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A novel polynomial filtering method for data smoothing in *Cognitive Radio* applications

Giovanni Betta [1], *Domenico Capriglione* [1], *Gianni Cerro* [1], *Luigi Ferrigno* [1], *Gianfranco Miele* [1]

[1] Department of Electrical and Information Engineering,
University of Cassino and Southern Lazio,
Via G. Di Biasio 43, 03043 Cassino (FR), Italy,
(betta, capriglione, g.cerro, ferrigno, g.miele) @unicas.it

Abstract - The smoothing process is a fundamental task in many application fields. This paper proposes a novel method to smooth raw data, based on the concept of polynomial fitting. It is thought to be effective in Cognitive Radio applications, especially focused on *spectrum sensing* tasks. The method is intended to be used instead of today's traditional smoothing filters, because of some advantages in terms of shaping retainment, data shifting problem avoidance, acceptable computational intensity, appreciable noise reduction property. The goodness of the proposal has been proved considering the H_1 norm operator as performance index.

Keywords: smoothing filtering, cognitive radio, polynomial fitting, noise reduction, spectrum management

1. INTRODUCTION

The data analysis, after a digital acquisition from a real system, is often affected by several different problems: quantization, oversampling, additive noise, spike samples and others. In order to reduce the negative effect of the acquisition process, and to find inside the data a particular trend or pattern, it is necessary to neglect some data samples and consider just a portion of them or their combination according to an analytical law. In particular, two main algorithm families can be recognized: the first one concerning the sample selection from manipulation of the original data (with average, median or other linear or non–linear operators); the second category dealing with the new sample extraction according to a particular curve to be fitted (e.g. quadratic, cubic approximations). In the first case, no fitting processes are involved, but samples are simply combined each other in order to obtain new samples. In the second approach, a target curve has to be reached and some kind of distance minimization is followed in order to find the best curve able to approximate the raw data samples. There are a lot of smoothing techniques present in literature: additive smoothing [1], moving average (M-A) [2], Savitzky–Golay (S-G) filter smoothing [3], exponential filtering [4], smoothing splines [5], just to cite a few. Each of the cited methods has its own application field, where its filtering properties well complain with the application specific requirements. As a general rule, the filtering abilities of a specific method are proportional to its computational burden: the higher the noise reduction and the shape retainment, the heavier the computational cost of the method.

In the present paper the second approach will be followed. In particular, a polynomial fitting method will be presented and its performance in a case study will be evaluated. The aim of the paper is to introduce a filtering approach achieving a good compromise between filtering ability and computational cost. The effectiveness of the proposed method is evaluated with respect to Cognitive Radio (CR) applications [6], with special regard on their spectrum sensing phase, as explained in section 3, since the authors have further experience on topic, as shown in [7], [8], [9], [10]. The paper is organized as follows: in section 2, brief notes about the filtering methods used as comparison terms are given; a case study concerning Cognitive Radio systems is presented in section 3; a theoretical approach to polynomial fitting problem and details about the proposal are provided in section 4.1. Finally, performance of the proposed method will be discussed in section 5 and conclusions will be given in section 6.

2. COMPARISON FILTERING METHODS

2.1. Linear Moving Average filters

A Linear Moving average filter (M–A) [2] is a method for data smoothing belonging to the first one of the previously explained categories. Let us suppose to have N data samples. Let us consider a data window, able to contain M pairs, where $M < N$. M–A uses such structure as a sliding window on the entire sample set, considering a one–sample right shift. The computation is performed until the window reaches the last sample in the set. For each fixed subset, it computes the arithmetic mean of the data (linear operation) and stores the result in a new dataset. The output for the k_{th} output sample is:

$$out[k] = \frac{1}{M}\sum_{j=0}^{M-1} in[k-j]; \tag{1}$$

where $out[\cdot]$ and $in[\cdot]$ are, respectively, the output and the input sequence of data. At the end of the process, the new dataset contains the smoothed data. The output set length (O_l) can be computed as follows: $O_l = N - M + 1$, since the first Y–values of the $M - 1$ pairs are returned as NaN, due to the incomplete window refilling.

