

## THE ACTIVE DC BRIDGE

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**Abstract** – The paper presents an original circuit of a resistance bridge balanced by means of an active element whose resistance is incorporated into one of the bridge's branches. In this solution the bridge balance condition includes the quotient of products of resistances in the bridge's appropriate arms being equal to the quotient of corresponding time-intervals. This facilitates resistance measurements employing a digital method without the use of an a/d converter.

**Keywords:** balanced bridge, precise resistance measurements, digital resistance measurements

### INTRODUCTION

The goal of this article is to present the features of an original balanced resistance bridge circuit and options of its implementation in novel measuring instruments, most notably portable and miniaturized ones.

The original element of this work is the method for balancing a resistance bridge by means of an active element, whose resistance is incorporated into one of the bridge's branches and the system for implementing this method termed the active balanced bridge (AMZ).

On the basis of the AMZ circuit can be built analogue or analog-digital transducers measuring resistance or relations between resistances.

These can be transducers of e.g.: a change (increment) in resistance, a relative change in resistance or the resistance quotient (e.g. the division ratio in a resistive voltage divider).

AMZs may also be employed for measurements of non-electrical quantities, such as temperature, pressure, force, mechanical stress, humidity, flow, as well as energy (e.g. thermal).

The paper presents a selected circuit intended for high-accuracy digital measurements of constant and slow-varying resistances.

AMZ circuits exhibit well-known properties of a balanced bridge and allow obtaining the measurement results in the digital form without the necessity for an A/D converter. They also have several new, advantageous metrological properties.

The work is organized as follows. Section 1 presents the active balanced bridge (AMZ) against the background of similar solutions known from literature, provides motives for developing the described solution and shows what merits it should have, and what disadvantages should be eliminated. Section 2 discusses the AMZ structure and its principle of

operation. Section 3 describes the basic, switchable AMZ structure and its use for resistance measurements. Section 4 lists the most essential general features of the switchable AMZ structure. Section 5 describes static properties of the selected AMZ structure. Section 6 provides the results of calibration of the measuring instrument based on the analysed circuit. Section 7 provides the analysis of dynamic properties of the selected AMZ structure. The last part of the paper is the conclusion section.

### 1. SHORTCOMINGS OF THE BALANCED WHEATSTONE BRIDGE

The commonly known four-arm resistance bridge (Fig. 1), invented by S. H. Christie in 1833, is balanced when the quotient of products of resistances in opposite bridge arms is constant and equal to 1 (1).

$$\frac{R_2 \cdot R_4}{R_1 \cdot R_3} = 1 \quad (1)$$

Such circuit does not provide a measuring-accessible signal (a current or voltage signal) that could be used for further processing, e.g. in an analog-to-digital converter.

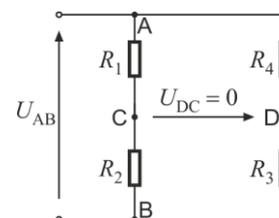


Fig. 1. The balanced Wheatstone bridge diagram with symbols used in this paper.

The measured value is found by reading the settings of the resistor being tuned to balance the bridge. The result can be obtained in digital form using digitally controlled resistors.

The AMZ circuits eliminate the major drawback of the balanced Wheatstone bridge that consists in the lack of an output signal. They also eliminate disadvantages of systems utilizing analog-to-digital converters. By substituting two conversion stages (R/U and a/d) with a single operation they make the measurement independent from imperfections of components that are not required in AMZ circuit (resistors, amplifiers).

AMZs have numerous advantages that are particularly evident in the case of measuring small resistance changes or

very small relative changes in resistances. They are characterized by a high sensitivity without the necessity to use a precision instrumentation amplifier. The direct measured quantity is the quotient of time-intervals which can be easily determined with high accuracy and is easily digitized.

## 2. THE ACTIVE BALANCED BRIDGE AMZ

The analysis of AMZ operation is based on the circuit diagram shown in Fig. 2.

One of the Wheatstone bridge arms (Fig. 2a) incorporates an active element controlled by the measurement diagonal (CD) signal in such a way, that the resistance this arm ( $R_{4A}$ ), further referred to as the active resistance, always takes the value satisfying the condition for the bridge balance (2).

$$R_{4A} = \frac{R_1 \cdot R_3}{R_2} \quad (2)$$

Functions of the active element can be performed by a differential amplifier (Fig. 2b) in the voltage follower configuration. The bridge is always balanced if the amplifier output current is not exceeded.

