval between calibrations, the number of calibration points in the measuring range.

Modeling the sequence of calibration and uncertainty assessment using Kalman was performed on the example of calibration of the ADC in five points. Maximum deviation (quantization and non-linearity) was 0,1 %, the correlation between neighboring points 0,94, the variability of the accuracy (matrix R_{wk} in (1) is $\sigma_w=0,01\%$ for $\Delta t=3$ months.

The simulation results are shown on figures 1, 2 for one calibration point (different intervals between calibrations) and for different number of points. The results confirm the real possibility of increasing the calibration accuracy by Kalman filtering. The filtering allows for a third consecutive calibration to improve the accuracy of estimates of 40-50 %.

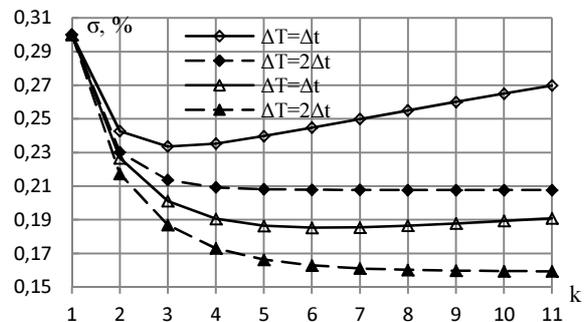


Fig.1.The standard deviations of the calibration results for one of measurement points when measurements at this point are executed (—) and not executed (---), ΔT - interval between calibrations.

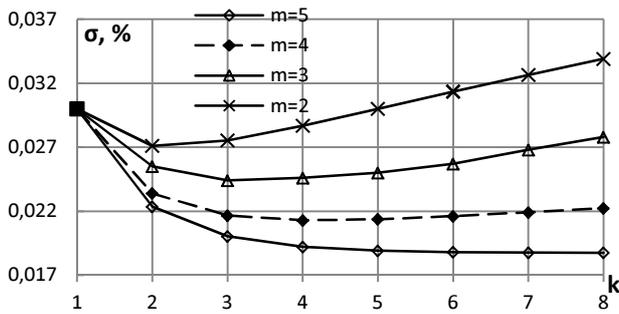


Fig.2. The standard deviations of the device calibration results for different number m of calibration points.

The issues of interest to the practice are the opportunities of using less precise standards, increase calibration intervals, reducing the number of measurement points. The use of less precise standards and the increase of calibration intervals reduce the cost of calibration. However the calibration accuracy is also reduced (figure 3).

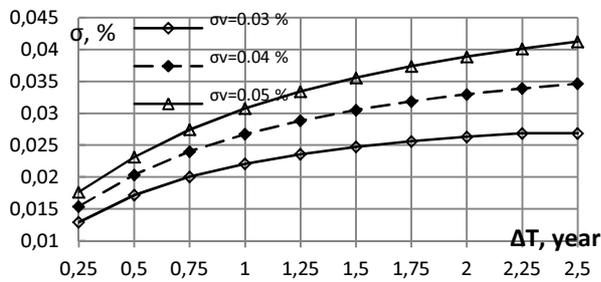


Fig.3. The standard deviations of the calibration results at intervals ΔT between calibrations and reference accuracy σ_v .

4. THE ESTIMATION OF TEST CHAMBER CALIBRATION UNCERTAINTY

Authors consider the calibrated temperature test chamber in the form of a multidimensional dynamic system with spatially-distributed temperature deviations U_k . Elements of the vector U_k are the differences between reproducible temperatures at some n points of working volume and true values at time t_k . The state vector U_k is estimated by the expression (2). According to the standard [2] temperatures are measured at 9 points of the working volume having the shape of a cube: one point is at the geometric center and the eight points are at the cube tops. Many factors affect the calibration results including performance of the chamber instability, the accuracy of the used measuring sets and the interval between calibrations [3]. Modeling sequences of calibrations have been performed according to the expressions (1) ... (4) for the chamber volume of 2 m³, temperature 70°C, $\sigma_v=0,3^\circ\text{C}$, $\sigma_w=0,15^\circ\text{C}$, the correlation coefficient for neighboring points of the working volume 0,94. The most topical issue is the possibility of reducing the number of measurement points for the calibration. If the temperatures are measured of all nine points the Kalman filtering improves the accuracy of calibration of 1.5 ... 1.7 times. When the number of measurement points is less we

get a loss of observability, the obsolescence of the correlation data and the lower accuracy of the calibration (Fig.4).

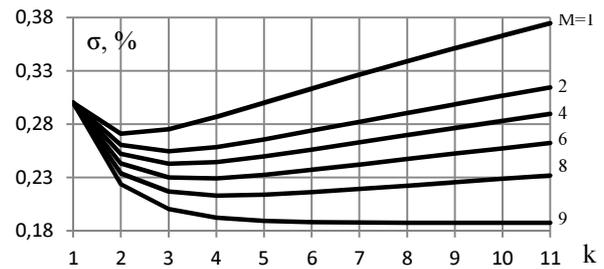


Fig.4. The standard deviations of the chamber calibration results at different numbers M of calibration points.

The accuracy of the calibration depends on the location of the calibration points in the working volume. Fig. 5 shows the simulation results for the three calibration points: at the center and two opposite vertices of the cube. Acceptable proposal for the number and location of calibration points should be looked for as the combination of different approaches.

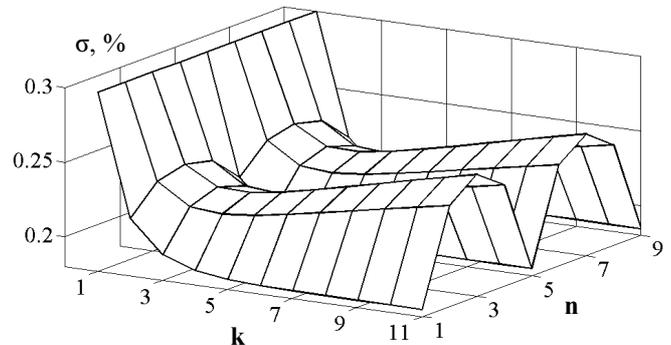


Fig.5. The standard deviations of the chamber calibration results at three points, n – point number.

5. CONCLUSIONS

Application of the Kalman filtering enables to take into account the individual characteristics of the calibrated device (correlation range and variability in time) and improves the accuracy of calibration in 1.5...1.7 times. The modeling of sequential calibrations allows to give recommendations on the number of measurement points, the calibration intervals and applicable standards.

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