2.2. Savitzky–Golay filters

A Savitzky–Golay filter (S–G) [3] belongs to the second category, which includes a polynomial fitting process in order to smooth a set of input data. In particular, for a given set of N input data samples, the method makes use of a sliding window as the previously described filter method (M–A), but it differs in the operating approach: for the M samples belonging to the actual subset in a certain time instant, S–G applies a polynomial fitting scheme (some theoretical hints are given in Subsection 4.1) in order to find the best approximating curve for such data samples. The process is based on a least square minimization procedure. Once the fitting curve has been found, it is evaluated at the central point of the sliding window and such value is stored in a new dataset, hosting the smoothed data. At the end of the process for the whole set of input data, the smoothed output length (O_l) is equal to the input data length, since fitting is performed also in non–full window condition, unlike M–A. The degrees of freedom of the S–G methods are the polynomial order (p) and the data window width. The more p higher, the best is the fitting process, corresponding to a better noise tracking and worse smoothing degree. On the other hand, the width of data window increases the method smoothing capabilities: the wider the data window, the higher is the smoothing degree, bringing as disadvantage the possible disregard of fast dynamics inside the smoothing window.

3. CASE STUDY: THE COGNITIVE WORLD

The suitability of a smoothing filter is strictly addicted to the application field in which it is demanded to operate. In order to show the goodness of the proposal, a Cognitive Radio environment [6] has been simulated. As for Cognitive technologies, it is meant to refer to wireless communications in dynamic scenarios, where the term "dynamic" is to indicate the capability to change the transmission features (carrier frequency, modulation paradigm, output power, occupied bandwidth) in accordance to the changes of the electromagnetic environment within which the communication takes place. Cognitive Devices (often known as Cognitive Radios) are license-exempt transmitters, having the aim of exploiting (for interweave approach [11]) the *frequency holes*, i.e. frequency intervals in which the actual licensees are not transmitting, in a certain moment and geographical location. According to recent studies[12], there are some spectral intervals (e.g. TV white spaces opportunities [13]) where primary users are effectively inactive for significant periods of time. If not reused by Cognitive Terminals, such spectral resources would remain unexploited for important amount of time, making the spectral usage highly inefficient. In order to well–exploit such *free bands*, spectrum sensing tasks [14] have to be performed. They consist of algorithms able to identify the spectral behavior of primary users (PUs–licensees) inside the frequency interval of interest. There are well-assessed techniques available in literature: energy detection [15], waveform–based detection [16], cyclostationarity–based detection [17], matched–filter [18], wavelet methods [19]. Most of them need to work on the signal shape (in time or frequency domain): it implies that any modification of the original shape (such as the presence of superimposed noise) heavily affects the accuracy of such algorithms. Hence, it is necessary to apply a filter stage to the trace in order to reduce noise and, at the same time, to keep sharp edges in correspondence with the signal edges in order to correctly detect the occupied bands. The authors of the present paper have also proposed some methods to sense the electromagnetic spectrum, whose effectiveness has been tested for DVB–T signal discovery [7] and VDSA communications [8].

4. THE PROPOSED METHOD

4.1. Polynomial fitting: a theoretical approach

Let $[Y,X] = \{(x_1, y_1), (x_2, y_2) \dots, (x_p, y_p)\}, p \in \mathbb{N}$ be p data pairs in $\mathbb{R}^2$.

The goal of the polynomial fitting process is to find a polynomial curve, which can be represented as:

$$f(x) = \sum_{i=0}^{n} a_i x^i; \tag{2}$$

such that its coefficients ($\mathbf{a}^\star$) are solution of the following least square minimization problem:

$$\mathbf{a}^\star = \underset{\mathbf{a}}{\text{argmin}} \left\{ h(\mathbf{a}) = \sum_{i=1}^{p} (y_i - f(x_i))^2 \right\}; \tag{3}$$

To solve a minimization problem, a possible solution can be the computation of the gradient of the cost function (h) w.r.t. its unknowns and setting it to the null-vector.

$$\begin{cases} \frac{\partial h}{\partial a_0} = 0 \\ \dots \\ \frac{\partial h}{\partial a_n} = 0 \end{cases} \tag{4}$$

The problem expressed in (4) admits solution **if and only if** $p \geq n$. On the other way, solutions are not uniquely determined, and the fitting process can converge to different fitting curves. Basically, it is necessary that the number of points to fit has to be greater than the Degrees of Freedom (DoFs) of the target curve, represented by its coefficients.

