

# SURFACE ANALYTICAL MODEL AND DESIGN METHOD OF MASS STANDARD BASED ON SELF ADAPTIVE ALGORITHM

*Xiaoping Ren<sup>1</sup>, Jian Wang<sup>1</sup>, Changqing Cai<sup>1</sup>, Qingxiong Ren<sup>2</sup>*

<sup>1</sup>Mass Laboratory, Division of Mechanics and Acoustics, National Institute of Metrology, Beijing, P.R.China, [renxp@nim.ac.cn](mailto:renxp@nim.ac.cn); [wjian@nim.ac.cn](mailto:wjian@nim.ac.cn); [caichq@nim.ac.cn](mailto:caichq@nim.ac.cn)

<sup>2</sup>Shanxi Province Institute of Metrology, Taiyuan 030002, P.R.China, [1716103068@qq.com](mailto:1716103068@qq.com)

**Abstract** – Surface analytical model is presented to observe the adsorption. Mass standards with special shapes are difficult to design due to complex parameters. Method based on self-adaptive algorithm is given out to help designing the best sorption artefacts. This algorithm helps to design best shape of cylinder-weight and stack-weight that are with same volume and different surface area.

**Keywords:** mass measurement; surface sorption corrected; mass dissemination; self-adaptive algorithm

## 1. INTRODUCTION

The unit of mass, the kilogram, is the last of the seven base units of the International System of Units (SI) to be defined according to an invariant of nature rather than in a material artefact<sup>[1]</sup>. Both the watt balance method and Avogadro method are making the new definition under the vacuum conditions whereas the current definition of the unit, from the International Prototype Kilogram (IPK), is realized and maintained in the air.

After the new definition, it is also necessary to consider traceability from the new realization in vacuum to the mass in air which supports the current working standard<sup>[2]</sup>. However, for the practical dissemination of the mass unit, the standards must be transferred from vacuum to air condition. During this transition, the mass of the standards is significantly affected by sorption phenomena, caused by atmospheric gases and humidity which then bring the limited stability of the mass value of the standards caused by atmospheric contamination over times<sup>[3]</sup>. In 1973, Takayoshi studied the problem of the surface water situation of metal artefacts<sup>[4]</sup>. R. Schwartz wrote a series of papers on adsorption isotherms in air<sup>[5]</sup> and sorption phenomena in vacuum<sup>[6]</sup>. Besides, a lot of investigations mainly on Pt/Ir<sup>[8]</sup>, stainless steel<sup>[5][6]</sup>, silicon<sup>[7]</sup> and also other materials (Au<sup>[8][9][10]</sup>) were performed. The other studies focused on the long-term stability<sup>[8][11]</sup>, water-vapor sorption and the performance of different cleaning procedures<sup>[12]</sup> and techniques, like: UV/ozone cleaning method<sup>[13]</sup>, hydrogen and oxygen low-pressure plasma cleaning method<sup>[8][14]</sup>. Storage method of mass standards is also an important research direction<sup>[15]</sup>.

This research method is making an indirect link between air and vacuum mass measurements by measure mass in vacuum and then characterizing the absorption layers of contaminants during vacuum to air transfer procedure<sup>[16]</sup>. In

[16] and [17], NIST and NRC scientist described a system based on a specially designed high precision mass balance for the simultaneous comparison of a mass in air to a mass in vacuum.

National Institute of Metrology (NIM) China is one of the institutes researching on the new definition of kilogram and mass dissemination. During the mass dissemination, there are many parameters to consider to design the mass standard, like: height, diameter, volume, surface area and mass value of the weight, besides, the measurement instrument characters are also important, including: size of the mass comparator and volume measurement instrument, electronic weighing capacity. In the paper, it describes an adaptive algorithm used to design the mass standard in the surface analytical model, which is of widely application in the surface analytical model of mass standard.

## 2. SURFACE ANALYTICAL MODEL

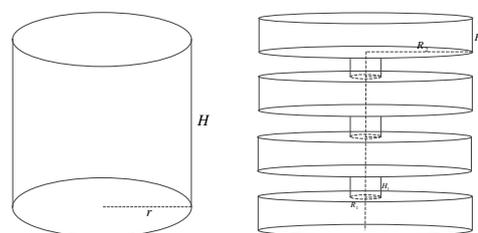


Fig. 1. Cylinder Prototype Fig. 2. Stack of Discs Prototype

In the mass measurement, the classical main uncertainties are due to the air buoyancy correction and the surface sorption correction. In order to improve the accuracy of surface analytical model of mass standard, we must try to reduce another influences-buoyancy correction-as great as possible. A special set of two 1 kg stainless sorption artefacts are shown in Fig 1 and Fig 2. The first artefact is a classical prototype (cylindrical with height equals diameter), The second is made in the form of a stack of discs separated from one another.

