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UNCERTAINTY EVALUATION FOR INDIRECT MEASUREMENT WITH DIFFERENT TYPES OF MEASURING DEVICES

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Abstract – The paper gives formulae for uncertainty evaluation of indirect measurement based on direct measurements by different types of measuring devices at different ranges as well as nominal values of quantities produced by measures. One type has specifications of total error (multimeters, e.g.), while other type has specifications of offset, gain and linearity errors (data acquisition devices, ADC). Choice of range for devices with specifications of separate errors is considered.

Keywords: indirect measurement, accuracy specifications, measurement uncertainty

1. INTRODUCTION

As well-known [1], the result of in direct measurement is given as a function of several variables:

$$X = f(X_1, X_2, ..., X_n)\,. \tag{1}$$

The values of these variables are found by direct measurements. The maximum possible absolute error of indirect measurement (the worst-case) can be found from (1) approximately as [1]

$$|\Delta X| = \left|\frac{\partial f}{\partial X_1}\right||\Delta X_1| + ... + \left|\frac{\partial f}{\partial X_n}\right||\Delta X_n|\,. \tag{2}$$

The accuracy of (2) is usually offered to find by means of high order derivatives [1].

Errors $\Delta X_1 .. \Delta X_n$ are usually supposed to be independent. The exception to the rule is given in [2]. All direct measurement are supposed to use the same ADC with the same maximum offset error U_0, maximum gain error U_G and maximum linearity error U_{inl}. The first two errors are supposed to have the same sign while linearity can change the sign for different direct measurements. Indirect measurement uncertainty is found in accordance with [3] for four cases: standard uncertainty and worst-case uncertainty (both absolute and relative). Absolute worst-case uncertainty of indirect measurement is then [2]

$$U(X) = U_0\left|\Sigma_{i=1}^{n} k_i\right| + U_G\left|\Sigma_{i=1}^{n} k_i X_i\right| + U_{inl}\sum_{i=1}^{n}|k_i|. \tag{3}$$

Coefficients k_i can be with different signs. For example, indirect measurement $X = X_1 - X_2$ gives $k_1 = -k_2 = 1$. Therefore, (3) can show less value than one found by (2).

This paper considers general case with different accuracy specifications of measuring devices used for direct measurements and different measures. Only one case, when the same device with the same range and the specification of U_0 and U_G is used for all direct measurements can give less uncertainty with regard to (2). This result is possible only for such indirect measurements, where (1) includes difference or/and ratio of variables. But this case is very important for many practical applications. Some examples are given in section 4.

According to any textbook, accuracy of direct measurement is increased if this measurement is realized as close as possible to the full scale of the corresponding meter. The choice of the full scale close to meter reading for direct measurements of $X_1 ... X_n$ is not always useful for indirect measurement. Sometimes the choice of the same range for all $X_1 ... X_n$ can give better accuracy even if the less range could be chosen. Conditions for choice of one range for two direct measurements are found in this paper.

Direct measurements of $X_1 ... X_n$ are often fulfilled by data acquisition devices. Three most popular sampling architectures are multiplexed, simultaneous sample and hold, multi-ADC structures [4]. Corresponding structures are shown in fig. 1. For the same accuracy and speed of used ADC, the multi-ADC architecture gives higher scan rate per channel [4]. According to given paper, many indirect measurements by multiplexed architecture can be more accurate with the same accuracy of ADCs.

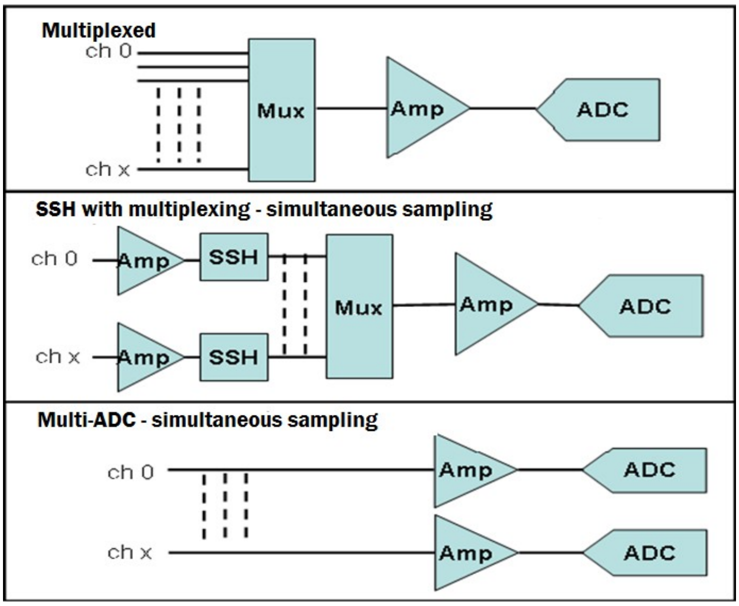

Fig.1. Simultaneous sampling architecture – simultaneous sample and hold (SSH).