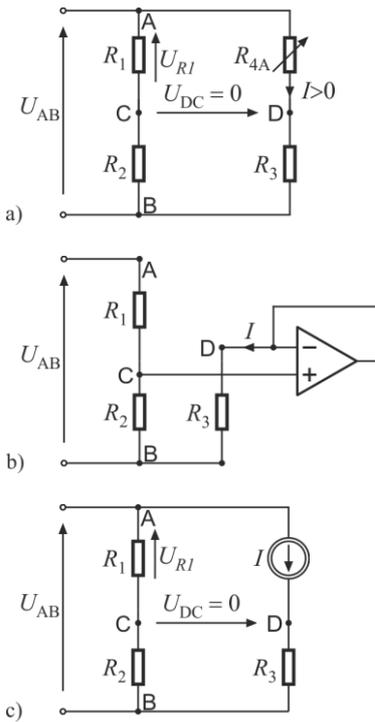


Fig. 2. The active balanced bridge: a) general diagram, b) the configuration employing an operational amplifier, c) equivalent circuit diagram

The value of the current  $I$  in the active resistance  $R_{4A}$  is determined by equation (3).

$$I = \frac{U_{R1}}{R_{4A}} = \frac{R_2}{R_3} \cdot \frac{U_{AB}}{R_1 + R_2} \quad (3)$$

Figure 3 shows the configuration where the active resistance  $R_{3A}$  is connected in parallel with resistor  $R_3$  (Fig. 3a) which satisfies the condition (4).

$$\frac{R_4}{R_3} < \frac{R_1}{R_2} \quad (4)$$

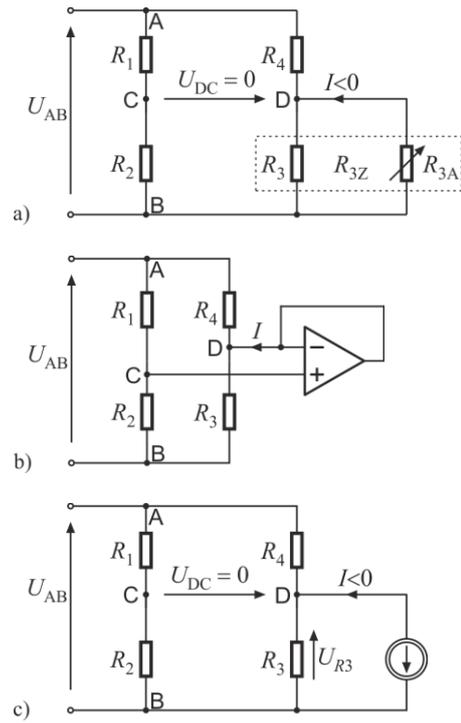


Fig. 3. The AMZ structure with resistor  $R_4$ : a) arms resistances for  $R_4/R_3 < R_1/R_2$ , b) the configuration employing an operational amplifier, c) equivalent circuit diagram

Thus, the equivalent resistance of arm DB (5):

$$R_{3Z} = \frac{R_3 \cdot R_{3A}}{R_3 + R_{3A}} \quad (5)$$

satisfies the condition of the bridge balance condition. The active resistance  $R_{3A}$  takes the value (6):

$$R_{3A} = \frac{R_2 R_3 R_4}{R_1 R_3 - R_2 R_4} \quad (6)$$

The current in the active resistance  $R_{3A}$  is  $I < 0$  and its value is determined by equation (7).

$$I = -\frac{U_{R3}}{R_{3A}} = \left( \frac{R_2}{R_3} - \frac{R_1}{R_4} \right) \cdot \frac{U_{AB}}{R_1 + R_2} \quad (7)$$

If the sense of the inequality (4) is reversed, the bridge is balanced by connecting the active resistance  $R_{4A}$  in parallel with the resistance  $R_4$ . This is achieved in the same configuration employing an operational amplifier as that shown in Fig. 3b. In the specific situation, where resistances  $R_1, R_2, R_3, R_4$  satisfy the balance condition (1), the active resistances are not connected to any of bridge's arms.

The differential amplifier in the voltage follower configuration (Fig. 3b) does not influence the circuit operation. The active resistances take infinite values ( $R_{3A} = R_{4A} = \infty$ ), and the current is  $I = 0$ .

The resistance bridge, balanced automatically by means of tuning the resistance of an active element incorporated into one of the bridge's branches, has been termed the active balanced bridge.

The characteristic feature of the AMZ bridge is the presence of a measuring-accessible current which can be measured without loading the bridge. Moreover, the measuring-accessible current flows through the active resistance, which automatically takes a value such, that the AMZ bridge is balanced. The active resistance value can be determined by measuring this current.

