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MEASUREMENT OF FREQUENCY SPECTRUM WITH INTERPOLATED ADAPTIVE CHIRP-Z TRANSFORMATION

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Abstract – In this paper Adaptive Chirp transform (ACT) and Chirp-z transform (CZT) for measurements of frequency spectrum are presented. Algorithms are adjusted according to IEC 61000-4-7 standard. Interpolated ACT with narrowed frequency span is used for Phase Lock Loop (PLL) synchronization. The CZT is used along with windows function, to obtain the amplitudes of signal harmonics and interharmonics. Window function length is defined by PLL. All parts of proposed algorithm are tested through various simulations with changing parameters of analysed signals. This paper shows that CZT can be adjusted to synchronization and measurement of frequency spectrum. Results are comparable to the traditional approach (DFT) and completely satisfy the needs according to standard IEC 61000-4-7.

Keywords: Chirp-Z transformation, interpolated DFT, PLL synchronization, frequency measurement

1. INTRODUCTION

Basic standard for defining harmonics, interharmonics and associated instrumentation is IEC 61000-4-7 *Testing and measurement techniques – general guide on harmonics and interharmonics measurement and instrumentation, for power supply systems and equipment connected thereto*. With growing number of new participants of the electrical grid (local renewable generators with integrated invertors) and with increasing number of nonlinear loads on grid, the need for detailed and unambiguous definitions of principles and parameters in power spectrum measurement occur. Exact definition of signal spectrum parameters seeks to ensure absolute comparability of two power quality instruments [1].

1.1. Grouping of harmonics and interharmonics

Last version of IEC 61000-4-7 standard was made in 2002 with appendix A1:2008. Standard describes additional requirements for harmonics and interharmonics measurement [2]. Spectrum parts which define harmonics and interharmonics cannot overlap. Spectrum measurement is divided into two frequency spans: first span up to 40th harmonic and second span from 40th harmonic to 9 kHz. According to Nyquist-Shannon sampling theorem, analysis of second span requests higher sampling frequencies and accurate AD converter. For analyzing 9 kHz span the sampling frequency must be at least twice higher. Analyzed signal data length is set to 200 ms regardless of the nominal frequency, which results in 10 periods of 50 Hz signal and 12 periods of 60 Hz signal. Data length defines 5 Hz frequency resolution of signal frequency spectrum. Leaking of energy on spectral bins around each harmonic still exists due to impossibility of absolute synchronization of data length with signal frequency.

Standard IEC 61000-4-7 solves the leaking problem by grouping basic harmonic with left and right contiguous frequency bins. Grouping is made by:

$$\mathrm{X_{g\_h}} = \sqrt{\sum_{k=-1}^{1} \mathrm{X}^2(\mathrm{g}+\mathrm{k})}, \qquad (1)$$

where X(h) is amplitude of h^{th} harmonic and $Xg_{\_h}$ is amplitude of grouped harmonic. Grouping of harmonics and interharmonics in first spectrum span (up to 2 kHz) is presented in Fig. 1.

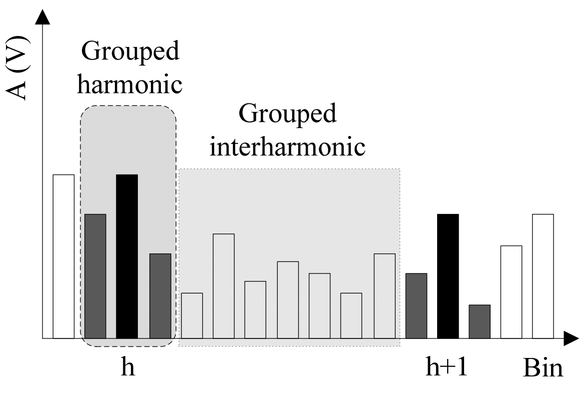

Fig. 1. Grouped harmonics and interharmonics.

Interharmonics are grouped similar to (1). Definitions of grouped harmonics and interharmonics do not overlap. Consequently the good synchronization needs to be performed. The data length should be strongly associated to signal frequency. Unsuitable synchronization could lead to mixing the harmonics group bins with interharmonics bins. Good synchronization need to assure the stability of defining 40th harmonic in the range of ±5 Hz. This condition results in maximum 0,125 Hz frequency measurement error of fundamental signal frequency. The bigger error will lead to mixing of harmonics and interharmonics and inability of defining amplitude of signal spectrum component.

