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THE LINEAR STATE ESTIMATION ALGORITHM TO IMPROVE THE ACCURACY OF MEASUREMENTS OF THE INSTANTANEOUS VALUES OF CURRENTS AND VOLTAGE IN ELECTRIC NETWORKS

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Abstract – The paper presents an algorithm which allows to reduce measurements' inaccuracy of an instantaneous values of currents and voltages in electrical networks. The considered approaches are used in the prototype stand for the non-sinusoidal power flows modeling based on LabVIEW programming environment using the facilities of modules NI-9225 and NI-9227. Experiment was performed to model non-sinusoidal load flow on the prototype stand for a simple AC 10 kV closed network.

Keywords: synchronous measurements, distribution network, smart grid, state estimation, non-sinusoidal load flow

1. INTRODUCTION

In recent years, the implementation of Phasor Measurement Units (PMU) and WAMS-technology built on these has become a forward-looking route for the development of tools for monitoring the power flow of power systems. Contemporaneously with implementation of the PMUs, the state estimation theory has been developing [1] in line with use of data obtained from such devices [2]-[5]. These works use measurements of the RMS values of currents, voltages and phase angles, which are synchronized with global universal time. Furthermore, as a rule, the power flow is considered to be stationary and sinusoidal. However, many models of PMU [6] and Intelligent Electronic Devices (IEDs) have the function of a memory oscilloscope, which allows for obtaining instantaneous values of currents and voltages digitalized with a certain sampling frequency $\boldsymbol{f_s}$. In addition, modern optical measurement current and voltage transformers, Merging Units (MUs) give a steady flow of measured values of $u(t)$ and pegged to Universal Coordinated Time (UTC) in accordance with the IEC-61850-9.2-sv protocol [7]. Currently, the use of this data in practice is problematic, due to the high requirements to capacity of telecommunications channels. However, intensive development of information technology allows for the assumption that the cost of PMU and IED in the near future will reduce noticeably; whereas the capability of high speed data transmission over optical fibre communication channels will expand [8]. This will result in the wide-scale use of synchronous measurement technology with practical implementation of the concept of "Smart Grids", including in distribution networks. The purpose of this article is to set out a theoretical foundation of the possibility of simulation of non-sinusoidal power flows on instantaneous synchronized values of currents and voltages in power grids and analysis of results obtained based on the example of a basic loop network.

2. FORMULATION OF THE PROBLEM

Let us consider a single-phase line with resistance R and inductance L. Let us assume that at the start and end of the line there are devices installed, to measure the instantaneous voltage values $u_1(t)$ and $u_2(t)$. The instantaneous current values in line $i_{12}(t)$ can be found using the following first-order differential equation [9]:

$$u_1(t) - u_2(t) = i_{12}(t) \cdot R + \frac{di_{12}(t)}{dt} \cdot L \,, \tag{1}$$

and representing current derivative in a finite-difference form:

$$\left.\frac{di_{12}(t)}{dt}\right|_{t=t} \approx \frac{i_{12} - i_{12}^{-}}{\Delta t} \,, \tag{2}$$

where i_{12} – instantaneous current value at current time t;

i_{12}^{-} – instantaneous current value at previous time t-Δt;

Δt – period of measurements, this is the reciprocal of the frequency sampling f_s.

The instantaneous current value i_{12} can be found from the equation obtained, if its value i^{-}_{12} at previous time is known. Such approach is very similar to a classic procedure of using Euler's method for the numerical solution of a differential equation. The difference is that under the classic approach functional dependencies of voltages $u_1(t)$ and $u_2(t)$ on time are known, whilst in this case their values are measured at discrete points in time. Therefore to increase the accuracy, instead of Euler's method, its various modifications or the Runge-Kutta methods [9] that are broadly used in transient analysis can be applied.