For instance, in the circuit shown in Fig. 2a, the active resistance  $R_{4A}$  value is interrelated with the current  $I$  by relation (8).

$$R_{4A} = \frac{R_1}{R_1 + R_2} \cdot \frac{U_{AB}}{I} \quad (8)$$

Fig. 4 shows the place of connection of a measuring system with the input impedance  $Z_0$ . When the current  $I$  is measured by means of an ammeter (then  $Z_0 = R_0$ ), its input resistance  $R_0$  should be selected so the voltage drop across that resistance will not exceed the limit value for the operational amplifier linear operation region. Where this condition is satisfied the system measuring the current  $I$  does not load the bridge: it neither affects the current value nor distorts the bridge balance.

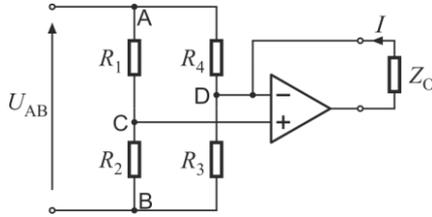


Fig. 4. The AMZ structure with indicated place of the current  $I$  measurement;  $Z_0$  – the input impedance of measuring system

Where certain conditions are satisfied the current  $I$  can also be measured by means of a circuit configuration with capacitive input. This option is described in the next section.

### 3. THE ACTIVE BALANCED BRIDGE WITH A SWITCHABLE STRUCTURE

An alternative solution is the active balanced bridge with a switchable structure. The basic AMZ circuit with a single switch is depicted in Fig. 5.

The structure of this circuit enables two different AMZ configurations, which differ in the polarity of the current  $I$  in the connected active resistance. When the resistance  $R_4$  is disconnected (Fig. 2) the current  $I = I_0 > 0$ . If the resistance  $R_4$  is connected (Fig. 3) the current  $I = I_z < 0$ .

In the configuration with the resistance  $R_4$  disconnected (Fig. 2) the charge carried by the current  $I = I_0 > 0$  during time interval  $t_0$  is  $Q_0$  (9).

$$Q_0 = I_0 \cdot t_0 \quad (9)$$

In the configuration with the resistance  $R_4$  connected (Fig. 3) the charge carried by the current  $I = I_z < 0$  during time interval  $t_z$  is  $Q_z$  (10).

$$Q_z = I_z \cdot t_z \quad (10)$$

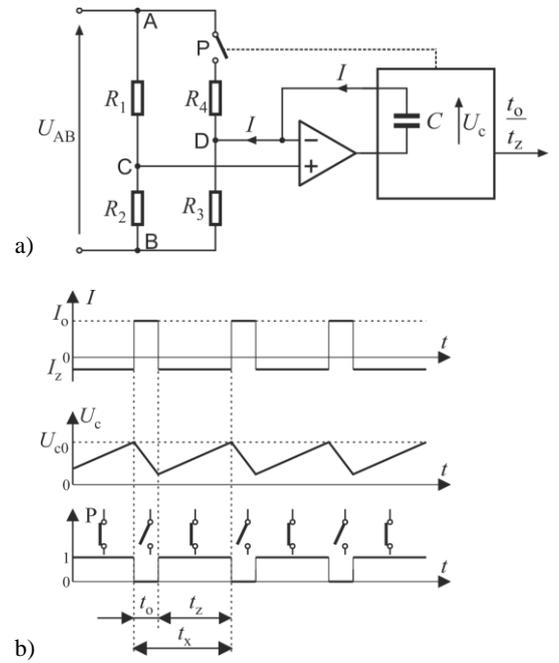


Fig. 5. The AMZ bridge with a single switch, which enables the connection or disconnection of the resistance  $R_4$ : a) equivalent circuit diagram, b) waveforms

The switchable AMZ circuit operation consists in cyclic switching between these two AMZ configurations in such a manner that in each of the two phases of a cycle through the active resistance flows a charge of the same value, but in the opposite direction. The sum of charges in both phases equals zero (11)

$$Q_0 + Q_z = 0 \quad (11)$$

This way of control allows to utilize the capacitor for determining equal amounts of charge. The capacitor  $C$  is connected in series with the active resistance (Fig. 5a) through which the current  $I$  is flowing. When the switch  $P$  is open, the current  $I = I_0 > 0$  is charging the capacitor over the time period  $t_0$  (Fig. 5b) with the charge  $Q_0$ . When the switch  $P$  is closed, the current  $I = I_z < 0$  discharges the capacitor  $C$  during the time period  $t_z$  with the charge  $Q_z$ .