In Fig.1, the surface area  $S$  of classical cylinder prototype is  $2\pi r^2 + 2\pi rH$  and the volume  $V$  of this prototype is  $\pi r^2 H$ , height  $H$  equals diameter  $D$  then  $r=D/2$  exists. Finally the volume and surface area of cylinder prototype is shown in equation (1) and (2).

$$S_{\text{cylinder}} = 2\pi r^2 + 2\pi rH = 2\pi\left(\frac{D}{2}\right)^2 + 2\pi\left(\frac{D}{2}\right)H \quad (1)$$

$$V_{\text{cylinder}} = \pi\left(\frac{D}{2}\right)^2 H \quad (2)$$

Stack of discs prototype is shown in Fig.2. Radius of large circle and height are  $R_2$  and  $H_2$ ; radius of small circle is  $R_1$  and  $H_1$ . Then the whole surface area of 4 level- stack of discs prototype is:

$$S_{\text{stack}} = 6\pi R_1 H_1 + 8\pi R_2 H_2 + 8\pi R_2^2 - 6\pi R_1^2 \quad (3)$$

And the volume of this 4 level-stack of discs is:

$$V_{\text{stack}} = 3\pi R_1^2 H_1 + 4\pi R_2^2 H_2 \quad (4)$$

In the research of surface analytical model, we expect these two types of mass standards with the same volume and different surface area. So the air buoyancy corrections applied during all the chains of comparison are minimized. That means equation (2) equals (4), and the larger difference between (1) and (3) is better for the mass measurement during transfer from vacuum to ambient.

In the measurement, first put these two weights both in the air condition and get the mass value  $m_{\text{stack-air}}$  and  $m_{\text{cylinder-air}}$ ; then transfer them to the vacuum condition and get the vacuum mass value  $m_{\text{stack-vacuum}}$  and  $m_{\text{cylinder-vacuum}}$ ; the surface area difference  $S_{\text{stack}} - S_{\text{cylinder}}$  is known, then we can get the surface adsorption ratio  $\eta_1$ :

$$\eta_1 = \frac{(m_{\text{stack-air}} - m_{\text{cylinder-air}}) - (m_{\text{stack-vacuum}} - m_{\text{cylinder-vacuum}})}{S_{\text{stack}} - S_{\text{cylinder}}} \quad (5)$$

Next transfer these two weights from vacuum condition to air condition, the absorption phenomenon happens again and in the short time the mass value will change. Observe the value  $m'_{\text{stack-air}}$ ,  $m'_{\text{cylinder-air}}$  under air and the surface desorption ratio  $\eta_2$  is:

$$\eta_2 = \frac{(m'_{\text{stack-air}} - m'_{\text{cylinder-air}}) - (m_{\text{stack-vacuum}} - m_{\text{cylinder-vacuum}})}{S_{\text{stack}} - S_{\text{cylinder}}} \quad (6)$$

Take consider with the two results  $\eta_1$  and  $\eta_2$ , we can get the sorption ratio by  $\eta = (\eta_1 + \eta_2) / 2$ . And then give the sorption result during transfer from vacuum to air condition of any cylinder prototype of the same materials by:

$$m_{\text{sorption-value}} = \eta \cdot S_{\text{cylinder}} \quad (7)$$

### 3. DESIGN METHOD BASED ON SELF ADAPTIVE ALGORITHM

In part 2, the four-level Stack of Discs Prototypes is an example to design the sorption artifact. Different number of levels can be adopted during design the sorption artifact depending on the sorption effect. The normal formula for volume and surface area of different levels-stack is shown in equation (8) and (9):

$$V = (n-1)\pi R_1^2 H_1 + n\pi R_2^2 H_2 \quad (8)$$

$$S = 2(n-1)\pi R_1 H_1 + 2n\pi R_2 H_2 + 2n\pi R_2^2 - 2(n-1)\pi R_1^2 \quad (9)$$

Besides, varied materials with different densities are also chosen in research. Here we also give a self-adaptive algorithm for design sorption mass standards.

For the same material with ideal density  $\rho$ , and the nominal mass value 1 kg. We can get both height  $H$  and radius  $r$  of cylinder prototype by equation (2). For the sorption purpose, we should let equation (8) equals the volume of cylinder. There are four parameters deciding the volume of n-level stack, which is  $R_1$ ,  $R_2$ ,  $H_1$  and  $H_2$ . The surface area  $S_{\text{cylinder}}$  of cylinder is also known, so the four parameters of stack vary until the difference between  $S_{\text{cylinder}}$  and  $S_{\text{stack}}$  is largest. However, these design scheme cannot neglect that the measurement equipment both for the mass value and volume measurement. All final artifacts should be put in these two instruments to be measured. The final practical density  $\rho$  is also need to be modified, because the ideal density is different from practical density.

