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Stochastic modelling for state estimation in medium-voltage electric grids

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Abstract - We present two stochastic modelling approaches to improve state estimation in low and medium-voltage grids. First, a dynamic model for a disturbance observer state estimation is proposed, which introduces a stochastic component into the disturbance dynamics. Second, an approach based on a stochastic electrical power generation model is considered. This model is based on PV data and incorporates information about the seasons and the type of the PV generation units.

Keywords: dynamic state estimation, Kalman filter, pseudo-measurements, nodal load estimation, nodal load observer.

1. Introduction

In electrical power distribution grids, state estimation is the main part of the supervisory control and data acquisition (SCADA) system. The aim is to determine an estimate of the system state based on a network model and the measurements available. The network state consists of voltage bus magnitudes and angles, line flows, loads, transformer taps, active and reactive power, power injections and generator outputs. In a typical state estimation scenario, the measurement data includes active and reactive power flows, power injections and voltage magnitude at the bus [1, 2]. In high voltage transportation grids redundant measurement data is available for state estimation. In low and medium-voltage grids the situation is much more difficult, since measurement points in the network are sparse owing to economical constraints. Thus, efficient modelling and state estimation techniques are required.

It is common practice to classify state estimation depending on the evolution of the estimation methods. For very large networks, typically multi-area state estimation is applied, which is based on breaking down large networks into small areas with a local state estimation processor in each area. Commonly, two classes of state estimation processors are considered — static and quasi-dynamic state estimation. Static state estimation is a procedure of obtaining network state variables at all system buses at a given point in time when the power network is in a steady state. Hence, a single set of measurements is used to estimate the system state at one instant in time.

Static state estimation is commonly applied on the high voltage level and has been applied successfully in practice for decades. However, this approach does not take into account the information contained in the evolution of the state over consecutive measurement instants. To this end, quasi-dynamic state estimation has been proposed which takes advantage of forecasting future network state variables using a dynamic model [3, 4]. The main assumptions are typically:

- the system states change slowly, and
- the uncertainties are described using white Gaussian noise with zero mean and known covariance.

In this paper we consider a recently proposed quasi-dynamic state estimation method, which is based on a disturbance observer [5, 6, 7, 8]. Its aim is to reconstruct electrical power (active and reactive) from available measurements. On the distribution level, where a high number of nodes is accompanied by a small number of measurements, the typical approach is to replace missing data with so-called pseudo-measurements. However, in this way the state estimation quality depends on the reliability of the pseudo-measurements. That means that in order to improve the accuracy and reliability of the state estimation result, one needs improved pseudo-measurement models [5, 7].

We investigate stochastic models to obtain reliable pseudo-measurements and to improve state estimation results. We thus utilize well-known autoregressive-moving-average models (ARMA) to model the error of pseudo-measurements. In principle, the employed disturbance observer state estimation method is then able to improve the quality of the pseudo-measurements based on assumed ARMA process coefficients. In addition, we consider measured photovoltaic generation data acquired by the Eindhoven University of Technology, from which we construct a generic model to generate pseudo-measurement data.

2. Generic state estimation

The method of quasi-dynamic state estimation relies on the following system:

$$\begin{aligned} x(k+1) &= A(k)x(k) + g(k) + w(k) \\ y(k) &= h(x(k)) + v(k) \end{aligned}, \qquad (1)$$

where k is the time instant, x is the state vector, $A(k)$ is the transition matrix, $g(k)$ is associated with the trend behaviour of the state trajectory, $y(k)$ is the measurement vector, and $h(k)$ is the load-flow function vector. The errors $w(k)$ and $v(k)$ are assumed to be independent Gaussian processes with known covariance matrices $Q(k)$ and $R(k)$, respectively.

The application of any state estimation technique requires knowledge about the (time varying) system matrices A, g and the covariance matrices Q and R. If these are not given *a priori* they have to be considered as model parameters, which can be estimated, for instance, using classical linear exponential smoothing techniques [9, 10].

