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TRACEABLE MASS DETERMINATION AND UNCERTAINTY CALCULATION

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Abstract – Functions for calibration of weights in compliance with international standards are implemented in the current series of Sartorius manual mass comparators. Their intuitive software leads the user step by step through the entire mass comparison procedure. As a result, a record is generated, which contains the relevant data, including measurement uncertainty analysis. The present article discusses the determination of the uncertainty components and a possible method for budgeting these components in order to verify the suitability of mass comparators for various accuracy classes.

Keywords: mass comparison, uncertainty, weight, conventional mass

1. INTRODUCTION

Numerous basic conditions are specified for weights and their mass comparisons in the OIML R 111-1 International Recommendation. Based on the maximum permissible errors for the nominal values of weights of different classes, this guideline specifies the allowed deviation of the weight under test from its nominal value and in context with the limit for the uncertainty of mass comparison. If all uncertainty components involved are considered as the total uncertainty of the weighing process – i.e., the uncertainty of the reference weight, the uncertainty of the buoyancy correction and the uncertainty of the balance – then the combined uncertainty can be calculated. Based on the combined uncertainty, mass comparison can be evaluated. Manufacturers of mass comparators need to ensure that the instruments themselves are suitable for mass comparisons of the specified nominal weights in the specific accuracy classes. This verification can also be done based on OIML R 111-1.

In order to ensure full compliance, a budget comprised of the different uncertainty components is necessary. Based on this budget, each uncertainty component can be calculated and checked separately. In this way, we can derive requirements for the standard deviation of a mass comparator. As the majority of mass comparisons are performed under atmospheric conditions, weights experience buoyancy effects, depending on their volume and the air density. This systematic effect on mass determination can be eliminated if the air density and the densities of the materials of the test and reference weights are known. Air density can be calculated if the temperature, barometric pressure and humidity as the key influence quantities are accurately measured. In order to determine the uncertainty of an air buoyancy correction, the densities and the corresponding uncertainties of the test and reference weight must also be known besides the air density and its uncertainty. Under these aspects, requirements can be placed on the uncertainty of climate data measurements. These unknown variables are accurately measured by the climate module implemented in the new Sartorius MCM mass comparator.

2. DETERMINING THE UNCERTAINTY IN COMPLIANCE WITH OIML R111-1

The expanded uncertainty of the conventional mass of a test weight $U(m_{\mathrm{ct}})$ is yielded by the expansion factor, or coverage factor, k and the combined standard uncertainty $u_{\mathrm{c}}(m_{\mathrm{ct}})$ according to Eq. C.6.5-3, [1] as follows:

$$U(m_{\mathrm{ct}}) = k \times u_{\mathrm{c}}(m_{\mathrm{ct}}) \tag{1}$$

In this equation, the coverage factor determines the confidence interval. As a rule, a confidence interval of 95 % is used, which corresponds to a coverage factor of $k = 2$. The expanded uncertainty of each weight must satisfy the following condition (cf. Eq. 5.2-1 [1]):

$$U(m_{\mathrm{ct}}) \leq 1/3\,\mathrm{MPE} \tag{2}$$

The maximum permissible errors, MPE, dependent on mass and accuracy class are indicated in OIML R 111-1 (cf. Table 1 [1]). The highest requirements are placed on weight class E1 for weights with a nominal mass m_0 greater than 100 g. The maximum permissible relative error for these weights is:

$$\frac{\mathrm{MPE}}{m_0} = 0.5 \cdot 10^{-6} \tag{3}$$

The combined standard uncertainty of the test weight is comprised of the standard uncertainty of the weighing process $u_{\mathrm{w}}(\overline{\Delta m_{\mathrm{c}}})$, the standard uncertainty of the reference weight $u(m_{\mathrm{cr}})$, the standard uncertainty of air buoyancy correction u_{b} and the combined standard uncertainty of the weighing instrument u_{ba}. According to Eq. C.6.5-1 [1], the combined standard uncertainty is calculated using the following formula:

$$u_c(m_{ct}) = \sqrt{u_w^2(\overline{\Delta m_c}) + u^2(m_{cr}) + u_b^2 + u_{ba}^2} \quad (4)$$