The accuracy of the solution of the equation (1) depends on any error in setting the passive line parameters, any error in the measurement of the instantaneous values of the voltages, the accuracy of their synchronization and period of measurements Δt. The error in setting the passive line parameters, in turn, is impacted by its temperature and the so-called "skin effect" (the increase in wire resistance for higher harmonics). Additionally to an error in the receivers of the satellite navigation systems, the accuracy of synchronization can influence the schematic diagram of PMUs. For instance, in order to cut costs in some cases PMU schemes with a multiplexed analogue-to-digital converter (ADC) can be applied. In this case, the value of instantaneous measurements will be shifted in time relative to each other, by a certain amount smaller than Δt. The selection of measurement frequency Δt is not a simple exercise. With the decreasing of Δt, in accordance with the theory of Kotelnikov, the number of highest considered harmonics increases. However, this leads to an increase in the error in the numerical calculation of derivatives in accordance with (2). The error increases due to a decrease in the numerator and denominator. In addition, the effective number of bits of many PMUs depends on the sampling frequency [10].

To increase the accuracy of simulation of network power flow in the presence of redundant measurements, the methods of the state estimation theory [1]-[5] are applied.

Under the non-sinusoidal power flow, two main approaches can be taken:

1. Estimation of the harmonic components of the parameters of state [2]-[5].
2. Estimation of the instantaneous values of currents and voltages [11]-[13].

The advantage of the first approach is that a smaller amount of initial information is transmitted. However, it cannot be applied in order to estimate the transient. The second approach allows for evaluating any network state, including transitional ones and those with subharmonics.

Let us consider the possibility of its application to a network of any configuration.

The mathematical formula of the problem of the state estimation of instantaneous values of voltages and currents can be shown as follows:

$$\varphi = \left[\overline{V} - V(X)\right]^T R_v^{-1} \left[\overline{V} - V(X)\right] \to \min, \quad (3)$$

where $\overline{V}$ – vector of measured parameters;

X – vector of estimated values;

$V(X)$ – the true values of the measured parameters, interconnected through equations of the first Kirchhoff's law in the instant currents for each network node and Ohm's law (1) for each branch network;

R_v – a square matrix called the covariance matrix. Its diagonal elements r_{ii} are equal to an error variance σ^2_i of corresponding measurements and the off-diagonal elements r_{ij} are equal to an error covariance of the i-th and j-th measurements.

Minimum of function φ is found by equating to zero the derivatives on components of the vector X.

$$\left(\frac{\partial V}{\partial X}\right)^T R_v^{-1} \left[\overline{V} - V(X)\right] = 0. \quad (4)$$

As $V(X)$-dependencies are linear, the system (4) can be transformed into the system of normal equations [14]:

$$A^T R_v^{-1} A X = R_v^{-1} B, \quad (5)$$

where A – matrix of dependence coefficients $V(X)$ with the unknowns;

B – vector of free members generated from the measurements.

Initial data in this problem formulation includes vector B, the covariance matrix of measurement errors R_v, and matrix A. Matrix A depends on the network topology and passive parameters of lines. The result of the solution is vector X that includes the estimated voltages at the nodes, the estimated load currents and estimated currents in the lines.

3. ALGORITHM

In practice, in order to enhance the accuracy of calculations with the help of a computer, it is advisable to combine the measurements of the instantaneous values over several time points at once into a system of equations (4). The more instantaneous values at the adjacent points in time are combined into one system, the more accurate the calculated results will be. We will call the set of measurements combined into such system a "portion", and the number of measurements of each parameter in this system – a "size" of the portion. All equations of each "portion" can be divided into three groups:

1. Equations of the first Kirchhoff's law, for each node in the network and for each considered point in time.
2. Equations of Ohm's law, for each network line and for each point in time.
3. Equations describing the measurements of the instantaneous values over the period under review.

The optimum size of "the portion" will depend on the sampling frequency, the processing power of the system and the requirements for speed and accuracy of the calculations. Let us consider the general case for a portion having a size of n. We will assume that the line resistance and the network topology remain constant for a period of time that corresponds to the measurements of a portion. In this case, the equations for all three groups mentioned above will be the same for each time point. This allows the calculation of the corresponding sub-matrix of coefficients for each group of the equations, and then, depending on the size of a portion, copying them the required number of times into the respective positions of matrix A. Such an approach allows for a considerable reduction in the computational costs incurred to form a matrix of coefficients with the unknowns for estimating the instantaneous values of currents and voltages, even when the size of a portion is large (from hundreds to thousands), since the calculation of each type of sub-matrices occurs only once.