4.2. The proposal

The proposal stems from polynomial fitting [20] theory, through the implementation of least square solutions. In particular, let N be the number of pairs (x_i, y_i), p the order of polynomial to be used. Let us subdivide the original N pairs in several subsets S_k, such that in each subset there are, at least, k pairs, with $k > p$. Let us introduce also another parameter, defined *overlap ratio* (O_r), whose value indicates the over-imposition percentage between one subset and its next one. The proposed method performs a polynomial

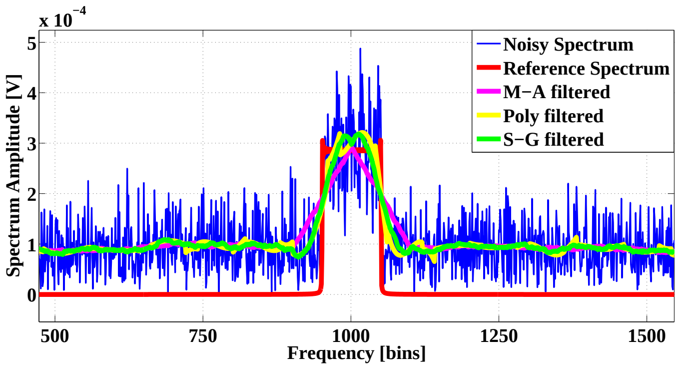


Fig. 1. The application of three smoothing filters on a frequency spectrum

fitting process in each subset, independently. The final fitted curve is the union of the obtained curves in each subset. The overlap ratio is necessary to make last points of a certain subset coincide with first ones of the next subset. In this way, even if independent, the fitting processes share some edge samples, and the continuity property can be respected.

With respect to S-G, the proposal has a very lighter computational cost, can be performed dramatically quicker, and gives similar results. Moreover, it is also preferred to standard smoothing methods, such as moving average, still widely used in many applications, such as Cognitive Radios' spectrum sensing [21], for its good response in shaping retainment, data shifting problem avoidance and appreciable noise reduction.

At the moment, a serial computing implementation has been carried out, but the problem structure is clearly suitable for a parallel architecture, giving a further increase to the quickness of the method. It is possible to appreciate the filtering effect of the proposed method (hereinafter named *Poly filtering*) in Fig. 1, where linear moving average and Savitzky-Golay filtering methods are also shown.

5. FILTERING PERFORMANCE

5.1. H_1 norm operator: theoretical description

Let $h(x)$ be a C^1 class function on its domain D, let $h'(x)$ be its first derivative. It is possible to define the H_1 norm as follows:

$$||h(x)||_{H_1} = \sqrt{||h(x)||_{L_2}^2 + ||h'(x)||_{L_2}^2} \tag{5}$$

where $||\cdot||_{L_2}$ is the Euclidean Norm.

This metric can also be applied to a function $h(x) = f(x) - g(x)$, in order to estimate the likelihood between the functions f and g. Such likelihood computation is preferable to the L_2 norm, since it takes into account not only the function values, but also its behavior in terms of shape, since a first derivative contribution is also present in the definition.

5.2. Scenarios' presentation

To evaluate performance of the presented method, a simulation campaign has been carried out. In particular, for a given frequency interval (defined in discrete mode as frequency bins), some different spectra have been realized, through the use of Matlab^{TM} software. They have been summarized in table 1 in which f_c and B denote the *carrier frequency* and the *occupied bandwidth* of the considered PU, respectively. The choice of the presented scenarios is due to the matter of fact that in CR application, it is usual to find as PUs to be detected wireless microphone and digital TV signals. That is why most scenarios in table 1 represent such trend, even if other scenarios (see Scenario 3) have been added in order to give a more general validity to the proposed method.