The uncertainty of a weighing process is determined using the standard deviation of the individual mass comparisons $s(\Delta m_c)$ and the number of weighing cycles n (cf. Eq. C.6.1-1 [1]):

$$u_w(\overline{\Delta m_c}) = s(\Delta m_{ci})/\sqrt{n} \quad (5)$$

The standard uncertainty of a reference weight is yielded by the following (cf. Eq. C.6.2-1 [1]):

$$u(m_{cr}) = \sqrt{(U/k)^2 + u_{inst}^2(m_{cr})}, \quad (6)$$

where U represents the expanded uncertainty of the reference weight given on the calibration certificate and $u_{inst}(m_{cr})$ is the standard uncertainty due to instability of the reference weight. This can be determined based on the calibration history of the particular weight.

According to OIML R111-1, the standard uncertainty of air buoyancy correction is calculated using Eq. C.6.3-1 [1]. Assuming that $\rho_{al} = \rho_0$, (cf. Eq. 34.67 [3]), this equation is simplified to the following:

$$u_b = m_{cr} \sqrt{\begin{array}{c}[(\rho_r - \rho_t)/(\rho_r \cdot \rho_t)\, u(\rho_a)]^2 + \\ (\rho_a - \rho_0)^2\, u(\rho_t)^2/\rho_t^4 + \\ (\rho_a - \rho_0)^2\, u(\rho_r)^2/\rho_r^4\end{array}} \quad (7)$$

Here, ρ_r and ρ_t are the densities of the materials of the reference weight and the test weight, respectively; ρ_a is the air density; and ρ_0 is the reference value of the air density, where $\rho_0 = 1.2\ \mathrm{kg\ m^{-3}}$.

The combined standard uncertainty of a balance is obtained according to Eq. C.6.4-5 [1]:

$$u_{ba} = \sqrt{u_s^2 + u_d^2 + u_E^2 + u_{ma}^2} \quad (8)$$

This result is comprised of the standard uncertainty of the sensitivity of the mass comparator u_s, the standard uncertainty of the display resolution u_d, the standard uncertainty due to eccentricity (off-center loading error) u_E and the standard uncertainty due to magnetism (magnetic properties and influences) u_{ma}. The standard uncertainty of the sensitivity is obtained according to Eq. C.6.4-1 [1]:

$$u_s = |\overline{\Delta m_c}|\sqrt{u^2(m_s)/m_s^2 + u^2(\Delta I_s)/\Delta I_s^2} \quad (9)$$

A possible, universally valid simplification is given in Sections 3.4.6.2–3.4.6.3 [3], where

$$u_s \approx |\overline{\Delta m_c}|\, u(m_s)/m_s \approx 5 \cdot 10^{-4} |\overline{\Delta m_c}| \quad (10)$$

The standard uncertainty of the display resolution is obtained using (cf. Eq. C.6.4-2 [1]):

$$u_d = d\sqrt{2}/(2\sqrt{3}) \quad (11)$$

In conventional measurements, the standard uncertainty due to eccentricity is already included in the standard uncertainty of the weighing process $u_w(\overline{\Delta m_c})$. As a result, the following is yielded with reference to Section C.6.4.4.1 [1]:

$$u_E = 0 \quad (12)$$

The uncertainty component due to magnetism can likewise be neglected to the extent that weights compliant to standards, such as OIML R 111-1, are used (cf. Section C.6.4.5 [1]).

$$u_{ma} = 0 \quad (13)$$

3. DETERMINING THE AIR DENSITY

The air density is an essential influence quantity in highly accurate mass comparisons. Systematic buoyancy effects can be corrected if the air density is known. To correct for the effect of air density, OIML R 111-1 specifies the following equation (cf. Eq. 10.2-1 [1]):

$$m_{ct} = m_{cr}\,(1 + C) + \overline{\Delta m_c} \quad (14)$$

with (cf. Eq. 10.2-2 [1]):

$$C = (\rho_a - \rho_0)\frac{\rho_t - \rho_r}{\rho_r\, \rho_t} \quad (15)$$

However, the uncertainty of air density results in an uncertainty component in the difference in mass determined. The air density can be calculated with a relative uncertainty of $u_F(\rho_a)/\rho_a = 22 \cdot 10^{-6}$ according to the CIPM-2007 equation [4]. As an alternative to this CIPM-2007-formula, the air density can be calculated using a simplified equation (cf. Eq. E.3-1 [1]):