Fig. 1 represents the structure of a matrix A and the vector of free terms B for a portion of size n. Each column of the matrices represents the vector of the estimated parameters at time points t, $t+\Delta t$, $t+2\Delta t,\ldots$, $t+(n-3)\Delta t$, $t+(n-2)\Delta t$, $t+(n-1)\Delta t$ respectively.

The letter "K" denotes the sub-matrices of the coefficients corresponding to the equations of the first Kirchhoff's law, written for each network node at the corresponding time point.

The letter "O" denotes the sub matrix of coefficients corresponding to the equations of Ohm's law for each network line. As can be seen from the figure, the equation of Ohm's law combines measurements in two adjacent points in time. This is due to the fact that these equations calculate the current derivatives in the current finite difference form through the current at current and previous time points. The only exception is the sub-matrix for Ohm's law at time t. As indicated above, the current value at the initial time point should be known. In this case, it is assumed to be zero.

The letter "M" denotes the sub-matrices of coefficients of equations corresponding to the measurements of instantaneous values at a specified time point (If there is a corresponding measurement, the coefficient is 1, 0 otherwise).

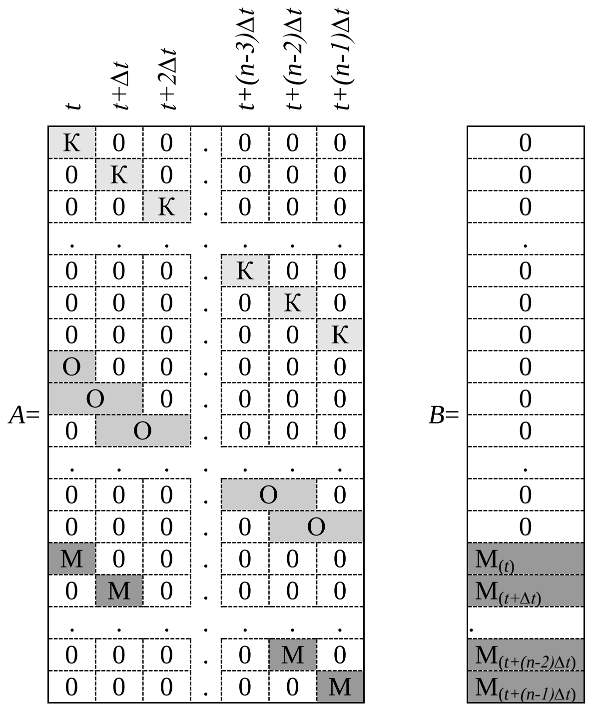

Fig. 1. The structure of the matrix of coefficients and the vector of free terms of the system of equations for estimating the instantaneous values of currents and voltages.

In general, the sub-matrices "K", "O", and "M" are rectangular. The remaining elements of Matrix A are equal to zero. Thus, the matrix of coefficients with the unknowns is extremely rarefied. Furthermore, the more adjacent measurements are combined in a system, the more zero elements it contains. This allows for the use of all the methods applied to register the weak occupancy of matrices [15], used in the classical theory of power flow calculation and state estimation.

Fig. 1 also represents the structure of a vector of free terms *B*. As can be seen from the figure, most of the elements of the vector *B* are also zero. Only the non-zero elements will correspond to the measurements of instantaneous values at specific time points.

Given all of the above described specific features, we propose the following algorithm for operating the software module that estimates the state based on instantaneous values of currents and voltages (Fig. 2):

1. To set the network topology and parameters of its elements.
2. To undertake the analysis of the network topology, based on which (F, which refers to analysis) sub-matrices "K" and "O" are formed.
4. On the basis of the list of measurements carried out sub-matrix "M" is formed, the dimension of which depends mainly on the number of PMUs or IEDs available in the network.