Table 1. Technical description of the presented scenarios

Scenario ID	No. PUs	f_{c1} [bins]	B_1 [bins]	f_{c2} [bins]	B_2 [bins]
1	1	1150	10	-	-
2	multiple	1500	30	-	-
3	multiple	1000	100	-	-
4	2	400	335	1000	335
5	1	1700	381	-	-

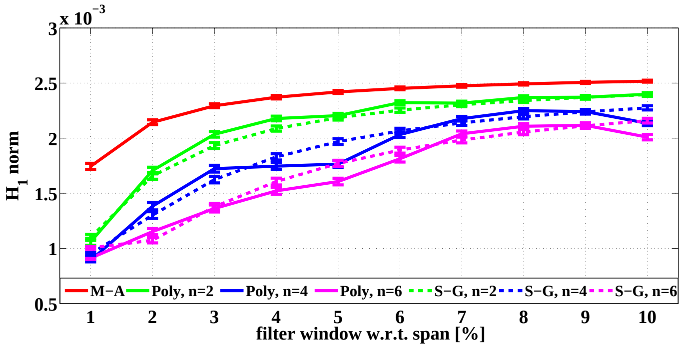
It is worth to highlight that the *frequency resolution*, in the simulated scenarios, is 20 kHz/bin.
Moreover the simulated scenarios can be seen as the following technology references:

- Scenario 1 corresponds to a wireless microphone (200 kHz band);
- Scenario 4 corresponds to two different DVB-T transmitter in 7 MHz mode;
- Scenario 5 is a DVB-T transmitter in an 8 MHz channels;
- Scenario 2 and 3 are created to evaluate the effect of filtering on intermediate cases, when more technologies are transmitting adjacently, such that an observer cannot distinguish them without further information.

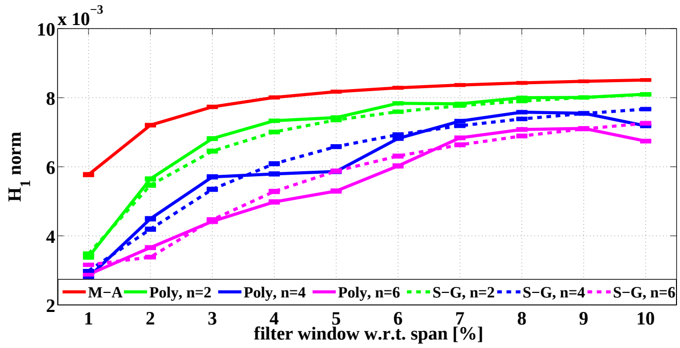
5.3. Results

The H_1 norm operator has been used to evaluate the *distance* between the filtered version of the spectrum under test and its original shape. Ideally, such distance should approach to zero in proportion to the capability of the filter to remove noise addition and maintain the original shape. In the following figures, some trial options have been chosen:

1. the polynomial order has been tested from 1^{st} to 7^{th}, since further increase would have provoked a very huge computational cost;
2. the filter length (the subset cardinality) has been chosen to cover the interval (1-10) % of the frequency interval under analysis;
3. the overlap ratio equal to 20% has allowed an acceptable continuity approximation;

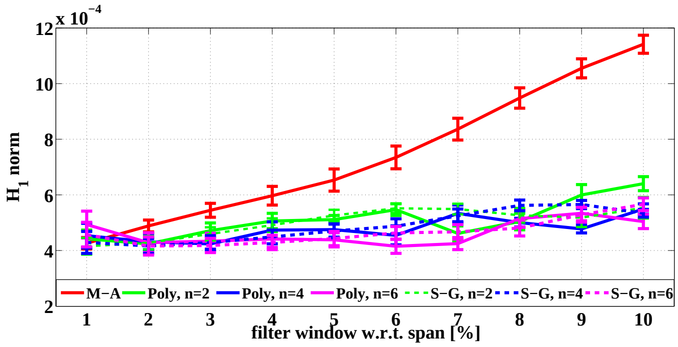


(a) SNR = -5 dB

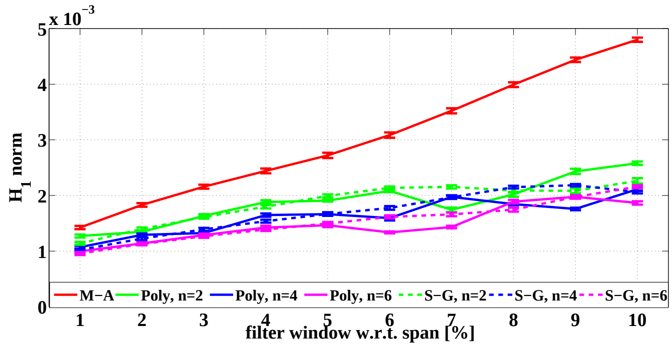


(b) SNR = 5 dB

Fig. 2. H_1 norm evaluation in Scenario 1



(a) SNR = -5 dB



(b) SNR = 5 dB

Fig. 3. H_1 norm evaluation in Scenario 3

4. four different signal-to-noise ratios (SNRs) from -10 dB to +5 dB have been tested to evaluate the goodness of filtering approach even in difficult operating conditions;

5. 100 iterations have been accomplished for each scenario, SNR, filter length, polynomial order combination.