Mux–multiplexer,
Amp–instrumental amplifier,
ADC-analog-to-digital converter,
SSH – simultaneous sample and hold.

For very complicated functions (1) some problems can be arisen to find (2) and its accuracy. The application of simulation to solve these problems can be offered.

2. ACCURACY SPECIICATIONS FOR DIFFERENT TYPES OF MEASURING DEVICES

Accuracy specifications of digital instruments are usually presented by total (maximum) absolute error. At the first approximation, maximum absolute measurement error for each variable X_i found by a digital instrument is

$$\left|\Delta X_i\right|_{DI} = (a + b\left|X_i\right|), \tag{4}$$

where a is a positive number with the same unit as X_i and b is a positive non-dimensional number.

Maximum absolute error of a digital instrument as a function of input signal X is shown in fig. 2. Two values of the input signal (X_1 and X_2) are considered. Maximum possible difference between the corresponding absolute errors (Δ_1 and Δ_2) is shown in fig. 2.

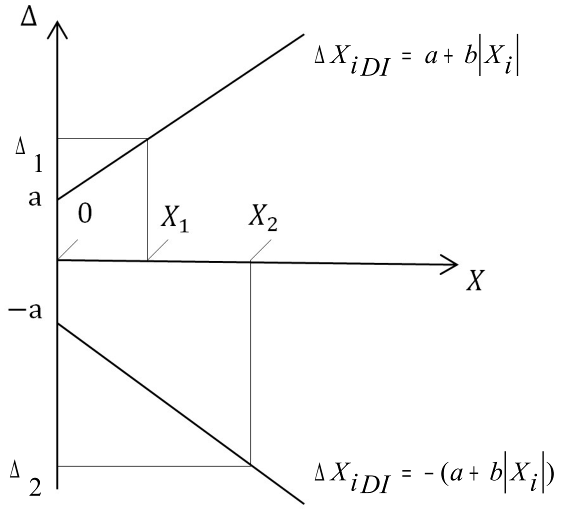

Fig.2. Maximum absolute measurement error vs. input signal for digital instruments.

Sometimes a is given in % of range ($X_{FS.i}$) and b is given in % of reading.

Accuracy specifications of ADS and data acquisition device are usually presented by maximum offset, gain and linearity errors. Quantization error and noise (random error) are usually included in a and b for digital instruments but can be specified separately for other devices. For simplicity we will not consider them within this paper. Only worst-case uncertainty will be calculated. Then, for data acquisition devices, maximum absolute error

$$\left|\Delta X_i\right|_{DA} = U_0 + U_G\left|X_i\right| + U_{inl}\,. \quad (5)$$

Maximum absolute error for ADC as a function of input signal X is shown in fig. 3. Two values of the input signal (X_1 and X_2) are considered. Maximum possible difference between corresponding absolute errors (Δ_1 and Δ_2) is shown in fig. 3. Linearity error is supposed to be zero at the ends of the range but can be equal to maximum value with any sign at any other points.

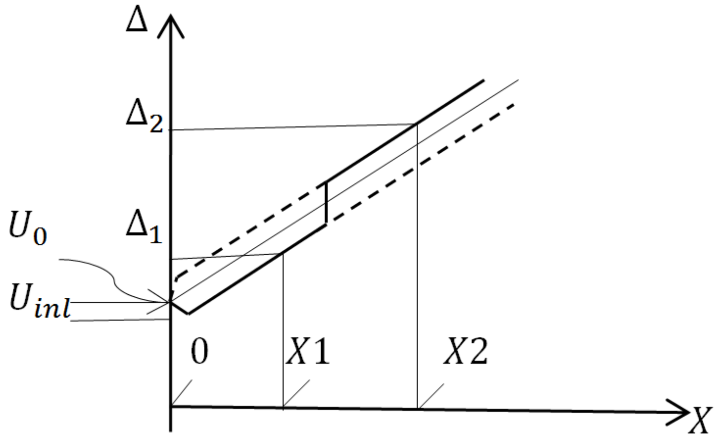

Fig. 3. Maximum absolute error vs. Input signal for data acquisition devices.