The majority of the state estimation techniques available in the literature use the extended Kalman filter technique [11, 12], for which estimated states are given by:

$$\hat{x}(k+1) = \tilde{x}(k+1) + L(k+1)[y(k+1) - h(\tilde{x}(k+1))], \quad (2)$$

with

$$\begin{aligned}
\tilde{x}(k+1) &= F(k)\hat{x}(k) + g(k) \\
L(k+1) &= \Sigma(k+1)H^T(k+1)R^{-1} \\
\Sigma(k+1) &= [H^T(k+1)R^{-1}H(k+1) + M^{-1}(k+1)]^{-1} \\
M(k+1) &= F(k)\Sigma(k)F^T(k) + Q
\end{aligned}$$

and with the Jacobian matrix $H(k+1)$ evaluated at $\tilde{x}(k+1)$.

3. Quasi-dynamic system with disturbance observer

The aim of the nodal load observer [5] considered here is to determine the grid state in medium-voltage distribution grids with incomplete measurement infrastructures. The procedure is based on measurements of voltage phasors at grid buses and forecasts of load and generation data. The main idea of the nodal load observer is to correct possibly incorrect pseudo-measurements, and then determine the grid state based on reconstructed and corrected values of nodal power and voltage. That means that rather than using the possibly incorrect pseudo-measurements directly for the determination of the grid state, the nodal load observer first uses all measurement information available to adapt the forecasts as much as possible to the actual situation of the grid. Even if only very few measurements are taken, major improvements of forecasts can be achieved in the vicinity of the measurement and as much improvement as possible can be attained for buses further away from it. Therefore, the vector of measured nodal power values in the network is written as

$$u(k) = D_m S_m(k) + D_{nm}\tilde{S}_{nm}(k), \quad (3)$$

with S_m the measured active and reactive power values, $\tilde{S}_{nm}$ the vector of pseudo-measurements, and with D_m and D_{nm} matrices to map the actual and pseudo-measurement values to the corresponding nodes in the network. The overall vector of nodal power in the network is given as

$$S(k) = u(k) + D_{nm}\Delta S_{nm}(k) \quad (4)$$

with ΔS_{nm} the difference between the actual, but unavailable measurements, and the pseudo-measurements at the corresponding nodes.

The error of the pseudo-measurements is considered as an unobserved disturbance of the network states. In order to estimate these disturbances, a model for their evolution over time is required. With $x(k) = \Delta S_{nm}(k)$ the mathematical representation of the quasi-dynamic system with a disturbance observer can then be written in the following way [6]

$$\begin{array}{rcl} x(k+1) & = & A(k)x(k) + g(k) + w(k) \\ y(k) & = & h(x(k), u(k)) + v(k) \end{array} . \quad (5)$$

The authors in [5] considered a simple type of dynamic behaviour for $x(k)$ via

$$x(k+1) = ax(k), \quad (6)$$

with $a \leq 1$. This model corresponds to an assumed constant (or decreasing for $a < 1$) deviation of the pseudo-measurements from the actual nodal power. However, as already mentioned in [5], more realistic models are required in order to improve the estimation quality of the nodal load observer. Moreover, model Eq. (7) does not take into account any coupling of the network states. This results in the network Eq. (5) not being observed, unless a voltage measurement $y_{meas}(k)$ is available at each node, which is unrealistic for medium- and low-voltage grids. To this end, we aim at an extension of the original model in order to improve the performance of the state estimation method. Therefore, we model the deviation between the load forecast and the actual nodal power as:

$$\begin{array}{rcl} z(k+1) & = & A(k)z(k) + g(k) \\ x(k) & = & C(k)z(k) + w(k), \end{array} \quad (7)$$

where the model matrices A, B and C, and the linear trend g are obtained from an ARMA(p, q) autoregressive-moving-average model of the order (p, q). The model consists of two parts, an autoregressive (AR) part of the order p:

$$x(k) = const + \sum_{i=1}^{p} \varphi(i)x(k-i) + w(k) \quad (8)$$

and a moving-average (MA) part of the order q:

$$x(k) = \mu + \sum_{i=1}^{q} \theta(i)w(k-i) + w(k), \quad (9)$$

where $w(k)$ is the white noise, μ is the expectation of $x(t)$ and $\varphi(i), \theta(i)$ are the parameters of the AR and MA models, respectively.