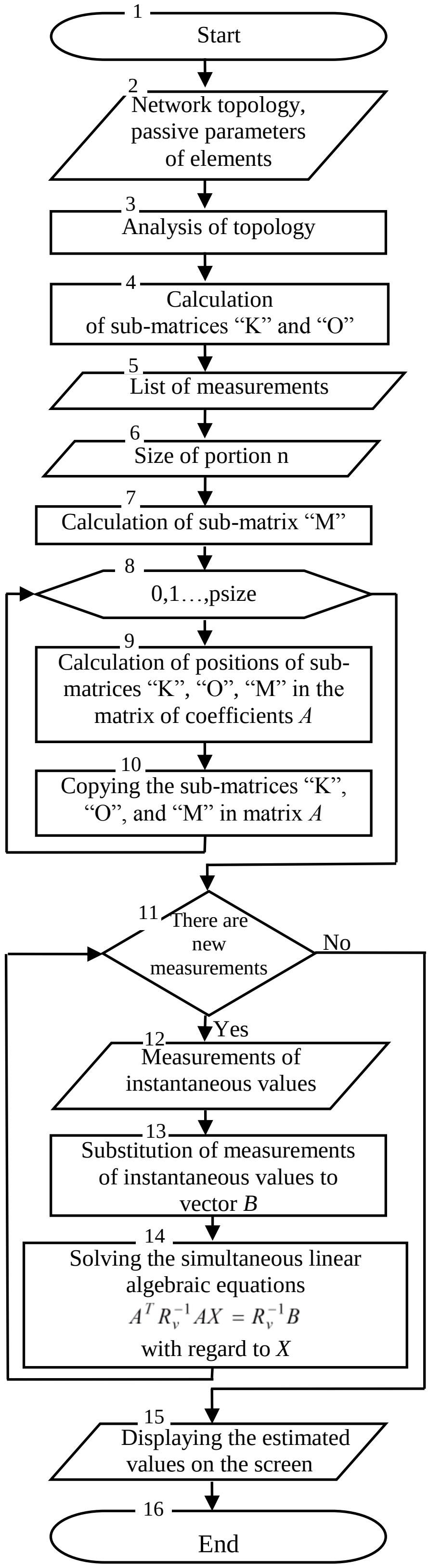

Fig. 2. Control-flow chart of estimating the instantaneous values of currents and voltages.

5. To select the size of the portion of data processed. This value affects the accuracy and time needed for

computation.

6. To mark the matrix of coefficients of the system of equations. This stage involves the calculation of the positions of each type of sub-matrices for the corresponding points in time.

7. To form the vector of free terms and the covariance matrix of measurement errors.

8. To solve the system (5) employing the Gauss method and using the techniques that take into account the sparseness of coefficient matrix [15].

9. The solution enables us to determine the estimated values of all network parameters: voltages at the nodes, the currents in the branches and injection currents.

10. Stages 1-9 to be repeated for the next portion of data. If the network topology and the passive parameters of its elements have not changed, stages 1-3 can be omitted. Likewise, if the portion size has also remained the same, the re-calculation of matrices of coefficients at stage 6 is not required. This reduces considerably the computational costs of the algorithm proposed. Therefore it is proposed to set the size of the portion in advance before making the calculations and not change it over the whole computational process.

4. SIMULATION RESULTS

In order to test the proposed method, an experimental software module that implements the algorithm discussed was developed. Development was carried out in the programming language C + + in the MS Visual Studio 2008 development environment. In order to assess the effectiveness of the proposed approach, an experiment was carried out on modelling the non-sinusoidal power flow on an AC table (Fig. 3). A model of a very simple 10 kV closed network was assembled. The network diagram is shown in Fig. 4.

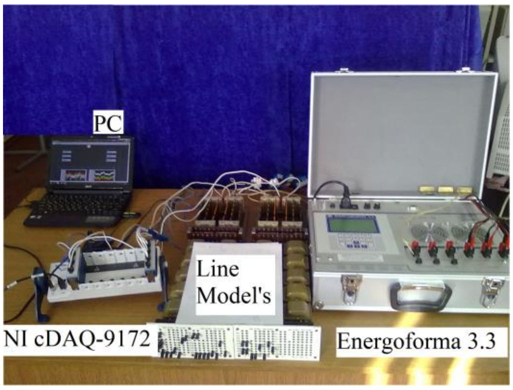

Fig. 3. AC table for modeling the non-sinusoidal power flow.

