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COPING WITH DISPERSION IN HOPKINSON-BARS

Felix Banda [2]; *Martin Brucke* [1], *Michael Mende* [1]

[1] SPEKTRA Schwingungstechnik und Akustik GmbH, Dresden, Germany, sales@spektra-dresden.com
[2] Technische Universität Dresden, Institut für Festkörpermechanik, Dresden, Germany

Abstract – Strain gauges can be used as cost-effective reference sensors in shock calibration systems based on Hopkinson-bar shock exciters. However, dispersion of the wave that's propagating through the bar limits the attainable measurement uncertainty since the strain gauges and the device under test (DUT) must be mounted at different positions. This paper suggests a method how to reduce the influence of the dispersion effect on the calibration results and thus improve the measurement uncertainty significantly.

Keywords: Hopkinson-bar, shock calibration, dispersion compensation, ISO 16062-22

1. INTRODUCTION

Hopkinson-bars are commonly used as calibration shock exciters if acceleration amplitudes above some 10 km/s^2 are required. The working principle of these exciters is based on the propagation of a compression wave in the bar that is reflected at the end where the DUT is mounted. This reflection leads to a movement at the front surface that acts as a high acceleration amplitude on the DUT. Fig. 1 shows the working principle of the Hopkinson-bar

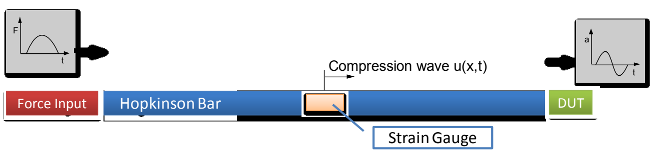

Fig. 1: Hopkinson-bar using a strain gauge as reference sensor (DUT = device under test)

As was shown in [1] strain gauges can be used as cost-effective reference sensors since the acceleration amplitude at the end of the bar is related via a defined physical relationship to the strain in the bar caused by the propagating compression wave if the wave can be assumed as a one dimensional plain wave (1) that propagates with a speed of sound c_0 in the bar. The theory of Hopkinson-bars is described in [3]

$$a(t) = 2c_0 * \frac{d\varepsilon(t)}{dt} \qquad (1)$$

In practice the force input device generates a wave package with a certain frequency spectrum that is degenerated by dispersion while propagating through the bar. The strain gauges are commonly attached in the middle of the bar while the DUT is mounted at the end of the bar. Due to the distance between strain gauge and DUT and the influence of the dispersion, the shape of the time signal a(t) derived from ε(t) (strain gauge measurement) and a(t) measured with the DUT will be different. In general it can be said that the dispersion decreases the acceleration amplitude while the shock duration increases. Thus the DUT mounted at the end of the bar 'sees' a lower acceleration amplitude than the strain gauges in the middle of the Hopkinson-bar. Since the sensitivity of a shock accelerometer is calculated by a comparison of the a(t) peak values from DUT and reference sensor, the DUT sensitivity measured with a strain gauge will be lower than the DUT sensitivity determined by a primary calibration according to ISO 16063-13 where the reference laser vibrometer measures close to the DUT at the end of the bar. Furthermore the deviation increases with the acceleration amplitude because the shock duration decreases with increasing amplitude thus shifting the spectrum to higher frequencies where the dispersion effect is more significant than at the lower end of the spectrum.

In [1] it was shown that this sensitivity deviation can be significantly reduced if the acceleration calculated by means of (1) from the strain gauges signal is multiplied by a correction factor that is determined from a reference measurement with a laser vibrometer at a certain shock amplitude. But it was also shown that the measurement uncertainty is still in the range of 5% to 8% due to the increasing influence of the dispersion at higher shock levels. So an improved method had to be found that can remove the influence of dispersion on the measurement results.

2. COPING WITH DISPERSION

Dispersion is a physical phenomenon, which occurs during wave propagation in the Hopkinson-bar. It can be described as the frequency dependence of the phase velocity. A wave package as generated by the force input device in the bar can be imagined as a superposition of multiple waves with specific frequencies. Due to different velocities of the frequency parts, the shape of the wave package changes during propagation. The pulse width increases while the amplitude decreases leading to the measurement error described above. Since this degeneration of the wave package can be described mathematically if the phase velocity c(f) as a function of the frequency f is known, it should be possible to remove the dispersion effect by a mathematical calculation.