Consider, for example, an ARMA(2,0) model. The representation of it in state space form is given as follows:

$$\begin{pmatrix} x(k+1) \\ x(k) \end{pmatrix} = \begin{pmatrix} \varphi(1) & \varphi(2) \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x(k) \\ x(k-1) \end{pmatrix} + \begin{pmatrix} const \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} w(k). \quad (10)$$

With the substitution of the disturbance Eq. (10) to the system Eq. (5) and using an appropriate replacement, we could rewrite the system equation in the following way, assuming that x is univariate:

$$\begin{pmatrix} x(k+1) \\ x(k) \end{pmatrix} = \begin{pmatrix} \varphi(1) & \varphi(2) \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x(k) \\ x(k-1) \end{pmatrix} + \begin{pmatrix} const \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} w(k) \quad (11)$$

$$y(k) = h(x(k), u(k)) + v(k).$$

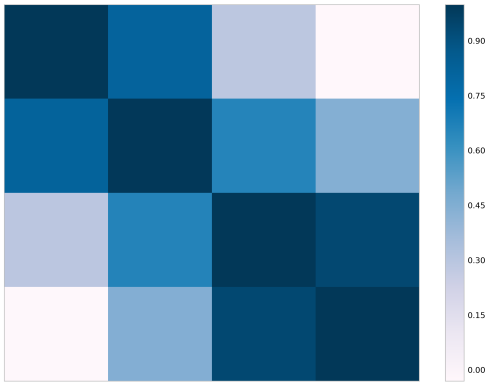

Fig. 1. Correlation between the error of the pseudo-measurements of nodal active power at nodes 1,2,3 and 8 in the network depicted in Figure 4.

The estimation of the state vector x can then be carried out, for instance, by an implementation of the extended Kalman filter Eq. (2). The parameters φ of the ARMA process considered can be obtained from test data by means of numerical optimization or using the technique of augmented states with the Kalman filter.

An important advantage of this modelling approach is the possibility to incorporate correlation between different buses by extending the above AR model to so called vector-AR (VAR) processes. That means that Eq. (11) is then extended to multivariate time series $\boldsymbol{x}$ so that

$$\begin{pmatrix} \boldsymbol{x}(k+1) \\ \boldsymbol{x}(k) \end{pmatrix} = \begin{pmatrix} \boldsymbol{A}_1 & \boldsymbol{A}_2 \\ \boldsymbol{I} & 0 \end{pmatrix} \begin{pmatrix} \boldsymbol{x}(k) \\ \boldsymbol{x}(k-1) \end{pmatrix} + \begin{pmatrix} const \\ 0 \end{pmatrix} + \begin{pmatrix} \boldsymbol{w}(k) \\ 0 \end{pmatrix} \tag{12}$$

with coefficient matrices $\boldsymbol{A}_1$ and $\boldsymbol{A}_2$ and multivariate noise process $\boldsymbol{w}(k)$. The incorporation of correlation into the dynamic model can help to overcome the issue of non-observability which originates from the lack of voltage measurements in the network, cf. [5]. The correlation in the difference between assumed pseudo-measurements and the actual measurements for a subset of the network in Figure 5 is shown in Figure 1. The non-zero correlation between the nodes introduces a structural dependence between the network nodes that can improve the observability of the dynamic system.

Another advantage of this approach is the possibility to obtain the state equation driving noise in Eq. (11) from the statistical model, instead of trying to choose suitable values by hand, cf. [7].

4. Photovoltaic measurements

In the previous section we proposed an approach to improve the estimation of the difference between the pseudo- and the actual measurements. One approach to obtain more dependable and improved pseudo-measurements is to rely on actual measurement data for the derivation of generic models. Therefore, in order to derive a realistic model for a PV generation unit, we used real photovoltaic power data from November 2012 till October 2013, recorded in the Netherlands by Eindhoven University of Technology. The available data consists of measurements of active power in consecutive steps of 15 minutes at five different PV generation units. The analysis of the behaviour of the data provided makes it possible to construct a realistic photovoltaic generation model.