Table 1. Comparison between ideal&practical size of cylinder prototype

Material	V(cm <sup>3</sup> )	h (mm)	D(mm)	A (mm <sup>2</sup> )	Note
Stainless Steel	125.3604	54.2446	54.2446	13866.1156	Ideal
	125.6514	54.42	54.22	13887.5881	Machining
	125.6312	/	/	/	Hydrostatic

Take stainless steel as an example. This material's density is 7977 kg/m<sup>3</sup>, so both the cylinder and stack's volume is 125.3604112 cm<sup>3</sup> and the height of cylinder is 54.24464138 mm, the radius of the bottom circle is 27.12232069 mm and the surface area of cylinder is 13866.1155820697 mm<sup>2</sup>. Important here is that the ideal size of height and radius is so precise but the machining ability can achieve 0.01 mm. All the desire size can be known only after the artefact is finished machining. Table 1 shows the comparison between the ideal design and machining result (measured by geometrical method). The most precise method for volume is hydrostatic method<sup>[18]</sup>, and the result is 125.6312cm<sup>3</sup>, which is close to the machining result.

According to the volume and surface area of cylinder prototype, take 4-level stack of discs prototype as an instance. In this design scheme, there are several limitations to be considered:

- $R_2$  cannot exceed the limitation of weight magazine of vacuum measurement instrument(50 mm);
- *Height* of weight cannot exceed the limitation of volume measurement instrument(90 mm);
- $V$  of 4-level stack should equal volume of cylinder prototype(in Table 1);
- $A$  of 4-level stack should larger than the surface area of cylinder prototype(in Table 1);
- Mass difference between cylinder and 4-level stack prototype cannot exceed the electronic weighing capacity( for vacuum measurement instrument is 1.5 g), or the instrument will stop working;
- Machining ability can only achieve 0.01 mm;

Table 2. Comparison between ideal & practical size of 4-level stack

$R_1$ (mm)	$R_2$ (mm)	$H_1$ (mm)	$H_2$ (mm)	Height (mm)	V (cm <sup>3</sup> )	A (mm <sup>2</sup> )	Note
18.87	28	8	10	64	125.3679	22874.8742	Ideal
18.93	27.99	7.96	10.02	/	125.4988	22825.0119	Machining
/	/	/	/	/	125.3460	/	Hydrostatic

There are four parameters  $R_1$ ,  $R_2$ ,  $H_1$ ,  $H_2$  and 6 constraint conditions to be considered in the sorption artifact. We should use self-adaptive algorithm to select the best parameters. Table 2 shows the comparison between ideal and practical size of 4-level stack. This self-adaptive analytical algorithm is based on simulated analyze function in Excel software, which is convenient and useful for our design.

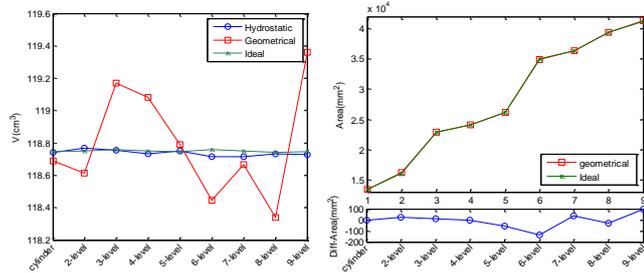


Fig. 3. Design of 2-level to 9-level sorption artifact (Copper)

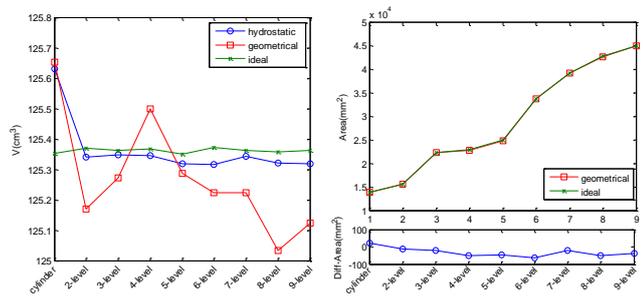


Fig. 4. Design of 2-level to 9-level sorption artifact (Stainless Steel)

Fig.3 and Fig.4 shows the final result from 2-level to 9-level sorption artifact with two different materials, copper and stainless steel). In the figure, it shows the difference between ideal size, geometrical method size and hydrostatic method. Ideal size is closer to geometrical method size.

#### 4. CONCLUSIONS

In order to provide practical means of disseminating the redefined kilogram realized in vacuum to the mass scale at ambient, processes such as air-vacuum transfer for standards must be studied. Effects such as adsorption and desorption are affecting the mass during transfer between vacuum and ambient condition, and need to be studied on artefact standards.

In this paper, we have presented a new surface analytical model combining artefact standards design method based on self-adaptive method which considering several important parameters to design the suitable cylinder prototype and

stack of discs Prototype. This can help metrologist to improve the accuracy of mass measurement. Besides, for more pre-selection of best shapes and materials studies can be carried out based on our method in the future.

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