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METHODS OF CALCULATING TEMPERATURE VALUES AND ESTIMATING ERRORS FOR IDENTIFICATION OF THE STATE OF TEMPERATURE TRANSDUCERS

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Abstract – In consideration, methods for calculating the temperature values and temperature errors are proposed. These methods are used for development of the evaluation criteria state of temperature transducers. The reliability of methods and efficiency of criteria were evaluated by a computational experiment both for the series of model functions and for the experimental data. The computational results confirm the efficiency of the proposed method and indicate the advantages of our approach.

Keywords: self-test, accuracy of measurement, error estimates, inverse problem, regularization method

1. INTRODUCTION

The devices for temperature measuring are widely used in actual practice. One of these devices is the temperature transducer with Resistance Temperature Detectors (RTD), which is employed for example, in petrochemical processing, heat power engineering, industrial engineering, power system. The accuracy both determining the temperature and estimating the temperature transducer state are very important for reliability and safety of system. [1]-[6].

The operating principle of RTD is based on the dependence of electrical resistance of materials on temperature. The main difficulty is that the temperature dependence of the resistance has been well studied [7], [8], but the inverse dependence has been represented as a system of algebraic equations whose left sides contain polynomials with unknown degrees and coefficients. We propose a method for determining the required temperature values using the results of solving the parametric identification problem and the method for temperature errors estimation. These methods provided the basis for development of the evaluation criteria for identification state of temperature transducers

2. STATEMENT OF THE PROBLEM

We consider mathematical aspect of the problem of identification state for a temperature transducer with two resistance temperature detectors, which are made of different metals. Let R_{1k} and R_{2k} be resistance values of the first thermometer and the second thermometer, measured at temperature t_k, $k \in \langle 1; n \rangle$. According to [8], [9] the mathematical model describing the dependence of temperature on resistances is represented as follows

$$\begin{cases} \sum_{m=0}^{M} A_m R_{1k}^m = t_k, \\ \sum_{l=0}^{L} B_l R_{2k}^l = t_k. \end{cases}, \quad k \in <1; n> \qquad (1)$$

where both degrees M and L and coefficients A_i and B_j are unknown. In addition, instead of the exact t_k, we are given temperature values T_k and the allowable temperature error at the calibration stage δ. Using these initial data, it is required to obtain both degrees and coefficients in (1). Next, using obtained parameters of the system, it is required to calculate the temperature values and to estimate their deviation from T_k, and then to develop the criteria for identification state of temperature transducers.

Another important feature of the problem (1) is the possible errors of the resistance measurements. If the errors are caused by system factors only, we assume that the values R_{1k} and R_{2k} are given exactly. However, the resistance values receive deviations, which are occur under operation conditions. The levels of these deviations are described by the quantities h_{1err} and h_{2err}. In this case, we assume the resistance values are given with some error levels h_{1err}, h_{2err} and denote the resistance values with deviations as R_{1k}^h and R_{2k}^h.

3. MATHEMATICAL FOUNDATION

The first stage of temperature values calculation involves solving the inverse parametric identification problem (1). To do this, we apply the idea proposed in [9]. This approach based on the regularization technique and allows it to obtain coefficients with guaranteed accuracy for the reasonable exact resistance measurements.

Denote the matrices Q_1 and Q_2 as follows:

$$Q_1=\begin{pmatrix} R_{11}^{M} & R_{11}^{M-1} \ldots \ldots & R_{11} & 1\\ R_{12}^{M} & R_{12}^{M-1} \ldots \ldots & R_{12} & 1\\ \vdots & \ddots & \vdots & \vdots\\ R_{1n}^{M} & R_{1n}^{M-1} \ldots \ldots & R_{1n} & 1 \end{pmatrix},\quad Q_2=\begin{pmatrix} R_{11}^{L} & R_{11}^{L-1} \ldots \ldots & R_{11} & 1\\ R_{12}^{L} & R_{12}^{L-1} \ldots \ldots & R_{12} & 1\\ \vdots & \ddots & \vdots & \vdots\\ R_{1n}^{L} & R_{1n}^{L-1} \ldots \ldots & R_{1n} & 1 \end{pmatrix}$$