1.2. Influence of fundamental frequency measurement

Spectrum of signal with 0,3 V amplitude on the 40th harmonic is presented in Fig. 2. In this case the synchronization error is set to the border value of 0,125 Hz. The amplitude of harmonics is well measured and error is not significant because the most of harmonic energy is moved to 399th bin which is included in calculation of 40th harmonic according to eq. (1).

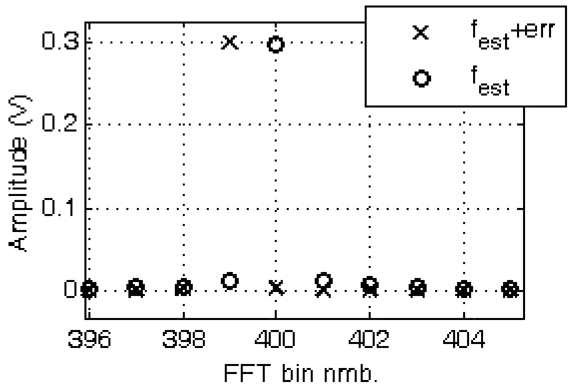

Fig. 2. 40th harmonic (includes bins: 399th, 400th and 401st), with good synchronization (f_{est}) and with synchronization error of 0,125 Hz (f_{est}+err).

With significant synchronization errors in estimation of signal frequency, the harmonics components can drift to the bins which define the interharmonics. Example with frequency synchronization error equal to 0,2 Hz is shown in Fig. 3.

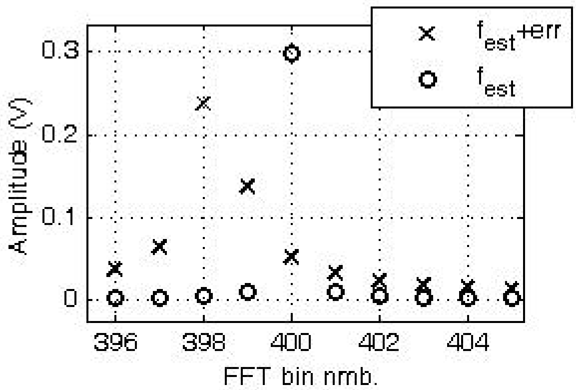

Fig. 3. 40th harmonic (includes bins: 399, 400 and 401), with good synchronization (f_{est}) and with synchronization error of 0,2 Hz (f_{est}+err).

Huge amount of signal energy disperses on the neighbor bins for the signal with inadequate synchronization. Part of dispersed energy will be used to define 39th interharmonics. Main spectral line is moved from 399th bin to 398th bin which is not included in 40th harmonic definition. This simple example presents the importance of good synchronization and good Phase Lock Loop (PLL) [2].

2. THE PROPOSED CZT BASED ALGORITHM FOR MEASUREMENT OF FREQUENCY SPECTRUM

First harmonic can be determined with some additional methods. IEC 61000-4-7 standard set the data length to ~200 ms. With such a long data set the best method to determine the first harmonic is Interpolated Discrete Fourier Transform (IpDFT) [4]. Our proposed method used Adaptive Chirp-z Transform (ACT) [5] for synchronization. Measurement setup according to IEC 61000-4-7 with ACT is presented in Fig. 4.

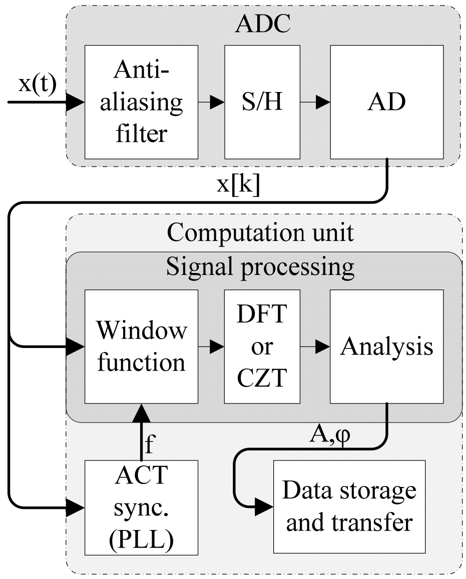

Fig. 4. The schematic diagram of the instrument for harmonics and RMS measurements.

In signal processing part the DFT is most commonly used for calculation of frequency spectrum components. Chirp-z transform (CZT) which is based on DFT is also frequently used for signal spectrum measurement. The CZT data length needs to be synchronized with analyzed signal in the same way as for DFT [6].

