XXI IMEKO World Congress "Measurement in Research and Industry"
August 30 – September 4, 2015, Prague, Czech Republic

PROBLEMS IN IMPLEMENTING NEW MEASUREMENT UNIT DEFINITIONS OF THE SI BY USING FUNDAMENTAL CONSTANTS

F. Pavese

Torino, Italy, email: frpavese@gmail.com

Abstract – The paper is a digest of some problems to be considered should the proposed use of fundamental constants in the definition of measurement units of the SI be implemented: (a) more base units being multi-dimensional, instead of fixing the present problems in this respect; (b) the multidimensionality in the definitions; (c) how the magnitude of the base units is established; (d) the use of CODATA adjusted values of the constants for this specific purpose; (e) formal issues in stipulating algebraic expressions of the definitions, and in respect to the rounding or truncation of the numerical values; (f) formal issues with the use of the integer number N_A; (g) limitations in new determinations that might arise from the stipulation of the values of several constants, for the CODATA to continue performing in future meaningful least squares adjustments of the fundamental constants taking into account future data; (h) implementation at the NMIs and Society level.

Keywords: SI, base units, New SI, fundamental constants

1. INTRODUCTION

The use of some 'fundamental constants' (c_0, h, e, k_B, N_A) with stipulated numerical values has been proposed for the re-definition of some of the present *base* units of the International System of Units, the SI [1, 15, 21, 22]. This proposal was submitted at its 2011 meeting to the CGPM, which decided in its Resolution 1 to "*take note of the intention of the CIPM*" for "*the possible future revision of the International System of Units*" [2]. This position was confirmed in 2014 and the present roadmap to 2018 is available in [23].

In [23] on the page "On the future revision of the SI" under the title "What" one finds the following statement:

"… *In brief, the* [new] SI will be [in [1] "will be" is replaced by "is"] *the system of units in which:*

• *the unperturbed ground state hyperfine splitting frequency of the caesium 133 atom* $\Delta\nu(^{133}Cs)_{hfs}$ *is exactly 9 192 631 770 hertz,*

• *[and so forth for each of the seven base units: the [specific constant] is exactly [numerical value] [SI unit], using c, h, e, k and* N_A*],* …

… *where the hertz, joule, coulomb, lumen, and watt, with unit symbols* Hz, J, C, lm, *and* W, *respectively, are related to the units second, metre, kilogram, ampere, kelvin, mole, and candela, with unit* symbols s, m, kg, A, K, mol, *and* cd, *respectively, …*".

This can be called the '*group-definition*' and considered the sufficient new SI definition.[1]

In addition, the statement follows that "*The SI may alternatively be defined by statements that explicitly define seven individual base units: the second, metre, kilogram, ampere, kelvin, mole, and candela. These correspond to the seven base quantities time, length, mass, electric current, thermodynamic temperature, amount of substance, and luminous intensity. All other units are then obtained as products of powers of the seven base units, which involve no numerical factors; these are called coherent derived units*". These would be the '*single-unit*' definitions of the *base* units, presently implemented in [1] (see Section 2.3). [2]

Thus, in the "New SI" proposal the *base units remain the present ones*.

Problems can be found in the literature over the past years concerning the proposal on the floor, related to four main areas: (i) how meaningful a progress can be achieved by using fundamental constants in the definition of the measurement units; (ii) which should the constants used for and how the definitions should use them; (iii) how the definitions should precisely be formulated; (iv) how to deal with future data.

In this paper, which is intended to be a digest of some of the problems, the following issues are summarised:

(1) How to handle more multi-dimensional base units instead of fixing the present problems in this respect;

(2) How to take into account the multi-dimensionality in the definitions;

(3) How do the fundamental constants establish the magnitude of the SI base units, considering that no definitional realisation method for the latter is anymore defined;

(4) How to use, or not to use, CODATA 'adjusted values' [3] of the constants for this specific purpose, being the main purpose of the adjustments rather to obtain the best measure of the consistency of a much wider set of constants;

(5) How to handle limitations that can arise from the stipulation of the values of several constants, namely for the CODATA Task Group to continue performing in future meaningful least squares adjustments of the fundamental constants, and taking into account future data;

---

[1] In fact, these constants should be considered as a group. [21]

[2] References dropped for limitations in the manuscript length.