Determining equal amounts of charge is performed by detecting identical values of the capacitor voltage  $U_c$ . The comparator which detects equal amounts of charge is connected at the input of the switch control circuit (KO+UC, Fig. 5).

From the balance of charges (11) it follows that (12):

$$\frac{t_0}{t_z} = -\frac{I_z}{I_0} = \left( \frac{R_1 \cdot R_3}{R_2 \cdot R_4} - 1 \right) \cdot \frac{U_{ABz}}{U_{AB0}} \quad (12)$$

If the supply voltage value is equal during both measurement phases ( $U_{ABz} = U_{AB0}$ ), equation (12) takes the form (13)

$$\frac{t_0}{t_z} = \frac{R_1 \cdot R_3}{R_2 \cdot R_4} - 1, \quad (13)$$

or (14)

$$\frac{R_2 \cdot R_4}{R_1 \cdot R_3} = \frac{t_z}{t_x}, \quad (14)$$

where:  $t_x = t_o + t_z$  is the measuring cycle duration.

Relation (14) defines the balance condition for the switchable AMZ with a single switch connected in series with resistance  $R_4$  in the arm AD (Fig. 5a).

The measurement of a resistance by means of the switchable AMZ consists in determining the quotient of time-intervals  $t_z/t_x$ . This can be achieved by measuring the duration of measuring cycle  $t_x$  for the set time  $t_z$ .

#### 4. GENERAL PROPERTIES OF AMZ WITH THE SWITCHABLE STRUCTURE

Metrological properties of AMZ were analysed comparing the switchable AMZ balance equation (14) with Wheatstone bridge balance equation (1).

Left-hand sides of both equations are identical. In both equations the quotient of products of the opposite bridge arms resistances (the left-hand side of equation) is the same, and is:

- constant
- independent on the bridge supply voltage.

These two properties mean the AMZ circuit has the commonly known advantages of DC balanced bridges.

Equations (1) and (14) differ, however, in their right-side-hand terms, namely:

- in the balanced Wheatstone bridge the left-hand side must be equal to 1,
- in the switchable AMZ the left-hand side is equal to the quotient of time intervals.

That means the Wheatstone bridge can only be balanced by tuning the arms' resistances, whereas the switchable AMZ can be balanced when zero voltage is attained across the diagonal (CD) for different values of arms' resistances. The balancing is performed automatically by tuning the quotient of time-intervals. That property gives rise to many original properties and possible applications of AMZ.

This bridge enables direct comparison of the resistances ratio with the ratio of time-intervals (14), without further processing.

The quotient of time-intervals  $t_z/t_x$  can be determined in several ways.

There is also possible to control the switchable AMZ circuits in such a way that charges of each of two phases are summed up during several consecutive cycles. Then, the sum of charges of the first phase equals the sum of charges of the second phase, for the same cycles. The balance of charge for several cycles equals zero (15)

$$\sum_i Q_{oi} + \sum_i Q_{zi} = 0. \quad (15)$$

Hence, instead of measuring a single cycle duration  $t_x$  the switching frequency  $f_x = \frac{1}{t_x}$  can be measured during a period comprising several cycles.

If the switch P control signal is appropriately synchronized (Fig. 6) with the measurement time T with a duration of several cycles thus, instead of measuring the frequency, the number of pulses can be counted over a

specified time. Then the switch P is not switched immediately upon change of the signal at the comparator K output but only at the instant of the synchronizing signal  $f_g$  occurrence. This causes that the switch P turn-on times ( $t_{zi}$ ) and turn-off times ( $t_{oj}$ ) are integral multiples of  $t_g$ . In the bridge controlled that way the balance of charge (15) can be written in the form of (16)

$$N_x \cdot I_z \cdot t_g + (k - N_x) \cdot I_o \cdot t_g = 0, \quad (16)$$

and the balance condition (14) takes the form (17):

$$\frac{R_2 \cdot R_4}{R_1 \cdot R_3} = \frac{N_x}{k}. \quad (17)$$

In this equation  $N_x$  is the number of the switch P switchings-on for the time-period  $t_g$  during a specified measurement time T (18).

$$T = k \cdot t_g, \quad (18)$$

where  $k$  is an exact number, programmed in the digital

system. The quotient of numbers  $\frac{N_x}{k}$  is equal to the quotient of: the sum of the switch P switch-on times during the measurement time T to the total measurement time T.