The exact values of t_k are denoted by $T_0=(t_1,t_2,...,t_n)$ and values of T_k are denoted by $T_\delta=(T_1,T_2,...,T_n)$. Then vectors $u=(A_M,A_{M-1},...,A_0)$ and $v=(B_L,B_{L-1},...,B_0)$ correspond to coefficients A_m and B_l, $m=\overline{0,M}$, $l=\overline{0,L}$ respectively. Next, it is required to avoid situation of the insufficient measurements or eliminate the situation of redundant measurements. To do this, we assume that $M>n$ and $L>n$, and then, we eliminate linearly dependent columns from matrices Q_1 and Q_2, and respective components of vectors u and v. Denote the obtained matrices as P_1 and P_2, and the obtained vectors as A and B. Then system (1) may be represented as:

$$\begin{cases} P_1A=T_\delta,\\ P_2B=T_\delta. \end{cases} \tag{2}$$

Since system (2) is overdetermined and condition number is rather high we use regularization method to solve this problem. To do this, we consider elements A,B,T_0,T_δ as the elements of non-zero separable Hilbert space H, and matrices P_1 and P_2 as finite-dimensional approximations of linear continuous injective operators P_1 and P_2. Assume that the exact solution $A_0\in M_{1r}$, $B_0\in M_{2r}$, where M_{1r}, M_{2r} are the spheres. The physical meaning of the problem allows us to do these assumptions.

To solve the problem (2), we use the approach proposed in [10], according to which the problem (2) is reduced to the variational problem:

$$\inf_A\left\{\|P_1A-T_\delta\|^2+\alpha_1\|A\|^2 : A\in H\right\},\quad \alpha_1>0,$$

$$\inf_B\left\{\|P_2B-T_\delta\|^2+\alpha_2\|B\|^2 : B\in H\right\},\quad \alpha_2>0,$$

where α_1 and α_2 are regularization parameters. The solutions to (3) are obtained via the regularizing operators and defined as follows.

Let σ_{1m} and σ_{2l} be the eigenvalues of the self-adjoint matrices $P_1^*P_1$ and $P_2^*P_2$, corresponding to their eigen elements e_{1k} and e_{2k}. From the Hilbert–Schmidt theorem [10] it follow that the regularized solution A_δ^α, B_δ^α of the problem (3) may be represented as the series

$$J_{\alpha_1}\left(P_1^*P_1\right)T_\delta=A_\delta^\alpha=\sum_{m=0}^{M}\frac{\sigma_{1m}\overline{T}_m}{\left(\sigma_{1m}\right)^2+\alpha_1}e_{1k},$$

$$J_{\alpha_2}\left(P_2^*P_2\right)T_\delta=B_\delta^\alpha=\sum_{l=1}^{L}\frac{\sigma_{2l}\tilde{T}_l}{\left(\sigma_{2l}\right)^2+\alpha_2}e_{2k},$$

where $\overline{T}=\left(\overline{T}_1,\overline{T}_2,...,\overline{T}_M\right)=P_1^*T_\delta$ and $\tilde{T}=\left(\tilde{T}_1,\tilde{T}_2,...,\tilde{T}_L\right)=P_2^*T_\delta$.

To choose regularization parameter and to estimate the values of coefficient errors in case, when $h_{1err}=h_{2err}=0$ we use functions, which are defined by the formulas:

$$\Delta_1=\sup_{A,T_\delta}\left\{\left\|J_{\alpha_1}\left(P_1^*P_1\right)T_\delta-A\right\| : A\in M_{1r},\|P_1A-T_\delta\|\le\delta\right\},$$

$$\Delta_2=\sup_{B,T_\delta}\left\{\left\|J_{\alpha_2}\left(P_2^*P_2\right)T_\delta-B\right\| : B\in M_{2r},\|P_2B-T_\delta\|\le\delta\right\},$$

Then we choose α_1 and α_2 as follow:

$$\alpha_1=\sup\left\{\alpha : \Delta_1\le\delta\|P_1\|\right\},\quad \alpha_2=\sup\left\{\alpha : \Delta_2\le\delta\|P_2\|\right\},$$

where $\|P_1\|$ and $\|P_2\|$ are maximal eigen values of the self-adjoint matrices $P_1^*P_1$ and $P_2^*P_2$.