(6) Implementing the New SI at the NMIs and Society level;

(7) How to deal with formal issues in stipulating algebraic expressions in the definitions, namely in respect to the rounding or truncation of the numerical values in their transformation from uncertain to exact values;

(8) How to deal with formal issues related to the integer number N_A.

For a discussion concerning alternative definitions of specific base units, and more on items (i) and (ii), see [16, 24, 28, 34-36, 40].

2. PROBLEMS OF PRINCIPLES

The aim of the International System of Units (SI) is to provide a system of units, one for each of a specified (and limited) list of quantities necessary to express measured properties in nature. Items (1)–(4) of the Introduction fall under this heading.

2.1. Meaning of 'base unit' of the SI

A long-lasting debate led to seven quantities that are called fundamental and whose units are called "base" [4]. Each of these units was originally aimed at being defined without having to resort to the unit of any other quantity. They were assumed to be dimensionally *independent* [5]. All the remaining are conversely called "derived units", since one has to express each of them as an algebraic expression of the seven base units.

There are some properties of unit systems that are desirable, and there are others that are essential. A property that is not essential but is desirable is that the system of units is *coherent*. For detail on this issue see [6].

A property that is mandatory for any system of units, and consequently often assumed achieved also by the SI, is internal *consistency*. For base units defined independently from each other this is *automatically* achieved. However, this principle is *presently not* implemented anymore for several base units of the SI (see section 2.2). In this situation, consistency cannot be assumed *a priori*, nor can be exactly implemented. For non-independent definitions, formal or logical consistency, in the sense of a syntactic property relating unit definitions, can be quite problematic to prove. There is another, more specifically metrological, meaning of the term consistency that is often used. It is defined in [7] as a kind of "metrological compatibility", a property of a set of experimental determinations affected by uncertainty. For details see [14].

With the new proposed definitions, the above property could become greatly complicated to prove. In fact, the constants c_0, h, e, k_B are *not* each the direct expression of a base SI unit: one has to express the relevant units as algebraic expressions of the constants (where the use of the *stipulated* values for the constants,[3] here indicated with an asterisk, is mandatory—for comments on their numerical values see later):

$$[c_0/\Delta\nu(^{133}\mathrm{Cs})] = [\mathrm{metre}], \qquad (1)$$
$$c_0^{*}/\Delta\nu(^{133}\mathrm{Cs})^{*} = 3.261\,225\,561\,106\,45\ldots \times 10^{-2}\ \mathrm{m};$$
$$[h\,\Delta\nu(^{133}\mathrm{Cs})/c_0^{2}] = [\mathrm{kilogram}],$$
$$h^{*}\,\Delta\nu(^{133}\mathrm{Cs})^{*}/c_0^{*2} = 6.777\,265\,181\,006\,83\ldots \times 10^{-41}\ \mathrm{kg};$$
$$[h\,\Delta\nu(^{133}\mathrm{Cs})/k_B] = [\mathrm{kelvin}],$$
$$h^{*}n\,(^{133}\mathrm{Cs})^{*}/k_B^{*} = 4.411\,764\,153\,914\,89\ldots \times 10^{-1}\ \mathrm{K};$$
$$[e\,\Delta\nu(^{133}\mathrm{Cs})] = [\mathrm{ampere}],$$
$$e^{*}\,\Delta\nu(^{133}\mathrm{Cs})^{*} = 1.472\,821\,956\,237\,59\ldots \times 10^{-9}\ \mathrm{A};$$
$$[1/N_A] = [\mathrm{mole}],$$
$$1/N_A^{*} = 1.660\,539\,000\,996\,49\ldots \times 10^{-24}\ \mathrm{mol}.$$

(N_A depends only on the unit mol—but see [17]; $\Delta\nu(^{133}\mathrm{Cs})$, and K_{cd} are not "fundamental constants")

2.2. Definition of a base unit making use of other base or derived units

As indicated before, already at present some base units are not dimensionally independent: the unit of length involves the second; the unit of amount of substance involves the kilogram; the unit of electric current involves the newton and the metre; the unit of luminous intensity involves the hertz, the watt and the steradian.