If (4) and (5) show the same results, then a and b can be found with given U_0, U_G, U_{inl} as

$$a = U_0 + U_{inl}, \tag{6}$$

$$b = U_G. \tag{7}$$

If a and b are specified, then U_0, U_G, U_{inl} cannot be calculated. Besides a and b, some digital instruments have the additional specification of linearity error. For example, linearity error is specified in [5] as

$$U_{inl} = A_L X_{FS} + B_L X_i. \tag{8}$$

One can find from (4), (5) and (8)

$$U_0 = a - A_L X_{FS}, \tag{9}$$

$$U_G = b - B_L. \tag{10}$$

Now maximum absolute error of the digital instrument can be written by the same way as it was given for data acquisition devices:

$$\left|\Delta X_i\right|_{DI} = U_0 + U_G\left|X_i\right| + A_L X_{FS} + B_L\left|X_i\right|. \tag{11}$$

Measures of voltage (digital-to-analog converter), resistance, time etc. can be specified both by (4) and (5).

3. GENERAL FORMULAE FOR INDIRECT MEASUREMENT UNCERTAINTY

Direct measurements with number of n used for indirect measurement can be divided into three parts. Absolute and relative worst-case uncertainty of indirect measurement is then

$$\begin{aligned} U(X) = {} & U_0\left|\sum_{i=1}^{n_1} k_i\right| + U_G\left|\sum_{i=1}^{n_1} k_i\left|X_i\right|\right| + U_{inl}\sum_{i=1}^{n_1}\left|k_i\right| + \\ & + \sum_{i=n_1+1}^{n_1+n_2} U_{0.i}\left|k_i\right| + \sum_{i=n_1+1}^{n_1+n_2} U_{G.i}\left|k_i X_i\right| + \sum_{i=n_1+1}^{n_1+n_2} U_{inl.i}\left|k_i\right| + \\ & + \sum_{i=n_1+n_2+1}^{n} (a_i + b_i\left|X_i\right|)\left|k_i\right|, \end{aligned} \tag{12}$$

$$\begin{aligned} U_r(X) = {} & U_0\left|\sum_{i=1}^{n_1} \frac{k_{ri}}{\left|X_i\right|}\right| + U_G\left|\sum_{i=1}^{n_1} k_{ri}\right| + U_{inl}\sum_{i=1}^{n_1}\left|\frac{k_{ri}}{X_i}\right| + \\ & + \sum_{i=n_1+1}^{n_1+n_2} U_{0.i}\left|\frac{k_{ri}}{X_i}\right| + \sum_{i=n_1+1}^{n_1+n_2} U_{G.i}\left|k_{ri}\right| + \sum_{i=n_1+1}^{n_1+n_2} U_{inl.i}\left|\frac{k_{ri}}{X_i}\right| + \\ & + \sum_{i=n_1+n_2+1}^{n} (\frac{a_i}{\left|X_i\right|} + b_i)\left|k_{ri}\right|. \end{aligned} \tag{13}$$

The first part of (12) and (13) includes n_1 direct measurements made by the measuring device with accuracy specification (5) used at the same range. In means that U_0, U_G, U_{inl} parameters are constant for all n_1 measurements. A value of n_1 corresponds to the condition

$$2 \le n_1 \le n . \tag{14}$$

The second part includes n_2 direct measurements made by the measuring device with accuracy specifications (5) used at different ranges, or made by different devices at any ranges.

The third part includes $n_3 = n - n_1 - n_2$ direct measurements made by any measuring device with accuracy specification (4) used at any ranges.

If $n_1 = n$, then (12) and (13) correspond to equations from [2]. The influence of offset and gain errors of direct measurements can be decreased for the evaluation of indirect measurement uncertainty in comparison with classical approach [1].

If $n_3 = n$, then influence of offset and gain errors of direct measurements are the same as in [1].

If $n_2 = n$, then influence of offset and gain errors of direct measurements cannot be decreased for the evaluation of indirect measurement uncertainty in comparison with approach [1] in the case of the worst-case method. Standard uncertainty of indirect measurement (probability P is less than 1, typically P=0.95) can be less than the result of classical approach [1]. Maximum difference up to $\sqrt{3}$ times can be found if influence of offset, gain and linearity error on uncertainty is equal.

All three parts of (12) and (13) can be given for values of measures (DAC, for examples).

If two or more devices with accuracy specification (5) are used for two or more direct measurements or two or more measures each at the same range, then the first parts of (12) and (13) must be written for each device.