We obtained the following coefficient error estimates for the reasonable exact resistance measurements

$$\frac{1}{2}\sqrt{r_1\delta}\le\Delta_1\le\sqrt{r_1\delta},\quad \frac{1}{2}\sqrt{r_2\delta}\le\Delta_2\le\sqrt{r_2\delta}.$$

If the resistance values are measured with some errors, we define the values of coefficient errors ΔA and ΔB by the formulas:

$$\Delta A=\sup_{A,P_1^h,T_\delta}\left\{\left\|J_{\alpha_1}\left(P_1^*P_1\right)T_\delta-A\right\| : A\in M_{1r},\right.$$
$$\left.\left\|P_1-P_1^h\right\|\le h_{err}^1,\ \|P_1A-T_\delta\|\le\delta\right\},$$

$$\Delta B=\sup_{B,P_2^h,T_\delta}\left\{\left\|J_{\alpha_2}\left(P_2^*P_2\right)T_\delta-B\right\| : B\in M_{2r},\right.$$
$$\left.\left\|P_2-P_2^h\right\|\le h_{err}^2,\ \|P_2B-T_\delta\|\le\delta\right\},$$

where matrices P_1^h and P_2^h are generated by R_{1k}^h and R_{2k}^h similarly as matrices P_1 and P_2.

We obtained the following coefficient error estimates

$$\Delta A\le\frac{\delta}{2\sqrt{r_1\|P_1\|h_{1err}+\delta}}+\frac{1}{2}\sqrt{r_1\|P_1\|h_{1err}+\delta}+\frac{r_1\|P_1\|}{\sqrt{r_1\|P_1\|h_{1err}+\delta}},$$

$$\Delta B\le\frac{\delta}{2\sqrt{r_2\|P_2\|h_{2err}+\delta}}+\frac{1}{2}\sqrt{r_2\|P_2\|h_{2err}+\delta}+\frac{r_2\|P_2\|}{\sqrt{r_2\|P_2\|h_{2err}+\delta}},$$

Thus, we proved, that for the reasonable exact resistance measurements the quantity of coefficient errors are dependent of accuracy on the calibration etalon only and this estimates are exact with respect to the order. This result allows it to calculate the temperature values with guaranteed accuracy. If the resistance values are measured with some errors we cannot to obtain temperature values with reasonable accuracy.

This feature of coefficient error estimates is used both for calculating temperature values under operating conditions and for the development methods to estimate the temperature errors.

3. TEMPERATURE VALUES CALCULATION AND TEMPERATURE ERRORS ESTIMATION

To calculate temperature values we substitute the obtained degrees and coefficients into the system (1). Let T_k^u and T_k^v be temperature values which are obtained for respective RTD due calibration process by the formulas.

$$T_k^u = \sum_{m=0}^{M} A_m R_{1k}^m, \quad T_k^v = \sum_{l=0}^{L} B_l R_{2k}^l, \qquad k \in <1;n> \quad (3)$$

The temperature values $T_{k\bar{\delta}}^u$ and $T_{k\bar{\delta}}^v$ are calculated likewise, but on the basis of resistance values measured in operation.

The average temperature values obtained from the system (1) for the first and second thermometers at the calibration stage are designated as $T_{Ni} = \left(T_1^u, T_2^u, ..., T_n^u\right)$ and $T_{Pt} = \left(T_1^v, T_2^v, ..., T_n^v\right)$ respectively. The temperature values obtained from the system (1) for the first and the second thermometers under operating condition are denoted as $T_{Ni}^u = \left(T_{1\bar{\delta}}^u, T_{2\bar{\delta}}^u, ..., T_{n\bar{\delta}}^u\right)$ and $T_{Pt}^v = \left(T_{1\bar{\delta}}^v, T_{2\bar{\delta}}^v, ..., T_{n\bar{\delta}}^v\right)$.