$$\rho_a = \frac{0.34848\,p - 0.009\,hr \cdot \exp(0.061\,t)}{273.15 + t}, \quad (16)$$

where hr is in % and t is in °C

The relative standard uncertainty of this simplified formula is as follows (cf. Section E.3 [1]):

$$u_F(\rho_a)/\rho_a = 2 \cdot 10^{-4} \quad (17)$$

If we calculate the air density from the quantities of temperature, barometric pressure and humidity, the standard uncertainty of air density is obtained according to Eq. C.6.3-3 [1]:

$$u(\rho_a) = \sqrt{\begin{array}{c}u_F^2 + \left(\partial\rho_a/\partial p\ u_p\right)^2 + \\ (\partial\rho_a/\partial t\ u_t)^2 + \\ (\partial\rho_a/\partial hr\ u_{hr})^2\end{array}} \quad (18)$$

The following values are given for the sensitivity coefficients under normal conditions (cf. Section 2.2 [4]):

$$\partial\rho_a/\partial p \approx +1\cdot 10^{-5}\,\rho_a\ \mathrm{Pa}^{-1}$$
$$\partial\rho_a/\partial t \approx -4\cdot 10^{-3}\,\rho_a\ \mathrm{K}^{-1}$$
$$\partial\rho_a/\partial hr \approx -9\cdot 10^{-3}\,\rho_a,\ \text{where } 0\le hr\le 1 \tag{19}$$

4. BUDGETING THE UNCERTAINTY COMPONENTS

It is necessary to budget the uncertainty components in order to generally verify whether a mass comparator is suitable for various accuracy classes. The following requirements have proven to be useful in practice:

$$u_w(\overline{\Delta m_c}) \le\ 4/5\,u_c(m_{ct}) \text{ and} \tag{20}$$

$$u(m_{cr}) = u_b = u_{ba} \le\ 1/3\,u_c(m_{ct}) \tag{21}$$

If these individual requirements are met, the combined uncertainty of a mass comparator will always be lower than the uncertainty limit prescribed.

4.1. Required repeatability

The required repeatability $s(\Delta m_c)$ of a mass comparator can likewise be derived from the required standard uncertainty of the weighing process $u_w(\overline{\Delta m_c})$.

$$u_w(\overline{\Delta m_c}) = \ s(\Delta m_c)/\sqrt{n} \le 4/5\,u_c(m_{ct}) = 2/5\,U(m_{ct}) = 2/15\ \mathrm{MPE};\ \text{i.e.,}\quad s(\Delta m_c) \le 2/15\ \mathrm{MPE}\sqrt{n} \tag{22}$$

The required number of weighing cycles n is specified by OIML and ASTM for the respective accuracy class of weights. Table 1 shows the minimum number of weighing cycles as a function of accuracy class along with the required repeatability for ABA cycles.

Table 1. Number of ABA Weighing Cycles.

OIML Class	E1	E2	F1	F2, M1, M2, M3
ASTM Class	0	1, 2	3, 4	5, 6, 7
n Cycles	5	3	2	1
$s_{max}(\Delta m_c)$	0.30 MPE	0.23 MPE	0.19 MPE	0.13 MPE

4.2. Required uncertainty of a reference weight

Based on the budget selected according to (21), the requirement on the uncertainty of the reference weight is yielded as follows:

$$u(m_{cr}) \le\ 1/3\,u_c(m_{ct}) = 1/6\,U(m_{ct}) = 1/18\ \mathrm{MPE} \tag{23}$$

Hence, the expanded uncertainty is calculated as:

$$U(m_{cr}) \le\ 1/9\ \mathrm{MPE} \tag{24}$$

Based on the equation in (25) (cf. Eq. 5.2-1 [1]) it follows that the expanded uncertainty of the reference weight is less than one third of the maximum permissible error of the reference weight $\mathrm{MPE_r}$.

$$U(m_{cr}) \le 1/3\ \mathrm{MPE_r} \tag{25}$$

This yields the requirement on the ratio of the maximum permissible errors for the reference and test weights as follows:

$$1/3\ \mathrm{MPE_r} \le 1/9\ \mathrm{MPE}\ \text{ or }\ \mathrm{MPE}/\mathrm{MPE_r} \ge 3 \tag{26}$$

The accuracy classes defined in OIML R 111-1 are progressively scaled so that a reference weight that is one class higher than the test weight meets this requirement. The only exceptions to this are the four weights of class E1 (1 mg, 2 mg, 5 mg, 10 mg).