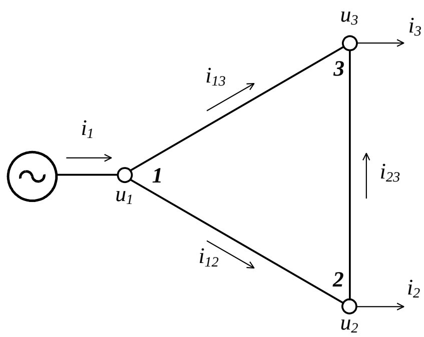

Fig. 4. Simple closed network.

The passive parameters of model: Z12=2.181+j5.446 Ohm, Z13=2.063+j4.097 Ohm, Z23=2.490+j4.318 Ohm. The power is supplied by node 1, the voltage at which was counted to be sinusoidal and equal to 10 kV, and the frequency was counted to be 50 Hz. Modelling was performed for one phase of the network. The following scales were adopted in developing the model: on current – 100 A original/A model; on voltage – 288 V original /V model; on resistance – 2,88 Ohm original/Ohm model.

The Energoforma 3.3 generator of non-sinusoidal waveforms was used to simulate the loads and the power supply [16]. It allows us to generate three currents and the three voltages of prescribed shape and amplitude in galvanically isolated channels. The research involved one voltage channel and two current channels. The voltage channel was connected to node 1 and ground. The current channels were connected in parallel to branches 1-2 and 1-3, respectively (Fig. 5). Thus, there was a possibility of independent control of the voltage at node 1 and the load currents at nodes 2 and 3. Taking into account the scale adopted, the generator voltage at node 1 was set to 20 V. The load currents contained the odd harmonics up to 9 (inclusive) (Table 1).

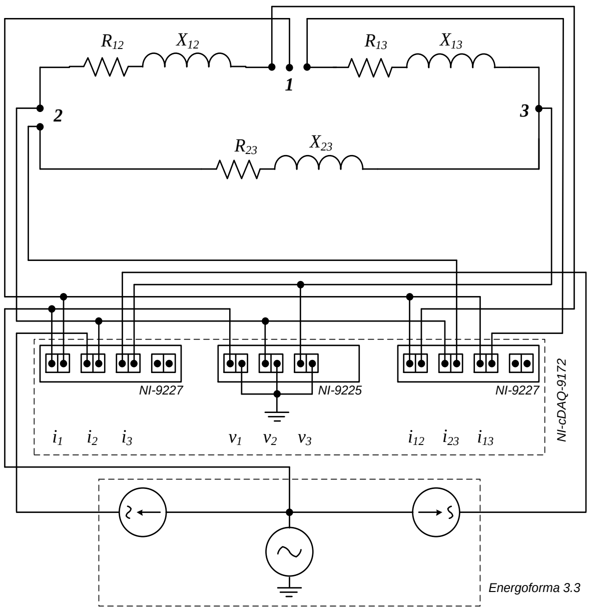

Fig. 5. Diagram of connecting generator of non-sinusoidal waveforms and measurement modules to the AC table.

Table 1. Parameters of load currents of modelled network.

Harmonics number	Load current at node 2		Load current at node 3	
	RMS value, A	Phase, deg.	RMS value, A	Phase, deg.
1	0.4044	-26.08	0.3036	-26.67
3	0.3081	52.55	0.0662	-8.93
5	0.1529	109.71	0.1238	27.78
7	0.0511	193.85	0.0545	47.88
9	0.011	259.21	0.0069	100.01

The voltage measurements were carried out using the three-channel 24-bit NI-9225 module (error of ± 0.05%). The current measurements were carried out using two fourchannel 24-bit NI-9227 modules (error of ± 0.1%). The measurement modules were installed in the *cDAQ NI*-9172 chassis [17], which was connected via USB to a PC. The measured parameters were recorded in a file using a virtual tool developed in the *LabVIEW* environment. Measurements were performed with a sampling frequency of 25 kHz over a 60 ms.

The measured voltage and currents instantaneous values in nodes 1, 2 and 3, and also currents in branches 1-3, 1-2 and 2-3 have been accepted as conditional-reference. To examine the efficiency of the suggested algorithm of a state estimation, the distortions in the form of white noise have been simulated and entered into the conditional-standard values. Distortions were modeled under the normal law of distribution with a standard deviation of σ =1,5V for voltage and σ =0,05 A for currents. Afterwards, the state estimation of the network's mode was performed on the basis of the distorted values of currents and voltage via the suggested method.