As shown in the expression (1) above, this fact is more extensive for the proposed new units. Only a *degree of consistency* can be obtained, with an uncertainty attached to it. In principle, the text of each definition should also make explicit reference to all other units used. This may result in being difficult, or even *practically impossible*, to write down a "single-unit" definition that is metrologically correct.

The definitions of the base units should *not* make use of any derived unit. For more details on base units see [20].

2.3. Magnitude of the proposed new base units in the lack of a definitional method [37]

The wording for the proposed new definitions [1] is of the following type: "*… its magnitude is set by fixing the numerical value of the [a constant] to be exactly [numerical value] when it is expressed in the SI unit for [kind-of-quantity] [new SI units]*".

Literally, what is stipulated is the value of the constant. Saying that the magnitude of the unit is set by fixing the value of the constant—should even that true, see below—does not mean, strictly speaking, that the magnitude is also stipulated (see also Section 3.1).

First of all, because the result of an algebraic operation on stipulated value is a rational number: this means that the number of digits is larger than the significant ones according to the uncertainty attached to it [1, 11], so being unsuitable for stipulation unless a further specific stipulation is done, which is not indicated in the proposed definitions.

Secondly, because the definitions refer only to the numerical values of the constants—obtained by using the present (old) units to set by stipulation their value in the proposed (new) definitions.

The numerical value of a quantity, so also the constants, is obviously determined by the magnitude of the units used in their determination. In principle, for the dimensionless

[3] The present numerical values originate from the set of measurement units that are *presently* defined—and as presently realised.

quantities, e.g. α, their numerical values are independent on the units used: they should treated as, for example, the mathematical constant π is. However, the numerical value is obtained experimentally, so, for example, the observed variability of the numerical value of α is shown in Fig. 1 as obtained from CODATA. [18, 8]

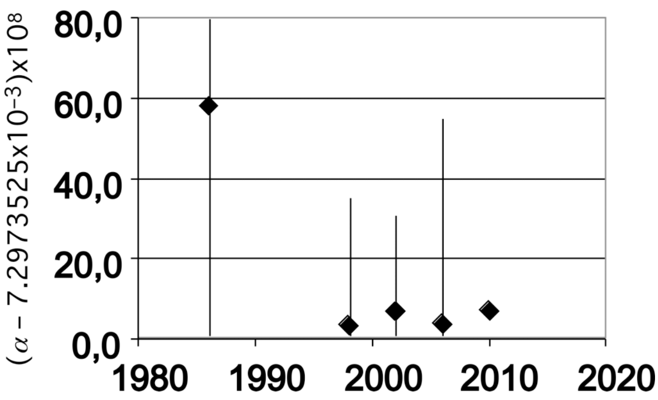

Figure 1. Values of dimensionless α as recommended by CODATA.

No definitional method or procedure for implementing the units is anymore indicated in the New SI definitions, so neither is explicitly a specific magnitude. [37]

Therefore, from an operational viewpoint, the proposed definitions do not make obsolete the previous standards, realised according to the former definitions. Consequently, at the moment of the change there is no contradiction between the new statement and the use of the previous (stable) standards (e.g., triple point of water, use of ^{12}C atoms, …). They become, *by definition*, part of the list of the allowed realisation procedures, at least initially. This *continuity requirement* subsequently holds in time until evidence of instability or inadequacy might emerge.

Continuity cannot be ensured exactly, but within the uncertainty level associated to the best realisations. According to [38], "*this residual uncertainty cannot be removed or reduced, being in fact the uncertainty of the related definition. With a better understanding of the problem we can reduce the uncertainty only by adopting a new independent definition and by changing consequently the unit*". However, "*In the case of constant-based definitions a certain discontinuity is unavoidable and one can only try to reduce its consequences. For instance, the present SI definition of the ampere suggests that two electric constants, namely μ_0 and R_K, have exact values. That is obvious from the relation $\mu_0/R_K = 2\alpha c$, where the fine structure constant is a dimensionless constant of Nature, which cannot be changed by a redefinition of the units.*

While at present the value of the magnetic constant μ_0 is adopted by the definition, the value of the von Klitzing constant is fixed by the same definition to an unknown value, a subject of measurement. The suggested definition adopts under certain conditions a value of R_K, while the value of the other constant, μ_0, becomes a subject of measurements.