In the literature several approaches can be found that try to describe c(f) pending on material parameters and geometry of the waveguide. For this work a simplified approach from Love was chosen that is described in [2]. According to this approach c(f) can be calculated as following

$$c(f) = \sqrt{c_0^2 - (kv2\pi f)^2} \qquad (2)$$

Where v is the Poisson ratio and k names the polar radius of gyration and can easily be calculated by (3),

$$k = \frac{d}{\sqrt{8}} \qquad (3)$$

Where d is the diameter of the bar and c_0 is the speed of sound in the bar. Using the comparison between the exact dispersion theory and Love's simplified approach as shown in [2], the lower limit of this approach can be estimated at shock durations in the range of 1.5 µs which are a factor 10 less than observed in the measurements.

Based on (2) the phase shift due to the different phase velocities for each frequency in the wave package can be calculated by (4)

$$\Phi = 2\pi f y \left(\frac{1}{c_0} - \frac{1}{c(f)}\right) \qquad (4)$$

Where y is the distance between the location of the strain gauges and the end of the bar where the DUT is mounted.

So the influence of dispersion can be compensated by the following procedure

1. Calculate FFT from the time signal
2. Adjust the phase shift for each frequency according to (4)
3. Transform signal back from the frequency domain to the time domain by means of inverse FFT
4. Perform the calibration according to ISO 16063-13 or ISO 16063-22 with the dispersion compensated time signal

For the calibration we have to determine the acceleration amplitude at the end of the bar where the DUT is mounted. Thus the dispersion compensation procedure described above has to be applied to the time signal of the strain gauges that serve as our reference sensors.

(1) Implies that the acceleration over time can be easily calculated from the measured output voltage of the strain gauge by means of

$$a(t) = \frac{2 * c_0}{U_S * K_{SG}} \frac{dU(t)}{dt} = S_{SG} * \frac{dU(t)}{dt} \qquad (5)$$

Where U_s is the excitation voltage of the measurement bridge and K_{SG} the nominal Sensitivity of the strain gauge. But as discussed in [1] it is not sufficient to calculate S_{SG} from the given or measured parameters c_0, U_s and K_{SG}. The material parameter c_0 as well as the sensitivity of the strain gauges are in general derived from results from static measurement methods. Thus experience showed that the calculated sensitivity S_{SG} and the sensitivity S^*_{SG} determined by means of a reference laser vibrometer can differ significantly. So the 'real' sensitivity S^*_{SG} of the strain gauges was determined by comparison of the dispersion compensated strain gauge time signal with the output of a laser vibrometer that measured the velocity amplitude at the end of the bar (see Fig. 2). In order to reduce amplitude linearity effects this calibration was performed at different acceleration amplitudes and the sensitivity values were averaged.

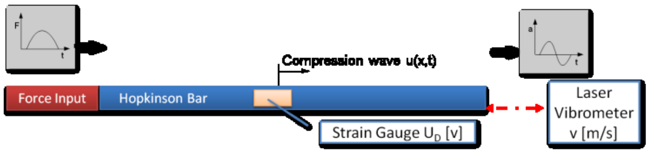

Fig. 2 Calibration of the Strain Gauge by means of a laser vibrometer

3. RESULTS

Measurements were performed on a SPEKTRA CS18P HS calibration system using a Titanium Hopkinson-bar with a diameter of 16 mm and a length of 2 m. Strain gauges were attached to the middle of the bar and a Polytech OFV 5000 laser vibrometer with VD-09 velocity decoder was used as primary reference sensor. An Endevco 7270-200k piezo-resistive shock accelerometer with a measurement range up to 2 000 km/s^2 served as DUT for the calibration measurements.

In a first step the capability of the dispersion compensation method was checked by the comparison of the shapes of the measured signals from DUT and reference strain gauges after the dispersion compensation was applied to the strain gauge signal. A result of such a measurement can be seen in Fig. 3. The dispersion compensated signal (Ref after correction) shows the same shape as the DUT signal.