Tables 2 and 3 show the results of calculations and comparisons of RMS values of currents and voltages for the conditional standard, the distorted, and the estimated values.

Table 2. Currents and voltages RMS values.

Parameter	Conditional standard	Distorted values	Estimated values
u1, V	19,9975	20,0168	20,0167
u2, V	19,6553	19,7132	19,7129
u3, V	19,7071	19,7459	19,7458
i1, A	0,5753	0,5788	0,5784
i2, A	0,3786	0,3835	0,3812
i3, A	0,2399	0,2466	0,2440
i12, A	0,2881	0,2914	0,2899
i13, A	0,2913	0,2952	0,2932
i23, A	0,1085	0,1182	0,1109

Table 3. Voltage and currents relative inaccuracy comparison, before and after the state estimation.

Para-meter	Relative inaccuracy before state estimation, %	Relative inaccuracy after state estimation, %	Inaccuracy change after state estimation, %
u1	0,097	0,096	-0,001
u2	0,295	0,293	-0,002
u3	0,197	0,196	-0,001
i1	0,608	0,539	-0,070
i2	1,294	0,687	-0,608
i3	2,793	1,709	-1,084
i12	1,145	0,625	-0,521
i13	1,339	0,652	-0,687
i23	8,940	2,212	-6,728

The tables show that the application of the proposed linear state estimation algorithm allows to reduce inaccuracy of measurements of voltage for 0,001-0,02%, and currents for 0,07-6,73%. The greatest effect in decrease in inaccuracy reduction is attained for the branch 2-3 current.

Taking into account that non-sinusoidal mode was modeled, comparison of correlation factors of voltage and current signals against the standard before and after application of the state estimation has been produced (Table 4).

Table 4. Comparison of correlation factors of voltage and current signals against the standard prior to and after the state estimation.

Parameter	Correlation factor prior to state estimation	Correlation factor after state estimation	Correlation factor change
u1	0,9972	0,9972	0
u2	0,9971	0,9971	0
u3	0,9970	0,9971	0,0001
i1	0,9964	0,9979	0,0015
i2	0,9915	0,9953	0,0038
i3	0,9795	0,9868	0,0073
i12	0,9851	0,9970	0,0119
i13	0,9858	0,9964	0,0106
i23	0,9070	0,9749	0,0679

The greatest increase in the correlation factor occurs in the current of a branch 2-3. The comparison of oscillograms of this branch's current at its greatest crest value is resulted in Fig 6.

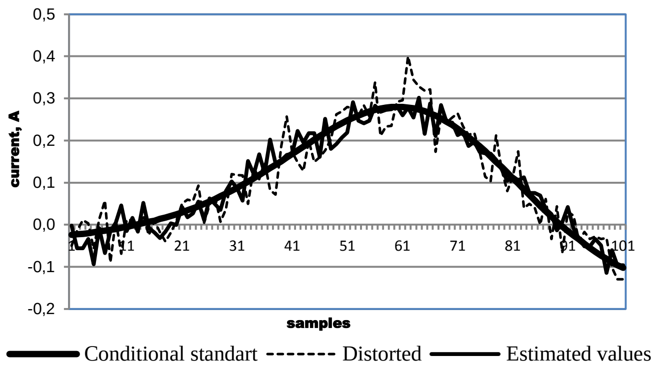

Fig. 6. Branch 2-3 current's oscillograms.

5. CONCLUSIONS

1. The proposed method of estimating the instantaneous values of currents and voltages allows us to model the non-sinusoidal and non-stationary power flow of electric networks based on highly-accurate synchronous measurements carried out as part of efforts to introduce the IEC-61850-9.2-sv protocol and Smart Grid concept. This issue is more pressing for distribution networks, where there is a problem of higher harmonics.

2. The software implementation of the proposed algorithm of estimating the state based on instantaneous values, was completed.

3. The results produced based on simple closed network of experimental studies have shown that the inaccuracy of measurements of an instantaneous value of currents and voltage can be considerably reduced through the use of the proposed linear state estimation algorithm.

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