We emphasize that each definition fixes in reality both constants exactly, with one of them known and the other to be measured. From the point of view of discontinuity, that means that we substitute one set of exact values by the other and there is no possibility to do such a substitution guessing the proper values to maintain their continuity.

From the practical point of view the discontinuity may be not seen immediately because of limited accuracy in measurements. However, while in the artefact-based scenario such an uncertainty is partly a residual one and cannot be reduced, in the case of a constant-based definition the improvement in accuracy is only a matter of time. In principle, with the suggested definition (if, e.g., one adopts certain values of e and h), earlier or later the magnetic constant μ_0 will be measured with such an accuracy that its departure from the value adopted in the present SI version should be clearly seen.

The CODATA results [8] reflect the best scientific knowledge on the subject to a certain date, nevertheless, we believe that CIPM should not try to adopt blindly values, the most close to the contemporary CODATA values, assuming that it may allow to avoid any discontinuity. We believe that CIPM should choose a strategy explained in [39] to reduce consequences of the unavoidable discontinuity".

On the other hand, the new definitions allow an extended range of realisations, to any measurement procedure having the potential to provide results metrologically compatible with the present definitions, in the sense that it would allow getting the same numerical values of the relevant constant(s) as a consequence.

2.4. Use of CODATA adjusted values of fundamental constants in the definition of a unit

For five base units, the relevant constant is one of the fundamental ones listed in the group-definition of the SI (c_0 having already been used since 1983 for the metre), included in the CODATA analysis.

For the second and the candela, the fixed numerical value is that of an invariant condition/physical state ($\Delta\nu$ (^{133}Cs) and K_{cd}, respectively), *not* presently included in the CODATA analysis.

Since 1973 a CODATA Task Group [8] performs a valuable check of consistency of a large set of 'fundamental constants' by means of the so-called Least Squares Adjustment (LSA) algorithm. At regular intervals, these evaluations produce a set of "adjusted values" and of the corresponding associated uncertainties (see in [8] for the full list of references). The aim of members of the CODATA Task Group, as expressed in their reports and publications is twofold: (i) "*a least squares adjustment (LSA) is one of the few ways in which the over-all consistency of physical theory can be systematically investigated. Moreover, it provides a consistent set of constants at a particular epoch which can be used by all workers requiring them*" [10]; (ii) to get "*particular numerical values obtained for a set of 'best' or 'recommended' set of constants …*" [9].

For aim (ii), the sentence reported above from [9] ends by saying that this feature "… *is only of secondary importance*", because the main importance is the "*information gained during the course of the critical review which necessarily accompanies* [precedes] *the adjustments*" [9]. Actually, extensive warnings in respect to the use of the LSA method form the whole Section C in [10] (see [8]). Another critical feature requiring attention is that pointed out in a "Warning!" in [9]: "*Because of the intimate relationship which exist among least-squares adjusted values of the fundamental constants, a significant shift in the numerical value of one will generally cause significant shifts in others. Consequently, for any critical application of these numbers, the user is urged to refer to the original article*[s] *as well as to the current literature* …". Their use in the definition of measurement units is certainly a very critical application.

Thus, the aim of the CODATA Task Group is:

(i) to use the least squares adjustment (LSA) method, as one of the few ways in which the over-all consistency of physical theory can be systematically investigated, to provide a consistent set of values of the constants at a particular epoch which can be used by all workers requiring them; [10]

(ii) to get particular numerical values obtained for a set of 'best' or 'recommended' set of constants. [9]

It is a fact that consistency is a most important requirement for a set of constants. However, its check is performed with tools suitable for that purpose, like the LSA, which are *different* from the tools used to obtain the numerical values of the chosen summary statistics directly evaluated from the—critically-selected—experimental data available for each constant.