The ratios of the maximum permissible error limits of consecutive OIML accuracy classes are shown in Fig. 1. The person performing calibration of ASTM weights has special responsibility for selecting the correct ratio as the ratios between adjacent accuracy classes do not generally satisfy the requirement of (26).

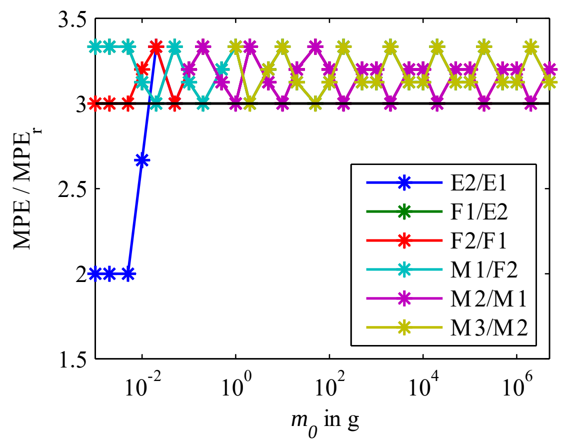

Fig. 1. Ratios of MPEs According to OIML for Consecutive Accuracy Classes

4.3. Required uncertainty of a balance

According to (8) and the restrictions derived from (12) and (13), the following requirement on the uncertainty of a balance is yielded for the budget of uncertainty components selected:

$$u_{ba} = \sqrt{u_s^2 + u_d^2} \le 1/18\ \mathrm{MPE} \tag{27}$$

The uncertainty component due to sensitivity can be calculated using (10). The maximum possible difference of the conventional masses $|\overline{\Delta m_c}|$ can be determined from the particular accuracy classes of the reference and test weights. The following applies to the reference weight based on Eq. (5.2-1) and (5.3-1) [1]:

$$|\overline{\Delta m_{\mathrm{cr}}}| \leq 2/3\,\mathrm{MPE_r} \quad (28)$$

The same also applies to the test weight with the required accuracy class as follows:

$$|\overline{\Delta m_{\mathrm{ct}}}| \leq 2/3\,\mathrm{MPE} \quad (29)$$

Hence, the following equation applies to the difference of the conventional masses:

$$|\overline{\Delta m_{\mathrm{c}}}| \leq 2/3\,\mathrm{MPE} + 2/3\,\mathrm{MPE_r} \quad (30)$$

Based on the requirement that the reference weight be one accuracy class higher than the test weight, it follows that:

$$|\overline{\Delta m_{\mathrm{c}}}| \leq 2/3\,\mathrm{MPE} + 2/3 \times 1/3\ \mathrm{MPE} \quad \text{or} \quad (31)$$

$$|\overline{\Delta m_{\mathrm{c}}}| \leq 8/9\,\mathrm{MPE} \approx \mathrm{MPE} \quad (32)$$

Therefore, the uncertainty of the sensitivity based on (10) is calculated as:

$$u_{\mathrm{s}} \approx 5 \cdot 10^{-4}\,\mathrm{MPE} \quad (33)$$

This notation reveals that the uncertainty contribution of the sensitivity is very low for a correctly adjusted balance, provided that weights with the same nominal value are compared. Using the ratio determined between the uncertainty of sensitivity and that of the maximum permissible error, a direct requirement on the digital resolution of a balance can be derived from (11) and (33).

$$d = u_{\mathrm{d}}\sqrt{6} \quad (34)$$

According to Eq. (27), it follows that:

$$u_{\mathrm{d}} \leq \sqrt{(1/18\,\mathrm{MPE})^2 + (5 \cdot 10^{-4}\,\mathrm{MPE})^2} \approx 1/18\,\mathrm{MPE} \quad (35)$$

Hence, the requirement on a balance's resolution is expressed as:

$$d \leq \sqrt{6}/18\,\mathrm{MPE} \quad (36)$$

4.4. Required uncertainty of air buoyancy correction

With reference to the budget selected according to (21), the uncertainty of an air buoyancy correction results in the following:

$$u_{\mathrm{b}} \leq 1/3\,u_{\mathrm{c}}(\mathrm{m_{ct}}) = 1/6\,U(m_{\mathrm{ct}}) = \mathrm{MPE}/18 \quad (37)$$