2.1. Adaptive Chirp-z-transform - ACT

ACT is presented in [5] and [7]. First step in calculation of ACT is narrowing the frequency band. First step is done by the adaptive P regulator which minimizes the error of non-linear model of the first harmonic signal. Adaptive filter in just one iteration decreases error of model function for observed frequency span. Amplitude and frequency errors are calculated for initial parameters and for first iteration of P regulation. Good initial parameters are defined with fast simple algorithm such as Zero-Crossing method. Algorithm flow is presented in Fig. 6.

The main parts of ACT are initializing fast methods for estimation of the beginning information (amplitude and frequency of first harmonic). This estimation produced the first parameter e_1 – estimation error (dimension 1x2). Next parameter is e_2 which is made by slightly change of estimated amplitude and frequency. According to these two pars of error information the adaptive regulation of signal estimated amplitude and frequency are made. Finally, with narrowed frequency span and according to the need of specific application, the corresponding frequency range and position of frequency bins for CZT are made.

2.2. Adjustment of CZT to Measurement of frequency spectrum

CZT is algorithm for calculation of z-transform of signal with *N* elements. It is widely used in energy measurement [8], rotor speed measurement [9], data filtering etc.

CZT is calculated by using DFT. Laplace transform and Fourier transform are only special cases of z-transformation:

$$X(z) = \sum_{n=0}^{N-1} x_n z^{-n}. \qquad (2)$$

For Laplace transformation:

$$z = e^{s\Delta t}, \qquad (3)$$

where Δt is time resolution defined by sampling rate. For DFT:

$$z(k) = e^{-\frac{j2\pi k}{N}}, k = 0, 1, \ldots, N-1. \qquad (4)$$

Generally z(k) can be written as:

$$\mathrm{z(k) = AW^{-k}, k = 0, 1, \ldots, M-1.} \qquad (5)$$

M is the number of bins in z-domain i.e. frequency spectrum.

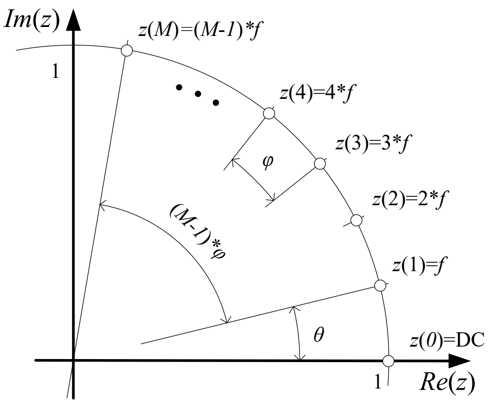

Fig. 5. Clip of |z|=1 circle. Points are set for both even and odd harmonics measuring. Parameter *f* is signal frequency.

Parameters A and W are complex numbers and they are defined as:

$$\mathrm{A = A_0 e^{j2\pi\theta}}, \ \mathrm{W = W_0 e^{j2\pi\varphi}}, \qquad (6)$$

where A defines starting point in z-domain and W amplitude modulation and exact position of *z(k)* points. A0 refer to the amplitude of starting point. Parameter θ is angle of starting point and φ is angle step between two sequential points as presented in Fig. 5.

All parameters A and W, and therefor the angles θ and φ, can be adjusted to the specific needs of application. Fig. 5. shows the exact position of CZT frequency bins for harmonics measurement. For measuring the values and contribution of signal harmonics according to IEC 61000-4-7 the neighbour bins also need to be taken into consideration. The base of CZT needs to be adjusted for this specific task. In this analysis the CZT bins are adjusted to have the same resolution as DFT. The ratio between two bins in frequency domain is 5 Hz.