Thus, confounding the results of the LSA with the several choices possible for the latter (e.g., mean, weighted mean, median, …) should be considered incorrect because [18]:

(a) The value of (at least) one member of the set—at arbitrary choice—is to be set fixed in the LSA, so that all the adjusted numerical values are relative to this choice (*relatively*-adjusted values)[4]—while any pair difference is invariant irrespective of the choice of the fixed member. If the CODATA adjusted values are used for the constants, as indicated in [1], it means that one considers them the best consistent set of these values according to the LSA criterion. This is true, but it does not necessarily means that they are suitable for the specific case of their application in the new units.

The LSA procedure used by CODATA has also been used, for example, by the Committee evaluating the data leading to the decision for possible values of the "atomic weights of the elements", in order to arrive at recommending a 'best set' of values [19]. However, there are basic differences between the atomic weight procedure and the CODATA procedure. In the former:

(i) all the analysed quantities are masses (of elements);

(ii) all mass values are relative to the mass of one atom of a specified isotope taken as the reference;

(iii) the assigned (relative) mass values do not need stipulation and retain their evaluated uncertainty (like the actual CODATA outcomes. This issue is not connected to the use of the LSA, but is useful to recall it here).

Thus, while the choice of the reference quantity that is kept fixed in LSA evaluation is arbitrary, in the case of the "atomic weights" there is an *additional condition* avoiding the intrinsic indeterminacy of the LSA method, i.e. the prior choice (decision) of a reference (the mass of one atom of ^{12}C).

On the contrary, in the CODATA use of the LSA, such an additional condition *does not exist*, hence the values of the constants are only *relatively* adjusted, the reference being taken for convenience, not dictated by any rule or need. All the adjusted values are relative to the choice of the reference, and they would be different for a different choice of reference (i.e., they are *biased*).

As reported in [18], it is recognised by the LSA users that it puts a relationship between all the analysed measured values. This fact constitutes the rationale for justifying the adjustment. In the case of the "atomic weights" this relationship fits the nature of the measured values involved, all mass values.

(b) In the case of the constants, not all of them are interlinked by physical relationships. The LSA adds this non-physical condition. This is proven by the fact that a constant may result subsequently adjusted even without any new measured values for it having become available since the previous adjustment: it is simply caused by new measured values for other constants having become available (examples in [18]).

The original values of the adjustable member(s) of the set depend on the chosen *inter*-subjective criteria, and are *altered* according to the LSA optimisation algorithm. The latter operation sets an "intimate relationship" [9] between *all* members of the set, which may have no physical meaning for some of them in the specific case of the fundamental constants. The issue here is that the obtained values only optimise the *consistency*, i.e. primarily the value of the statistical parameter used for evaluating the *uncertainty* of the set. For this there is a cost for the values: divorcing by a certain amount from the original physics world. This cost is generally irrelevant in other applications of the CODATA analysis. Also, it is not a characteristic of the LSA only, but of all methods having the same purpose, e.g. the methods with fixed effects. However, in the specific case of the measurement units, the CIPM may want (or need) to take advantage of the best accuracy allowed by the current experimental data, and this is a critical condition making incorrect the use of the CODATA relatively-adjusted *values*, and that may even result in missing the aimed goal [15].

In addition, the *continuity requirement*, discussed hereinafter in Section 3.1, is *more important than the consistency of the set of units*. Any adjustment of the initial statistical 'best value' should be considered extraneous to that requirement: one evidence is that an adjustment, due to

---

[4] Actually, α (or $1/\alpha$) would be a more natural choice than μ_0, being dimensionless.

the overall interdependence set by the LSA, may depend in principle on constants that do not have any dimensional link to the relevant best value concerned. The *continuity requirement for the set of base units puts constraints that have no relationship whatsoever with the set of constraints set by the LSA evaluation*.

(c) The above difficulties are generally also involving the *adjusted uncertainties* typically lower than the experimental ones, associated with the relatively-adjusted values: it may happen that the uncertainty aimed level is satisfied by some adjusted constants, but not yet achieved in actual experimental determinations.

The LSA outcome comes as a pair of parameters for each constant: a value and an uncertainty (the target of the LSA being the minimisation of the overall uncertainty of the set). It looks that stripping out the latter, as required by the stipulation of the value, after which the value is then considered exact irrespective of its uncertainty, substantially alters the justification of the LSA procedure, or, at least, clearly indicates that the stipulated value cannot have a generalised meaning, but it is exclusively valid for the metrological applications.