To estimate temperature errors we introduce functions $\Delta(T), \left(err\left(T_{\bar{\delta}}^u\right)\right), \left(err\left(T_{\bar{\delta}}^v\right)\right)$ as follow. We define

$$\Delta_k = \sum_{m=0}^{M} A_m R_{1k}^m - \sum_{l=0}^{L} B_l R_{2k}^l \quad , \quad k \in <1;n> . \quad (4)$$

Then the quantity $\Delta(T)$ takes on the value

$$\Delta(T) = \left| \frac{\max_k \Delta_k}{\min_k \Delta_k} \right| \quad (5)$$

We denote values $\Delta(T)$ obtained in the stage of calibration as *krN* and the values $\Delta(T)$, which are calculated under operation condition as *krU*.

The functions $\left(err\left(T_{\bar{\delta}}^u\right)\right)$ and $\left(err\left(T_{\bar{\delta}}^v\right)\right)$ describe affiliation of calculated temperature to the tolerance class. They are determined by formulas:

$$\left(err\left(T_{\bar{\delta}}^u\right)\right) = \max_k \left| \left(T_{k\bar{\delta}}^u - T_k^u\right) \right|,$$
$$\left(err\left(T_{\bar{\delta}}^v\right)\right) = \max_k \left| \left(T_{\bar{\delta}k}^v - T_k^v\right) \right|, \quad (6)$$

The functions $\Delta(T), \left(err\left(T_{\bar{\delta}}^u\right)\right), \left(err\left(T_{\bar{\delta}}^v\right)\right)$ provide the basis for the development of criteria to identifying the state of devices.

To evaluate the quality of the Validated Measurement Value, we apply classification, proposed in [6]. According tothis approach, distinguish the following measurement value statuses (MVS): CLEAR, BLURRED, DAZZLED, BLIND, SECURE and UNVALIDATED. To identify MVS we proposed the following criteria:

- CLEAR indicates that there is no fault, and Validated Measurement Value(VMV) has been calculate normally. The conditions *krU=const* and $|krU - krN| \le \varepsilon$ are indicate to this status. The quantity ε characterizes the calibration accuracy.
- BLURRED indicates that the measurement has been partially impaired by the presence of a sensor fault and that a correction has been applied in the calculation of the VMV. To indicate this status the both conditions *krU=const* and $\varepsilon < |krU - krN| < 3\varepsilon$ together with inequality $\left(err\left(T_{\bar{\delta}}^u\right)\right) < \bar{\delta}_1$, $\left(err\left(T_{\bar{\delta}}^v\right)\right) < \bar{\delta}_2$ must be hold. The quantities $\bar{\delta}_1$ and $\bar{\delta}_2$ correspond to temperature tolerance limits for respective materials.
- DAZZLED is a temporary status used when transducer data is clearly erroneous, but there is insufficient internal evidence to confirm that a substantial fault has occurred. To indicate this status the conditions $krU \ne const$ together with inequality $\left(err\left(T_{\bar{\delta}}^u\right)\right) < \bar{\delta}_1$, $\left(err\left(T_{\bar{\delta}}^v\right)\right) < \bar{\delta}_2$ must be fulfilled.
- BLIND indicates that a diagnosed fault has occurred which has a severe impact upon the measurement, and so the current VMV is projected from historical data, not live transducer data. To indicate this status the conditions $|krU - krN| > 3\varepsilon$ together with inequality $\left(err\left(T_{\bar{\delta}}^u\right)\right) \le \bar{\delta}_1$, $\left(err\left(T_{\bar{\delta}}^v\right)\right) \le \bar{\delta}_2$ must be hold.
- SECURE indicates that the VMV has been generated from redundant transducers or sensors, all of which are in nominal condition. The conditions *krU=const*, $|krU - krN| \le \varepsilon$ and $\left(err\left(T_{\bar{\delta}}^u\right)\right) > \bar{\delta}_1$, $\left(err\left(T_{\bar{\delta}}^v\right)\right) > \bar{\delta}_2$ together with $\left|\left(err\left(T_{\bar{\delta}}^u\right)\right) - \left(err\left(T_{\bar{\delta}}^v\right)\right)\right| < \varepsilon$ are indicate to this status.
- UNVALIDATED indicates that validation has not been in operation in the sensor which generated the measurement. To indicate this status it is sufficed that the condition $|krU - krN| > 3\varepsilon$ and one of the following inequality $\left(err\left(T_{\bar{\delta}}^u\right)\right) > \bar{\delta}_1$ or $\left(err\left(T_{\bar{\delta}}^v\right)\right) > \bar{\delta}_2$ must be fulfilled.