Therefore, the maximum permissible uncertainty of air buoyancy correction for class E1 weights with a nominal value greater than 100 g is yielded by:

$$u_{\mathrm{b}} \leq \frac{0.5 \cdot 10^{-6}}{18}\,m_0 = 2.7\bar{7} \cdot 10^{-8}\,m_0 \quad (38)$$

The standard uncertainty of the air buoyancy correction is given in (7), which can be split into three individual formulas.

$$u_{\mathrm{b}}^2/{m_{\mathrm{cr}}}^2 = u_{\mathrm{b1}}^2 + u_{\mathrm{b2}}^2 + u_{\mathrm{b3}}^2 \quad \text{, where} \quad (39)$$

$$u_{\mathrm{b1}} = \frac{|\rho_{\mathrm{r}} - \rho_{\mathrm{t}}|}{\rho_{\mathrm{r}}\,\rho_t}\,u(\rho_{\mathrm{a}})$$

$$u_{\mathrm{b2}} = |\rho_{\mathrm{a}} - \rho_0|\,\frac{u(\rho_{\mathrm{t}})}{\rho_{\mathrm{t}}^2}$$

$$u_{\mathrm{b3}} = |\rho_{\mathrm{a}} - \rho_0|\,\frac{u(\rho_{\mathrm{r}})}{\rho_{\mathrm{r}}^2}$$

The term u_{b1} depends on the density difference between the reference and the test weight and on the uncertainty of the air density measured. Both terms u_{b2} and u_{b3} are each yielded by the product of the deviation of air density from the reference value $\rho_0 = 1.2\ \mathrm{kg/m^3}$ and the uncertainty of the density of the test weight and of the reference weight.
Taking the problematic nature of this calculation into general consideration, we assumed that all three terms would yield the same uncertainty contribution to the air buoyancy correction; i.e.

$$u_{\mathrm{b1}} = u_{\mathrm{b2}} = u_{\mathrm{b3}} = u_{\mathrm{b}}/\sqrt{3} \leq 1.6 \cdot 10^{-8} \quad (40)$$

This assumption enables us to derive the requirements on the uncertainty of the air density measured and, therefore, on the uncertainty of measurement of the climate quantities of temperature, barometric pressure and humidity.

5. REQUIREMENTS ON CLIMATE DATA MEASUREMENT

The starting point for considering these conditions is the requirement on term u_{b1}:

$$u_{\mathrm{b1}} = |\rho_{\mathrm{r}} - \rho_{\mathrm{t}}|/(\rho_{\mathrm{r}}\,\rho_{\mathrm{t}})\,u(\rho_{\mathrm{a}}) \leq 1.6 \cdot 10^{-8} \quad (41)$$

Hence, the following is obtained as the required uncertainty of measurement for air density:

$$u(\rho_{\mathrm{a}}) \leq 1.6 \cdot 10^{-8} \rho_{\mathrm{r}}\,\rho_{\mathrm{t}}/|\rho_{\mathrm{r}} - \rho_{\mathrm{t}}| \quad (42)$$

The permissible ranges for the density of the materials of weights are provided in OIML R 111-1 (cf. Table 5 [1]). For weights of accuracy class E1 with a nominal mass of at least 100 g, the density must be within the range of $7{,}934\ \mathrm{kg/m^3} \leq \rho \leq 8{,}067\ \mathrm{kg/m^3}$. If we use such maximum permissible errors for measurement of the density in (41), the following requirements will be yielded for the uncertainty of air density:

$$u(\rho_{\mathrm{a}}) \leq 7.7 \cdot 10^{-3}\ \mathrm{kg/m^3} \quad (43)$$

and for the relative uncertainty:

$$u(\rho_a)/\rho_a \leq 7.7 \cdot 10^{-3}/\rho_a = 6.4 \cdot 10^{-3}\,, \quad \text{where } \rho_a = \rho_0 \tag{44}$$

The maximum permissible errors on measurement of the density are higher for weights smaller than 100 g. However, this error on measurement of the density is established as such: "… that a deviation of 10% from the specified air density does not produce an error exceeding one-quarter of the absolute value of the maximum permissible error given in Table 1" [1, p. 17]. The ratio of the maximum permissible relative error of the weights to their maximum permissible errors on measurement of their density is therefore approximately constant.