(d) the LSA should be considered as a mathematical tool (it is not a statistical tool since it does not involve any inference), useful only and specifically to evaluate the *degree of consistency* of a set of measured values the consistency of which remain unknown. For this function it is largely used and useful in many fields, because it fits a specific purpose.

For more details on issues (a-d) see [18]. On the other hand, the competent critical review performed by CODATA on all the available measured values prior to the application of the LSA is also an evaluation of great value. However, these sound aspects of the CODATA task should not be thought of to generate the possibility of a more generalised use of its outcomes.

3. PROBLEMS IN PERSPECTIVE FOR SCIENCE AND METROLOGY

The item (5) in the Introduction falls under this heading.

3.1. Future possibility of considering new determinations of the fundamental constants: how to avoid circularity

A question arises: is it useful (or correct, depending on the viewpoints) that CODATA analysis and outcomes always incorporate constraints arising from the basically regulatory field needed in metrology, for decisions concerning the measurement units? In general science, the scientists are basically interested in consistency of the physical theories, but the present and future LSA evaluations of consistency are *biased* by metrological issues. See [18] for the reason of bias.

If the CGPM will eventually adopt the CIPM proposal, more constants will have their numerical values stipulated. Whence, many more will also get fixed numerical values because of the inter-relationship between constants placed by the LSA method, considerably reducing the number of the freely-adjustable ones according to the present philosophy. The number of the remaining constants might be so small that the whole CODATA task could become irrelevant or even terminated.

This situation would also place delicate question marks on the handling of future experimental determinations of the constants, namely of the stipulated ones, since an opinion is frequently found that no new determinations of the constants will be possible after the stipulation of their values, without incurring in a circular process invalidating any new realisation.

However, the literal meaning of the wording of the *single-unit* definitions derived from the group-definition, as reported above in Section 2.3, can be spelled out as follows: *According to the new definition, the magnitude of each of the base units is 'such that' the chosen stipulated numerical value of the relevant constant apply.*

As said in [18]: "*By assuming that significant discontinuities between the new and the previous units are avoided, the experimenters can then treat the local realizations of the constants as they were with the former units, and compute the new actual values as they have done so far*". More explicitly, one can say that in metrology a specific requirement must be met: to avoid significant changes in the magnitude of the units when changing from the old and the new units.

This is the universally accepted *requirement of "continuity"*.

The consequence in that the circularity induced by stipulation is *removed* by the above condition, because [37]:

- At the time of the change, the old and the new units have the same magnitude, so the old (stable) and the new standards realising them are *interchangeable* in measurement with the current uncertainty;
- Subsequently, the continuity condition *holds in time* until one can obtain sufficient evidence of the fact that significant drifts in time of the values of some of the realisations of the old units have occurred.

Thus, future further determinations of the values of the constants are possible. On the other hand, nobody can prove that the stipulated values are "ultimate" [28] (see more of [28] in [41]). Will new determinations be taken into account in future? Possibly, it will not be in the interest of metrology to adjourn the stipulated value of a specific constant. On the other hand, it would certainly be advantageous for general science to have this new information taken in due account, even by the CODATA Task Group, according to the normal scientific method.

3.2. Future of the remaining fundamental constants

A statement is often found in the literature (e.g. [33]) that the use of five fundamental constants in the definition of the units, thanks to their stipulation, will nullify the uncertainty associated with other constants depending on the former. On the basis of the fact that a stipulation cannot propagate (Section 5.1), such statement looks not founded.

4. PROBLEMS IN PERSPECTIVE FOR THE NMIs AND THE SOCIETY

The item (6) in the introduction falls under this heading.

4.1. Realisation of the base unit definitions, metrological traceability and calibrations at the NMIs

The meaning of 'realisation' of the proposed base units is made ambiguous by a conflict between the literal meaning of the definitions and what is claimed in the BIPM documents [1, 23] and in most literature of the advocates of the proposed definitions—that fixed numerical values of a fundamental constants are the *ultimate* resource in physics (including quantum physics) to define a measurement unit. The latter statement is found also expressed as follows (FAQ#8 on the kilogram [23]): "*By fixing its [of a constant] numerical value we define the magnitude of the unit in which we measure that constant*". This is not true because:

− The unit of mass will not depend on a single constant, though presently dominant in determining its magnitude. The magnitude depends on all involved constants;

− The present text of the definitions, as said in Section 2.3, only resorts to the 'continuity requirement', stipulating the present value born from the *old* (present) units. There is no (explicit) indication of a stipulated magnitude of the units. In fact, as indicated in Section 2.3, the definition does not indicate a definitional method or procedure for implementing the units.