To describe the sensor's own assessment of its need for maintenance, in [6] proposed the following Device Statuses*:*

- *OK:* no maintenance requested.
- *Low Priority.* The device for temperature measuring is relatively fine, but needs maintenance.
- *High Priority.* It indicates that a diagnosed fault has occurred which has a severe impact upon the measurement, and so subsequent operation without maintenance is not reasonable
- *Critical:* the sensor has detected a condition which may lead to damage beyond the sensor itself.

To indicate these Device Statuses we proposed following criteria:

- The conditions $|krU - krN| < \varepsilon$ together with inequality $\left(err\left(T_{\bar{\delta}}^u\right)\right) < \bar{\delta}_1$, $\left(err\left(T_{\bar{\delta}}^v\right)\right) < \bar{\delta}_2$ are correspond to *OK* status.
- To define *Low Priority* Device status the condition $\varepsilon < |krU - krN| < 3\varepsilon$ together with inequality $\left(err\left(T_{\bar{\delta}}^u\right)\right) < \bar{\delta}_1$, $\left(err\left(T_{\bar{\delta}}^v\right)\right) < \bar{\delta}_2$ must be hold.
- If one of the following equalities $\left(err\left(T_{\bar{\delta}}^u\right)\right) = \bar{\delta}_1$ or $\left(err\left(T_{\bar{\delta}}^v\right)\right) = \bar{\delta}_2$ together with $|krU - krN| > 3\delta$ are fulfilled, then *High Priority* Device status occur.
- If the condition $|krU - krN| > 3\varepsilon$ and one of the following inequality $\left(err\left(T_{\bar{\delta}}^u\right)\right) > \bar{\delta}_1$ or $\left(err\left(T_{\bar{\delta}}^v\right)\right) > \bar{\delta}_2$ hold, then *Critical* Device status occur

To evaluate the effectiveness of the estimation method, computational experiments were carried out both for the series of model functions and for the experimental data.

4. COMPUTATIONAL RESULTS

In the experiments the inverse parametric identification problem for system (1) was solved, the functions T_{Ni}, T_{Pt}, T^u_{Ni} and T^v_{Ni} were obtained and their error estimates were calculated.

The computational results for some test function are illustrated in the Figures below. The same notations are used in all the Figures. Each Figure illustrates the temperature error functions. The deviation of the calculating values from the calibrated values T_δ is described via functions $\left(T_{Ni}-T_\delta\right)$, $\left(T_{Pt}-T_\delta\right)$, $\left(T^u_{Ni}-T_\delta\right)$, $\left(T^u_{Pt}-T_\delta\right)$, where T_δ is the temperature measured by the standard.

and T_{Ni}, T_{Pt}, T^u_{Ni} T^v_{Ni} are calculated via the formulas (3) .

Circle-marked lines correspond to applicable tolerance limit for the first resistance thermometer and rectangle-marked lines correspond to applicable tolerance limit for the second resistance thermometer, respectively.

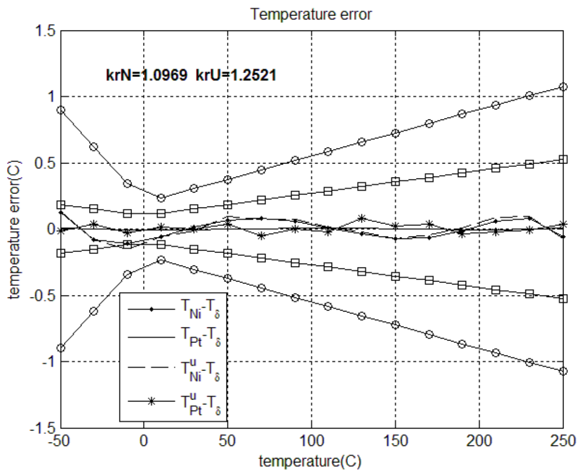

Fig. 1 Temperature functions obtained on the basis system (1) and relation (3). This Figure illustrate BLURRED MVS and LOW priority devices status.