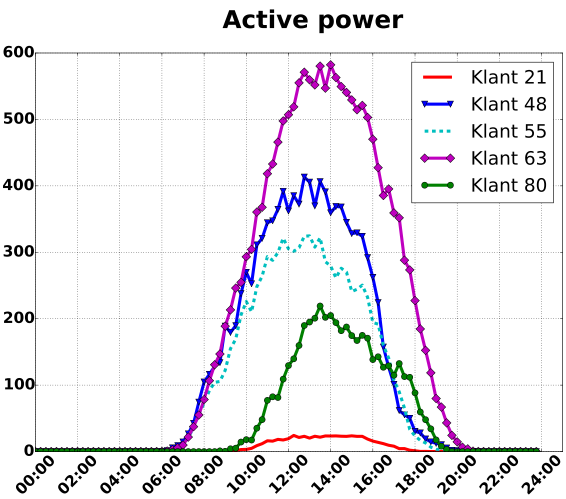

Fig. 2. Average photovoltaic values for five different nodes measured in May 2013 at Eindhoven University of Technology.

The monthly average power values are presented in Table 1, showing that the highest values are achieved in April and the lowest in December. The values in Table 1 also show that not only the season has to be considered in a modelling approach, but also the type of the PV unit.

Table 1. Average monthly values of active power for five different nodes.

	Klant n^o 21	Klant n^o 48	Klant n^o 55	Klant n^o 63	Klant n^o 80
Nov	5	115	77	139	63
Dec	2	49	51	74	35
Jan	2	100	79	157	62
Feb	5	176	114	249	96
Mar	14	316	244	397	176
Apr	34	448	345	681	292
May	24	413	324	582	219
Jun	27	449	335	575	260
Jul	31	459	333	610	236
Aug	31	443	324	617	258
Sep	18	301	203	437	181
Oct	10	236	170	300	113

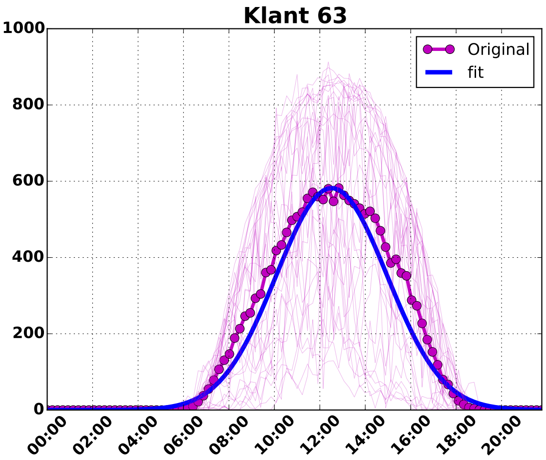

Fig. 3. Photovoltaic values for node 63 measured in May 2013 at Eindhoven University of Technology. The average value is depicted by the dotted line, whereas the thick solid line represents the fitted Gaussian function.

In Figure 2 we depicted the average active power value for all nodes in May 2013. It is clearly visible that the obtained behaviour resembles a Gaussian function

$$f(x) = a \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right), \tag{13}$$

with a being the maximum amplitude, μ the time of maximum power generation over the course of one day, and σ modelling the width of the power generation curve. An example of a fitted model to the available data is depicted in Figure 3.

In order to simplify modelling, we consider three types for the power value generation: low (behaviour similar to Klant 21), normal (behaviour similar to Klant 48, 55, 80) and high (behaviour similar to Klant 63). Fitted values for the amplitude, mean and standard deviation for the Gaussian function are presented in Table 2.

Uncertainties in the fitted parameters, which result from the variability in the available data, can be utilized to quantify reliability of the fitted models. In addition, the uncertainties provide a natural way to derive a stochastic model for the power generation at PV units.

Next, one of the United Kingdom Generic Distribution Systems (UKGDS) [13] which provides daily domestic electricity load profiles is used to illustrate the work of the nodal load observer with photovoltaic load data. We have chosen a 77 bus medium-voltage test grid with a distribution voltage of 11kV and interconnections provided to the adjacent 33kV. Here it was decided to take only past of the network for the analysis, in particular, 11 branches and 12 buses with a generator at node number 5. This part of the general scheme of the power grid is presented in Figure 4.

To illustrate the influence of available voltage measurement data on the estimation result for the nodal

Table 2. The values for the amplitude and (mean, standard deviation) for the Gaussian function.