Thus, the present meaning of "mise en pratique" would be extended to *all the actual realisations of the units*, with the only constraint that they have the potential to allow the verification that the stipulated values of the constants are obtained.

However, in that respect, the realisation procedures could not be called anymore "mises en pratique" according to the present definition of this expression, [31] because the hierarchy [29] between the definitional and the 'other' realisations vanishes due to the formal lack of the former. *These realisations have a different status* with respect to the *mise en pratique* of the present definitions: by missing a default method/procedure in the unit definitions, they *all become true realisations of the definition* of the unit, not 'practical' lower-rank procedures (*mises en pratique*).

This is maybe the most important change in the practical features of the SI, possibly even deserving a new name.

On the other hand, the lack of a unique definitional method introduces, in principle, a definitional uncertainty: it consists of the non-uniqueness of the different realisations. In turn, the non-uniqueness corresponds to a change of magnitude in the realised unit.

This fact changes in many respects the duties of the NMIs wanting to realise independently their own National standards of the base units, as discussed in extent in [29]. Basically, since at the top of the metrological traceability pyramid there are (only) the very definitions of the units, this would require each NMI to provide its own determination of all the relevant constants, as the only means to demonstrate that its chain of standards comply with the condition set by the definitions.

Then, should the new definitions become "immaterial", how could a transfer standard be available that could be "calibrated" at (B) and used at (A)? Certainly it cannot be under the definition of "calibration" in VIM clause 2.39 in [7]. The only solution might seem to rely on one of the "old fashioned" standards included in the previous *mise en pratique*. However, as already pointed out, the issue here is not necessarily getting a smaller uncertainty, but the fact that what is transferred by (B) to (A) would not be the very definition of the unit, but a proxy. The very definition of the fundamental constant-based units looks to be possibly not able to be propagated among laboratories. Somebody should explain what could be considered a mass standard "calibrated" in order to ensure metrological traceability to exactly the value $6.777\,264\,729 \times 10^{-41}$ kg? Or, for a unit requiring a scale, how a calibrated thermometer could be considered traceable to a stipulated numerical value of the Boltzmann constant [and, at the same time, of the Planck constant and of $\Delta\nu(^{133}\mathrm{Cs})$]?

4.2. The hierarchical issue introduced into the Metre Treaty by the proposed formulation

The issues in the previous Section brings to another problem, not strictly scientific, but certainty of basic importance in the metrological frame: what will happen to the lack of hierarchy between the NMIs that the Metre Treaty ensures for the Countries preferring to avoid a dependence on another Country for a particular measurement unit, namely the base units.

At present, a hierarchy involves the *mises en pratique* as the first step of the pyramid below the unit definition. Quoting from [29], using the metre as an example, "*the mise en pratique lists the very definition of the metre as the allowed method (a) [the definition of the unit]. It is very possible—even, one would say, normal—that a definition can be directly implemented in practice. However, this fact should not be taken as implying that the other methods included in the mise en pratique are hierarchically at the same level of the definition of the unit, being the latter unique*".

The proposed new definitions for the base units, as they presently stand, still require each NMI to provide facts supporting evidence of metrological traceability up to the level of the new definitions, for the units that the NMI intends to realise nationally. This would imply the mandatory metrological need for future direct measurements of (some of) those constants at the NMIs [29]. This also is a basic change, since only a handful of NMIs in the entire world are likely to be able to stay at the top of the hierarchy. This fact has already been raised in [32] as a "loss of accreditation".

The latter issue is probably the most sensitive difficulty to overcome, in order to progress toward a widest acceptance of the proposal.[5]

[5] Items (7) to (8) indicated in the Introduction are dropped for space limitations in the manuscript length.