The relative uncertainty of $2 \cdot 10^{-4}$ in simplified equation (16) for calculating the air density is lower by a factor of 32 than the required uncertainty for air buoyancy correction. Hence, the uncertainty contribution of this equation is negligible. The relative uncertainty of air density, along with the corresponding sensitivities, is obtained using the three essential influence quantities for determining the air density, temperature, barometric pressure and relative humidity:

$$u^2(\rho_a)/\rho_a^2 = \left(-4 \cdot 10^{-3}\,\text{K}^{-1} \cdot u(t)\right)^2 + \left(1 \cdot 10^{-5}\,\text{Pa}^{-1} \cdot u(p)\right)^2 + (-9 \cdot 10^{-3} \cdot u(hr))^2 \leq (6.4 \cdot 10^{-3})^2 \tag{45}$$

Assuming that all three climate quantities have the same uncertainty contribution, the following is yielded:

$$\begin{aligned} &4 \cdot 10^{-3}\,\text{K}^{-1} \cdot u(t) \leq 6.4 \cdot 10^{-3}/\sqrt{3}, \text{ or } u(t) \leq 0.92\,\text{K} \\ &1 \cdot 10^{-5}\,\text{Pa}^{-1} \cdot u(p) \leq 6.4 \cdot 10^{-3}/\sqrt{3}, \text{ or } u(p) \leq 3.7\,\text{hPa} \\ &9 \cdot 10^{-3} \cdot u(hr) \leq 6.4 \cdot 10^{-3}/\sqrt{3}, \text{ or } u(hr) \leq 0.41 \end{aligned} \tag{46}$$

The requirements calculated for the uncertainty of climate data measurement can be met using conventional climate data sensors. The uncertainty of the relative humidity of 0.41 means that the relative humidity may be measured using a standard uncertainty of 41%. Under this aspect, it would not be mandatory to measure the humidity.

6. INFLUENCE OF HEIGHT ABOVE SEA LEVEL

E1 laboratories place high requirements on air conditioning. Such requirements usually cover devices for controlling the temperature and relative humidity. However, the barometric pressure is not controlled. The average barometric pressure essentially depends on the height of a laboratory above sea level. Therefore, temperature and humidity play a subordinate role for determining the mean value of air density. Fig. 2 indicates the correlation between air density and height above sea level (cf. Eq. E.3-2 [1]).

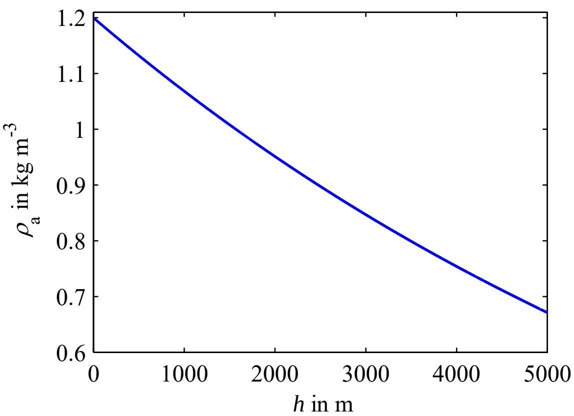

Fig. 2. Air Density as a Function of Height above Sea Level

The deviation of the air density from the reference value of 1.2 kg/m^3, in conjunction with the uncertainty of the density for the test weight and the reference weight, yields an uncertainty component for air buoyancy correction (7). This correlation is clearly indicated by the two terms u_{b2} and u_{b3} in (39). A maximum permissible error of $1.6 \cdot 10^{-8}$ is required for the relative uncertainty of air buoyancy correction according to (40) for the three uncertainty components u_{b1}, u_{b2} and u_{b3}. For instance, if we use a value for the density uncertainty of $u(\rho_r) = u(\rho_t) = 5\,\text{kg/m}^3$, (see also OIML R 111-1, Table B5 Estimated typical uncertainties on page 43), the curve depicted in Fig. 3 is yielded for the terms u_{b2} and u_{b3} as a function of the height above sea level. The dashed line represents the required maximum permissible error for the uncertainty of the air buoyancy correction.