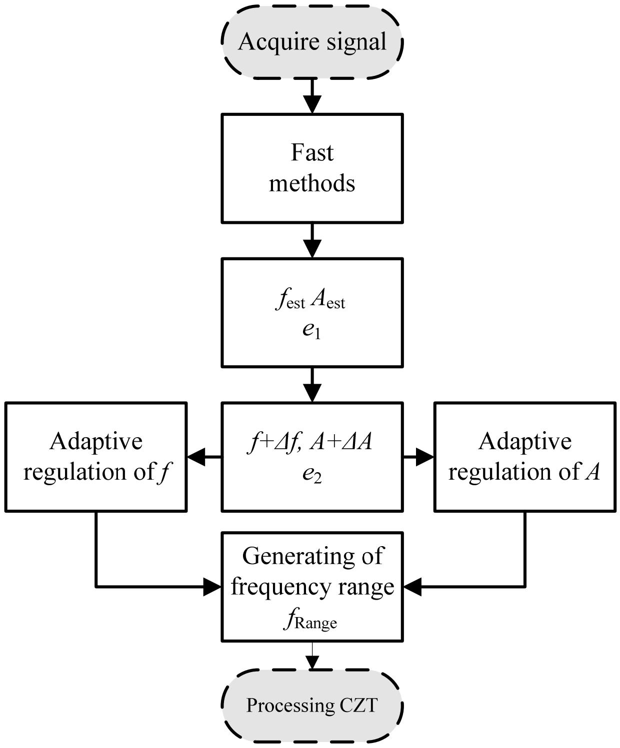

Fig. 6. ACT algorithm flow (reproduced from [5]).

3. RESULTS OF DFT AND CZT COMPARISON

ACT (CZT), inside PLL, needs to be adjusted for synchronization. In this case the f_{Range} for power frequency metering, the point A and W from (6) and number of bins *M* (Fig. 5.), are set to the corresponding values:

$$\mathrm{A = e^{-\frac{j2\pi}{N}}}, \ \mathrm{W = e^{shift\frac{2j\pi}{N}}}, \ shift = 7, \ M = 7. \qquad (7)$$

In this way ACT creates, for the PLL, the fixed frequency range [35, 65] Hz with 5 Hz step. After processing CZT on input data the 3-point interpolation for used Hann window function is calculated, according to [10]:

$$f_{apx.} = 2\frac{(X_{\mathrm{CZT}}(i_{\mathrm{MAX}}+1) - X_{CZT}(i_{\mathrm{MAX}}-1))}{(X_{\mathrm{CZT}}(i_{\mathrm{MAX}}+1) + 2X_{\mathrm{CZT}}(i_{\mathrm{MAX}}) - X_{\mathrm{CZT}}(i_{\mathrm{MAX}}-1))}. \qquad (8)$$

Where X_{CZT} is the CZT of signal *x* and the index i_{MAX} is the maximum of transformed signal. Now with knowing approximated frequency ($f_{\mathrm{apx.}}$) of input signal the window length for harmonics measurement can be determent.

According to IEC 61000-4-7 standard for harmonics measurement in z-domain 408 bins should be created.

ACT frequency spectrum of signal with changing first harmonic frequency, from 44 to 56 Hz, is presented in Fig. 7.

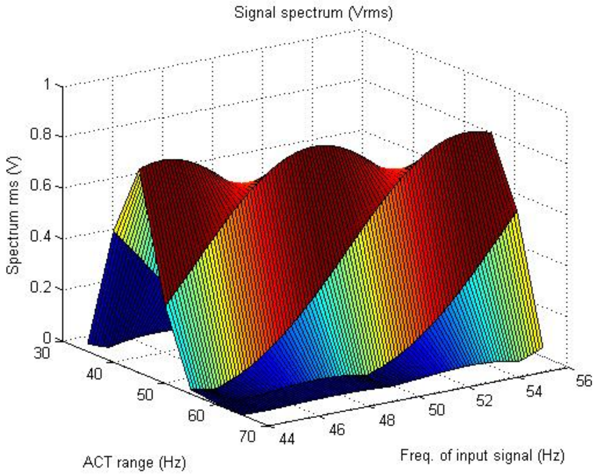

Fig. 7. Frequency spectrum of first harmonic measured with ACT.

The level of amplitude of ACT spectrum characteristic is presented in Fig 8.

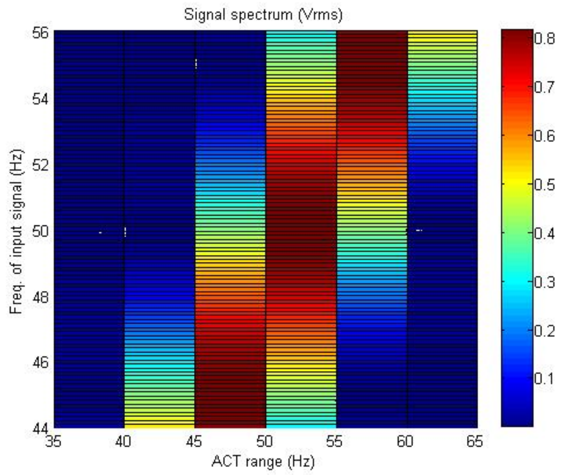

Fig.8. 2D presentation of first harmonic amplitude transition over specified ACT frequency range.