4. TYPICAL INDIRECT MEASUREMENTS

The simplest indirect measurement is described by a function $Y = X_2 - X_1$. For this function $k_1 = -k_2 = -1$. Absolute worst-case uncertainty of indirect measurement for application of one device at the same range ($n_1 = n = 2$) and $X_2 \ge X_1 \ge 0$ in accordance with (12) is

$$U_I(X) = U_G(X_2 - X_1) + 2U_{inl} . \tag{15}$$

Absolute worst-case uncertainty of indirect measurement for application of two devices by the same type at the same range ($n_2 = n = 2$, $U_{0.1} = U_{0.2} = U_0$, $U_{inl.1} = U_{inl.2} = U_{inl}$, $U_{G.1} = U_{G.2} = U_G$) in accordance with (12) is

$$U_{II}(X) = 2U_0 + U_G(X_1 + X_2) + 2U_{inl} . \tag{16}$$

The maximum difference of results found by (16) and (15) will be at $X_1 \approx X_2 \approx X_{FS}$:

$$\frac{U_{II}(X)}{U_I(X)} = 1 + \frac{U_0 + U_G X_{FS}}{U_{inl}} . \tag{17}$$

Let us use (17) to compare uncertainty of multiplexed and multi-ADC structures (discussed in section 1) for implementation of the function $Y = X_2 - X_1$. Model PCI-6250, used in multiplexed structure, includes ADC with, for example, following parameters [6]: X_{FS} = 10 V, U_0 = $2 \cdot 10^{-4}$ V, U_G = $6 \cdot 10^{-5}$, U_{inl} = $6 \cdot 10^{-4}$ V. If we use the same ADC in multi-ADC structure, then, according to (17), the worst-case uncertainty will be by 2.3 times more.

If $X_1 << X_2$, then two ranges can be used for each direct measurement. In this case the absolute worst-case uncertainty of indirect measurement $Y = X_2 - X_1$ for (5) is

$$U_{III}(X) = U_{0.1} + U_{0.2} + U_{G.1}X_1 + U_{G.2}X_2 + U_{inl.1} + U_{inl.2}. \tag{18}$$

The application of one range for both direct measurements will be better, if (16) gives less result in comparison with (18).

Let us consider an example of PCI-6250 used for indirect measurement $Y = X_2 - X_1$ with X_1 =5 V and X_2 =10 V. Corresponding specifications for $X_1 \approx X_{FS.1}$ = 5 V are $U_{0.1}$ = $1 \cdot 10^{-4}$ V, $U_{G.1}$ = $7 \cdot 10^{-5}$ V and $U_{inl.1}$ = $3 \cdot 10^{-4}$ V. The specifications for $X_2 \approx X_{FS.1}$ = 10 V were given before. Using PCI-6250 at one 10 V range, we get from (16) $U_I(X)$ = 1.5 mV. If we use two ranges (5 V for X_1 and 10 V for X_2), the result is $U_{III}(X)$=2.15 V. It means that application of only one range gives 1.4 times better result.

5. CONCLUSIONS

There are two main types of specifications for measuring devices: maximum total error (4) and maximum offset, gain and linearity errors (5). The second type of specifications allows user to evaluate maximum total error (6, 7). Opposite action is not possible in general case. Sometimes linearity error is specified besides (4). The formulae for offset (9) and gain (10) uncertainties for this case are offered in the paper.

Indirect measurement is based not only on direct measurement, as it is usually supposed [2, 3], but also on values of measures (DAC, for example) used in indirect measurement.

General formulae for absolute (12) and relative (13) indirect worst-case uncertainties are found as functions of three parts of variables. These parts include application for indirect measurement of one or several devices at one or several ranges with specification of total error or maximum offset, gain and linearity errors separately. Only the first part of (12) and (13) can give less value for some types of indirect measurement in comparison with classical approach (2). in the case of the worst case method. Formulae published before are the private cases of (12) and (13). The second part of (12) and (13) was not discussed in [1, 2]. This part can give less value in comparison with classical approach (2) only for the standard uncertainty. Formulae (12) and (13) can be used also to take into account application of measures (DAC, for example) for indirect measurements.

Accuracy of (12) and (13) can be found not only by means of high order derivatives [1] but also by simulation of (1) giving corresponding changes of variables $X_1, X_2, ..., X_n$.

Applications of offered approaches are given in the 4 section. The advantage of multiplexed structure vs. multi-ADC structure for indirect measurement from static accuracy point of view is shown though the multi-ADC structure is better from dynamics point of view [4].

Conditions for choice of one range for typical indirect measurement are found.

6. REFERENCES

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