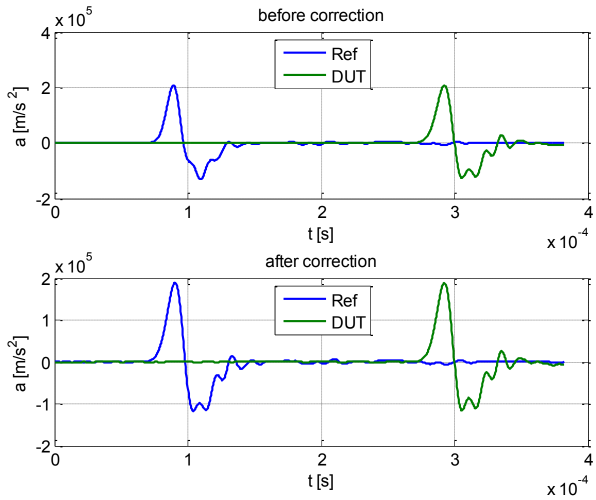

Fig. 3 Time signals of strain gauges (REF) and DUT before and after dispersion compensation

In a second step the DUT accelerometer was calibrated with the laser vibrometer as reference sensor according to ISO 16063-13 [4] and with a strain gauge according to ISO 16063-22. From the strain gauge measurements the DUT sensitivity values were determined with and without dispersion compensation. For the calibrations without dispersion compensation the strain gauge sensitivity S^*_{SG} was determined with a laser vibrometer reference measurement at an amplitude of 100 km/s^2.

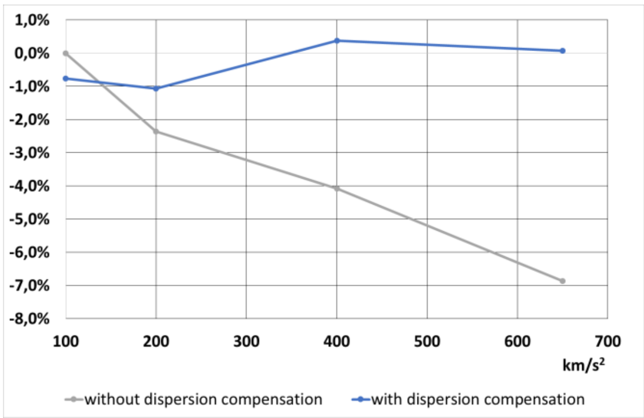

Fig. 4 Deviation of calibration results from strain gauge measurements compared to laser vibrometer measurements

The increasing deviations between the calibration with reference laser vibrometer and the calibration with reference strain gauge without dispersion compensation comply well with the results that were observed and published in [1]. With dispersion compensation applied the influence of the dispersion is significantly reduced. In the example measurements shown above with the best available high shock accelerometer glued to the Hopkinson-bar (thus few mounting issues) the deviation between both calibration methods was in the range of 1% and less at amplitudes below 700 km/s^2. For other types of accelerometers the measured deviation was sometimes larger because the properties of the accelerometer and mounting issues (e.g. influence of studs) were dominating influence factors.

4. CONCLUSIONS

It was shown earlier that strain gauges can be used as cost-effective reference sensors for the calibration of accelerometers with Hopkinson-bars. However, dispersion of the elastic waves in the bar can lead to a high measurement uncertainty in the range of 10% and more. This is mainly caused because the reference sensor and the DUT must be mounted in a certain distance at the bar due to practical reasons. Thus both sensors see the wave in different stages of ‘deformation’ due to dispersion.

This paper suggests an approach how to consider dispersion of the wave by adding mathematically a dispersion component to the time signal of the reference strain gauge. Thus the transformed time signal is similar to the signal at the end of the bar where the DUT is mounted. Measurements show that due to this transformation the shape of the time signal becomes similar to the shape of the signal measured with the DUT. The comparison of calibration results obtained with a laser vibrometer as reference compared to results from calibrations with a strain gauge as reference sensor showed the significant improvement that can be achieved with this method. The deviation of the measured DUT sensitivity was less than 1% between both calibration methods. This should allow to manufacture Hopkinson-bar based shock calibration systems using cost-efficient strain gauges as reference sensors with a measurement uncertainty close to primary systems according to ISO 16063-13.

Future measurements at higher amplitudes have to proof if the simplified dispersion theory that was used here will be sufficient even at amplitudes above 2000 km/s^2. If required, a much more complex approach may be considered alternatively in the future.

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