In Figs. 2a, 2b, results for Scenario 1 are shown. It is possible to note that the proposed method shows comparable results w.r.t. Savitzky–Golay filter, while it represents an important improvement w.r.t. Moving average. The latter gives a linear shape to the filtered data, thus losing the sharpness of the edges. On the other hand, the S-G maintains good properties in terms of signal reconstruction but has a heavy computational cost, in terms of flops' intensity. The proposed method, instead, shows good performance and it is able to keep the computational effort on an acceptable level, as better highlighted in subsection 5.4. Results are shown only for 2^{nd}, 4^{th} and 6^{th} polynomial order, but the trend is respected also by the other tests. The higher the polynomial order, the lower the H_1 norm, meaning that it is better, if possible, to increase the order to reach better reconstructions. On the other hand, if such an order is too high, the computational burden becomes no more negligible, also for the proposed method.

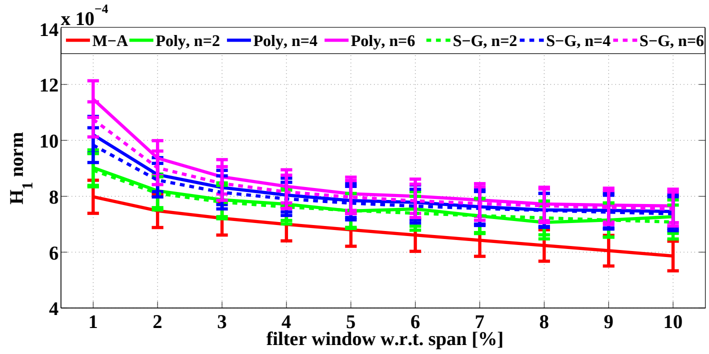
Results are also confirmed in Figs. 3a, 3b, where filtering performance concerning scenario 3 is presented. For the scenario under test, S–G and Poly filtering show very similar outcomes, while M–A has an almost linear increasing norm w.r.t. the filter window. The trend is confirmed both for low (-5dB) and high (5dB) SNR. The only difference has to be found in the vertical error bars (representing the standard deviation over the 100 tests), showing a more stable behavior when SNR increases.

The situation trend changes for Scenario 4, illustrated in Figs. 4a, 4b, where two users are simultaneously present on the analyzed spectrum. The global occupied bandwidth is about 28% of the entire span. In this chance, for low SNRs (Fig. 4a), where noise strongly corrupts the signal, the shape reconstruction is not a simple task, and denoising becomes the main operation. In such a case, our method maintains compatibility with other methods in capability of removing noise from trace. When SNR becomes appreciable (Fig. 4b), the proposed method still performs the best attitude.

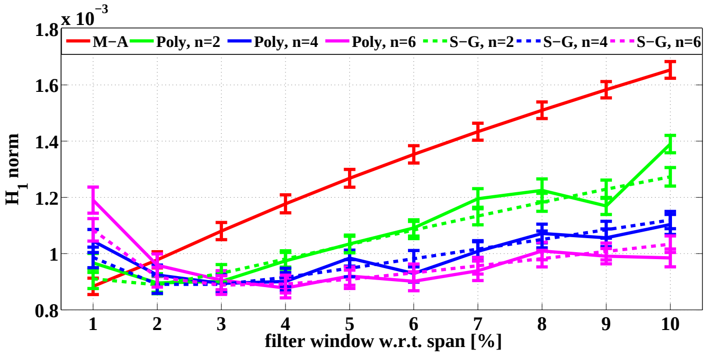
Finally, it is important to underline that in all situations, when SNR increases, the order of magnitude of H_1 norm results increases. It is only due to the increase of the signal power, leading to an augmentation in the absolute differences. It does not deal with the filtering abilities, witnessed by the error bars, showing a greater stability when SNR gets higher.