	Low $a, (\mu, \sigma)$	Normal $a, (\mu, \sigma)$	High $a, (\mu, \sigma)$
Winter	3, (49,4)	85, (50,5)	160, (51,6)
Spring	24, (53,7)	309, (52,8)	553, (53,9)
Summer	26, (54,7)	344, (53,9)	600, (55,10)
Autumn	11, (50, 5)	162, (51,6)	292, (52,7)

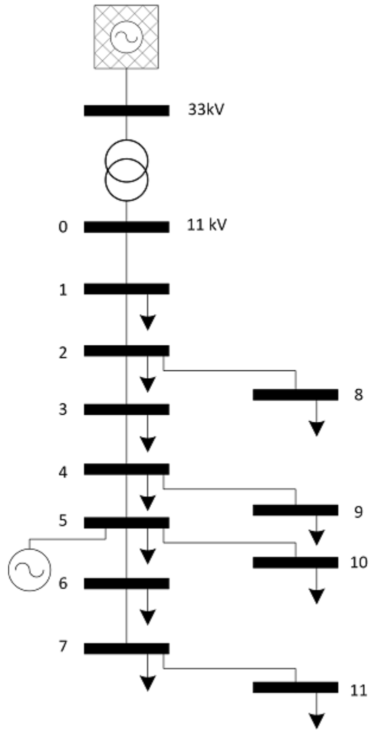

Fig. 4. Chosen sample of the UKGDS grid used for the analysis.

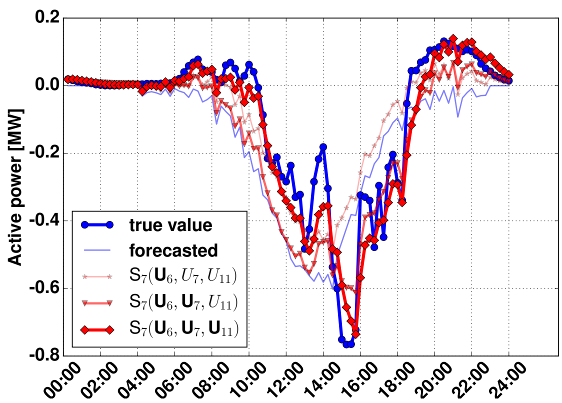

Fig. 5. Estimation of active power at node 7, forecasted value (light blue), actual value (dotted blue), and estimated values with observed active power at node 6 only (stars), at nodes 6 and 7 (triangles) and nodes 6,7 and 11 (square).

load observer we analyze three different cases for node 7 in the network considered. As visible from the grid structure in Figure 4, node 7 is connected with nodes 6 and 11. Therefore, the three conditions are:

1. Voltage is measured at node $\{6\}$, active power is not measured at any nodes;
2. Voltage is measured at node $\{6, 7\}$, active power is not measured at any nodes;
3. Voltage is measured at node $\{6,\ 7,\ 11\}$ (fully observable), active power is not measured at any nodes.

As our actual value we have chosen the load photovoltaic data from 20th May 2013 at Klant 63. The forecasted values were generated according to Eq. (13) for set parameters {"Spring", "High"} respectively. The estimation value is closer to the real value of power for the case of a fully observable system. This is also visible from Figure 6, where the difference between the true value and the estimate are presented.

5. Conclusion and outlook

For low- and medium-voltage networks forecasting-aided state estimations like the nodal load observer considered here have to rely on suitable dynamic models and good pseudo-measurements. For the former we proposed (vector) autoregressive stochastic processes, as a versatile model with the possibility to include correlations between network nodes. For improved pseudo-measurements we presented a generic mathematical model derived from reference data. Such a model provides realistic pseudo-measurements with flexible parameters to adapt to specific scenarios. The proposed approaches provide stochastic expressions for their reliability, which can be utilized in the extended Kalman filter to assign the state covariance matrix, instead of choosing arbitrary values as has been done before. Future research will focus on assessing the improved estimation quality of actual network data.

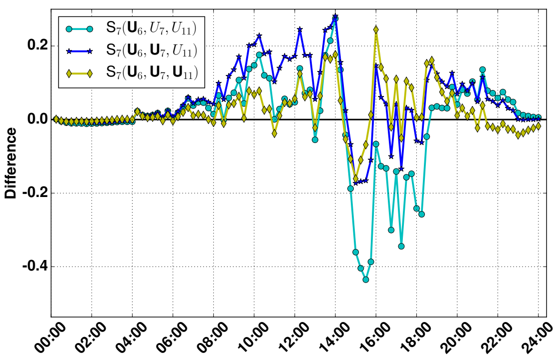

Fig. 6. Difference between estimated and true value for node 7 in the case of three various observations.

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