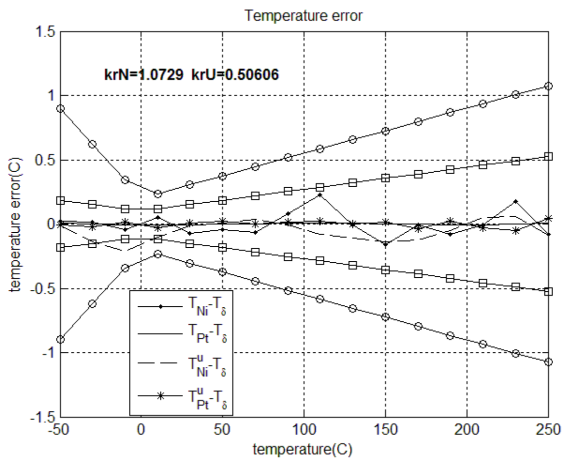

Fig. 2 Temperature functions obtained via system (1) and (3). This Figure illustrates DAZZLED MVS and LOW priority devices status.

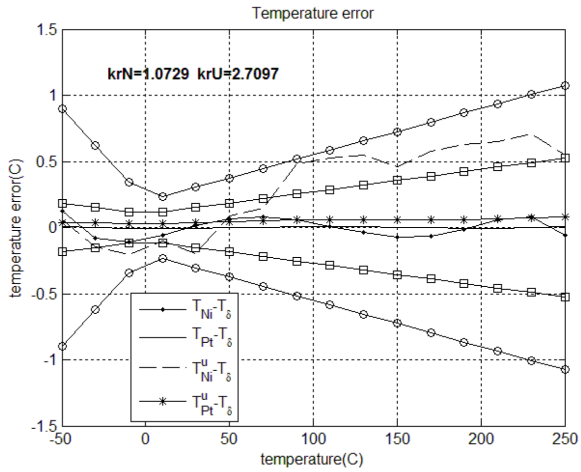

Fig. 3 Temperature functions obtained via system (1) and relation (3). This Figure illustrates BLIND MVS and HIGHT priority devices status.

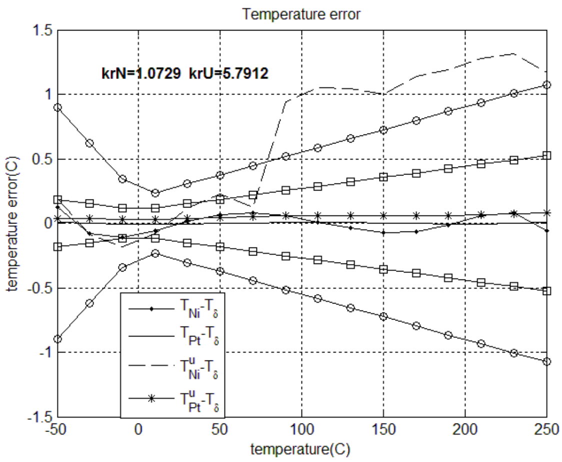

Fig. 4 Temperature functions obtained via system (1) and relation (3). This Figure illustrates UNVALIDATED MVS and High priority devices status.

4. CONCLUSIONS

The methods for calculation of the temperature values and temperature errors are proposed. Both methods are sensitive to measurement errors and are uniform for materials, temperature range and for temperature affiliation to tolerance limits.

The temperature errors estimates provide the basis for the development of the criteria to self-test of device.

The reliability of the obtained temperature values and accuracy of error estimates were evaluated by a computational experiment. The computational results confirm the efficiency the proposed methods and indicate the advantages of our approaches.

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