5.4. Flops intensity hints

In order to make explicit the computational advantages of the proposed method w.r.t. Savitzky–Golay approach, belonging to the same filter category, an estimate of the *floating point operations* has been performed through

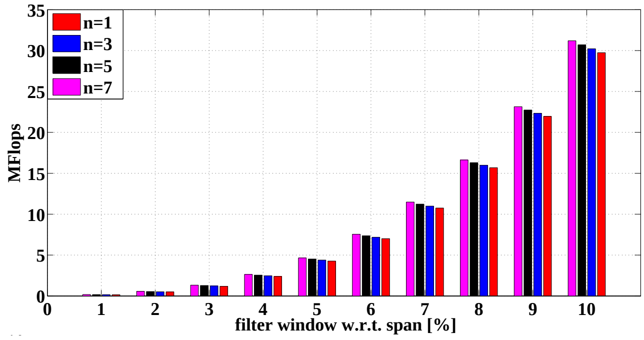


(a) SNR = -5 dB

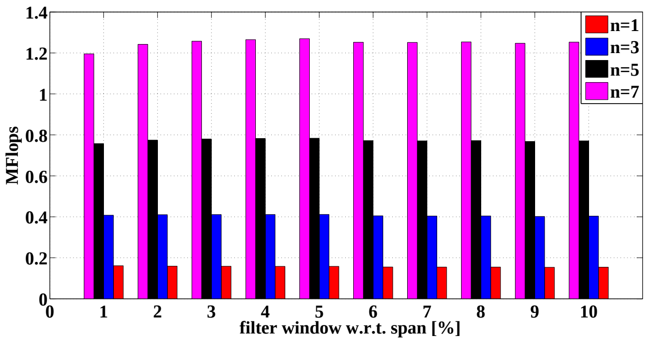


(b) SNR = 5 dB

Fig. 4. H_1 norm evaluation in Scenario 4



(a) S–G Flops' estimation



(b) Proposed method Flops' estimation

Fig. 5. Flops estimation for the filtering methods

Matlab™ software. The results can be observed by Figs. 5a, 5b, and they have been summarized in Table 2. All measurements are to be intended in Mfl (Mega Flops). In Fig. 5a the S–G flops account is presented. The different columns of the histogram represent different polynomial order, while the X–axis describes the filter window percentage variation. Finally, Y–axis displays the estimated number of Mega Flops in each condition. In the same manner, Fig. 5b describes the Poly Filter flops estimation. It is possible to see a different trend between the Poly and S–G methods: the first is heavily influenced by the polynomial order, while it is basically constant w.r.t. the filter window; the second is almost order independent, whilst it suffers the filter window variation. As an overall result, the different Y–scales in Figs. 5a, 5b remark that the proposed Poly Filtering Method is definitely much lighter in terms of computational burden (at least one order of magnitude) than S–G method, as shown in Table 2. That implies that the proposed method is clearly more suitable to be employed in low cost devices, having reduced computational capabilities.

6. CONCLUSIONS

In this paper a novel data smoothing technique has been presented. It takes its origin from the polynomial fitting method. It can be compared to Savitzky–Golay filter, which represents one of the most promising smoothing method in *CR* applications, as shown by the authors in [8], but

Table 2. Mega Flops' estimation for S-G and the proposed method

		Filter window w.r.t. span		
		2 %	6 %	10 %
Polynomial order p	1^{st}	S-G: 0.49	S-G: 7	S-G: 29.7
		POL: 0.16	POL: 0.16	POL: 0.15
	3^{rd}	S-G: 0.52	S-G: 7.18	S-G: 30.2
		POL: 0.41	POL: 0.41	POL: 0.41
	5^{th}	S-G: 0.54	S-G: 7.36	S-G: 30.7
		POL: 0.77	POL: 0.77	POL: 0.77
	7^{th}	S-G: 0.56	S-G: 7.55	S-G: 31.2
		POL: 1.24	POL: 1.25	POL: 1.25

the different implementation has been shown to be more efficient and computational lighter, while preserving almost the same output results. The H_1 norm has been implemented to test its effectiveness, and satisfying results have been obtained. It can be still improved and optimized in order to be executed in a parallel architecture. Moreover, its ability to reconstruct original noisy data can surely be exploited for further applications, not only in *CR* field. The authors claim to show, in future works, how such a filter can be used to achieve good performance in spectrum sensing, also in very poor SNR conditions.

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