Results for first harmonic frequency estimation are presented in Fig. 9. Signal is disturbed with 40 dB of white noise. Simulation is made for wide range of signal frequency and repeated for 100 times, averaged results both for interpolated DFT and interpolated ACT are presented. In Fig. 9. can be seen that there is no significant difference in results.

4. CONCLUSION

This paper presents capability of CZT and ACT transforms in measurement of frequency spectrum. Frequency spectrum is based on harmonics and interharmonics measurement and grouping the energy distribution of spectral components.

CZT and ACT have proved to be equally effective as DFT based algorithms. Main contribution of this paper is using interpolation from IpDFT on CZT results.

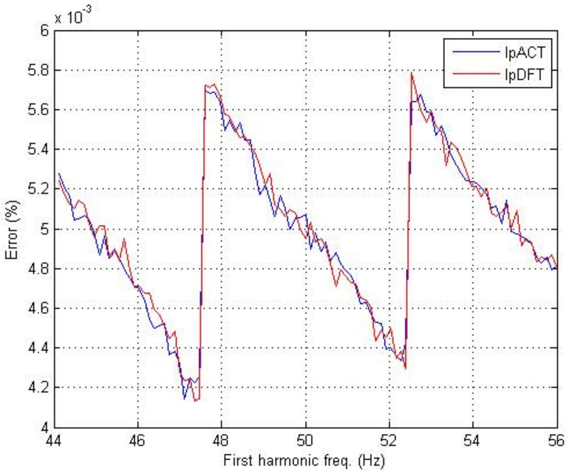

Fig. 9. Result of DFT and ACT synchronization comparison.

Also it is shown that ACT can be both adjusted for PLL synchronization an for signal spectrum measurement.

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REFERENCES

[1] P. Bilik, "Measurement of voltage and current harmonics for frequencies up to 9 kHz according to IEC61000-4-7", 10th International Conference on Electrical Power Quality and Utilisation, EPQU 2009. Str. 1. – 5., 2009.

[2] H. C. Lin, C. H. Chen, L. Y. Lui, "Harmonics and Interharmonics Measurement using Group-harmonic Energy Distribution Minimizing Algorithm", Engineering Letters, 19:3, 2010.

[3] A. Antonelli, S. Giarnetti, F. Leccese, "PLL System for Harmonic Analysis", 10th International conference on environment and electrical Engineering (EEEIC), pp. 1 – 5, May 2011.

[4] T. Radil, P. M. Ramos, A. C. Serra,: "New Spectrum Leakage Correction Algorithm For Frequency Estimation Of Power System Signals", IEEE trans. On Instrumentation and measurement, vol. 58., No. 5., May 2009.

[5] P. Mostarac, R. Malarić, H. Hegeduš, "Adaptive Chirp Transform for frequency measurement, Elsevier Measurement", Vol. 45, No. 3, pp. 268 – 275, April 2012.

[6] K. J. Runtz, D. Hack, A Multistage DFT – FFT – CZT Approach for Accurate Efficient Analysis of Sparsely Distributed Spectra, Proceedings of the 2002 IEEE Canadian Conference of Electrical & Computing Engineering, pp. 127 – 132, 2002.

[7] P. Mostarac, R. Malarić, H. Hegeduš, "Measurement uncertainty of adaptive Chirp-z transform", Elsevier Measurement, Vol. 46, No. 9, pp. 268 – 275, November 2013.

[8] M. Aiello, A. Cataliotti, S. Nuccio, "A chirp-z transform-based synchronizer for power system measurements", IEEE Transactions on Instrumentation and Measurement, vol. 54., No 3. pp. 1025 – 1032, 2005.

[9] M. Aiello, A. Cataliotti, S. Nuccio, "An induction motor speed measurement method based on current harmonic analysis with the chirp-Z transform", IEEE Transactions on

Instrumentation and Measurement, vol. 54., No. 5 , pp. 1811 – 1819, 2005.
[10] D. Agrež, “Weighted Multipoint Interpolated DFT to Improve Amplitude Estimation of Multifrequency Signal”, IEEE Trans. Instrum. And Meas. Vol. 51. br. 2, travanj 2001.