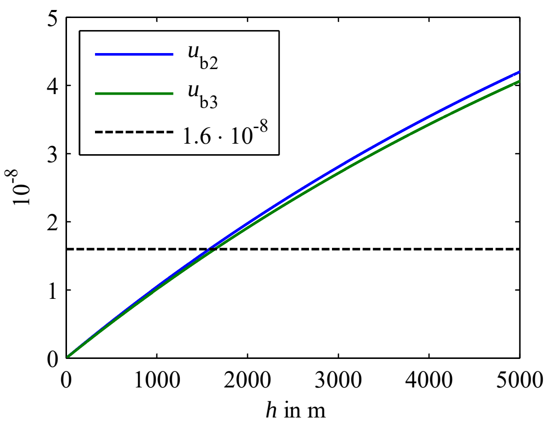

Fig. 3. Uncertainty Contribution $\boldsymbol{u_{b2}}$ and $\boldsymbol{u_{b3}}$ as a Function of Height above Sea Level

It can be seen that independently of the uncertainty of climate data measurements, the required maximum permissible error for the uncertainty of air buoyancy correction is exceeded at an elevation of approximately

1,600 m. On the other hand, an unambiguous requirement can be derived for the uncertainty of test weight and reference weight density measurements as a function of height above sea level. This correlation is shown in Fig. 4. The calculation of this density is based on a density of $8{,}000\ \mathrm{kg/m^3}$ for the material of a weight.

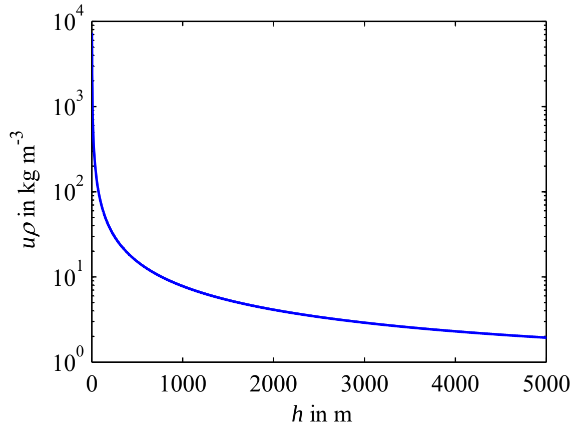

Fig. 4. Required Uncertainty of the Density of the Weights as a Function of Height above Sea Level

To meet the requirement placed on the uncertainty of air buoyancy correction at higher elevations, the uncertainty of the density measurement for the test weight and the reference weight has to be reduced. For example, to fulfill the requirement on this uncertainty at elevations of up to 5,000 m, the standard uncertainty of the density measurement for both the test weight and the reference weight must be less than $2\ \mathrm{kg/m^3}$.

7. CONCLUSION

The present article describes the essential steps for performing uncertainty analysis of direct mass comparisons under atmospheric conditions in conformance with recognized standards. This procedure begins by budgeting of the uncertainty components, an approach that has proved to be practical and able to be generalized. Such budgeting enables calculation of the direct requirements on metrologically important parameters of mass comparators, such as repeatability, resolution and uncertainty of the adjustment weight or the adjustment process. Furthermore, the requirements on the uncertainties of the reference weight used and on the climate measurement necessary for air buoyancy correction, respectively, can also be derived using the equations given.

This expertise has been incorporated in the current series of manual mass comparators manufactured by Sartorius. These instruments have a number of additional functions integrated that facilitate the user's measurement procedures and ensure reliable results. Using a built-in calibratable climate module, the mass comparators measure the climate quantities present during a weighing process: temperature, barometric pressure and relative humidity. They then use these data to determine the associated air density and to calculate the air buoyancy correction fully automatically.

In combination with special application software for performing mass comparisons in compliance with recognized standards, the instruments permit exceptionally convenient and accurate comparison procedures by displaying precisely timed user guidance prompts.

These functions go well beyond those provided by manually operated mass comparators available so far. In addition to indicating the actual result of a mass comparison, i.e., the difference in mass determined between the test weight and the reference weight, the mass comparators generate a structured record of the results. This record includes information on the climate data actually measured, the air density, the densities of the materials of the test and reference weights and a complete uncertainty analysis of each mass comparison carried out.

Fig. 5. Cubis® MCM2004 Manual Mass Comparator

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