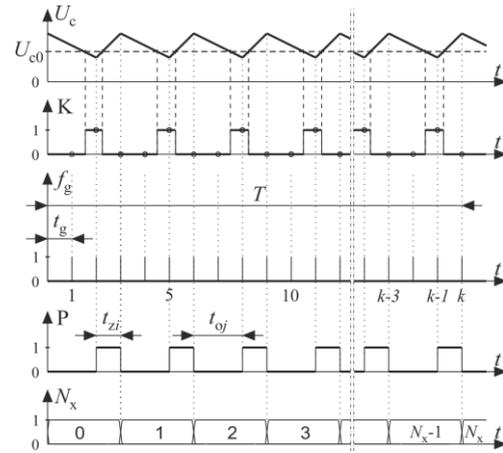


Fig. 6. Waveforms in the synchronously switched AMZ

The author's analysis indicates that switchable AMZ structures have the properties of balanced bridges. Their output signal is a function of resistances of the bridge arms (14). Moreover, the output signal can have a digital form (17) without the use of an additional analog-to-digital converter.

#### 5. STATIC PROPERTIES OF THE SELECTED AMZ STRUCTURE

In the bridge that satisfies the balance condition (17):

- the static characteristic is described by equation (19)

$$N_x = k \frac{R_2 \cdot R_4}{R_1 \cdot R_3}, \quad (19)$$

- the output quantity  $N_x$  (the number of pulses switching on the switch for a set time-period  $t_z$  during the measurement time  $T = k \cdot t_z$ ):

- is exclusively a function of the bridge arms resistances,
  - it does not depend on the circuit supply voltage value, or any reference voltages,
  - it does not depend on the generator frequency which determines the switch-on time and measurement time,
  - it is provided in the digital form.
- the number  $k$  (exact) is a parameter programmed in the digital system.

The bridge enables:

- a direct indication of a resistance, conductance or a ratio of resistances (or conductances),
- a direct comparison of two resistive voltage dividers having different ratios (20):

$$\frac{R_1}{R_2} = \frac{k}{N_x} \frac{R_4}{R_3}. \quad (20)$$

The compared ratios of resistances or conductances need not be nominally equal; they may differ e.g. by an order of magnitude. That enables us to compare directly two different ratios of resistances with resolution resulting from the choice of numbers  $N_x$  and  $k$ .

## 6. EXAMPLES OF AMZ APPLICATIONS

The analysed AMZ circuit with static characteristic described by equation (19), has been used, inter alia, in construction of a low resistances meter.

The meter employs the bridge (Fig. 5) supplied with voltage  $U_{AB} = 5V$ . The measured resistor is connected in the arm CB ( $R_x = R_2$ ). The measuring current flowing in the measured resistor is determined by selecting the resistance  $R_1 = 1 \text{ k}\Omega$ . Thus the equation (19) takes the form (21):

$$N_x = 10^{-3} k \frac{R_4}{R_1} R_x. \quad (21)$$

The resistances of resistors  $R_1$  and  $R_4$  and the number  $k$  are selected so that the equation (21) takes the form (22):

$$N_x = R_x. \quad (22)$$

The measuring instrument was subjected to tests in a calibration laboratory accredited by the Polish Centre for Accreditation (PCA). The calibration results are tabularized in Table 1.

Table 1. Results of the resistance meter calibration.  $W_n$  – nominal value,  $W_w \text{ } \acute{s}r$  – average indication of the calibrated instrument,  $W_r$  – conventional true value,  $U$  – uncertainty of measurement

No.	$W_n$ [mΩ]	$W_w \text{ } \acute{s}r$ [mΩ]	$W_r$ [mΩ]	$W_w \text{ } \acute{s}r - W_r$ [mΩ]	$U$ [mΩ]
1	100	100	100.0	0.0	0.6
2	200	229	229.3	-0.3	0.6
3	500	528	528.8	-0.8	0.7
4	1000	1029	1029.7	-0.7	0.8
5	1500	1528	1529.4	-1.4	0.9
6	1900	1930	1932.9	-2.9	0.9

From the results provided in Table 1 it can be concluded that the standard uncertainty of resistance measurement does not exceed the value of 0.9 mΩ within the measuring range 100 mΩ to 1900 mΩ

## 7. ASSESSMENT OF DYNAMIC PROPERTIES OF THE SELECTED AMZ STRUCTURE

The structure of an AMZ bridge with static characteristic described by equation (19) enables high-resolution resistance measurements with the resolution determined by selecting the number  $k$ . On the other hand, increasing the number  $k$  extends the measurement duration  $T$  (18) and reduces the readout frequency.

Analysis of dynamic properties was performed for the circuit with the measured resistor ( $R_x = R_2$ ) connected in the arm CB. This configuration has resolution  $k$ , and its measuring range is 0 to  $N_{x\max}$  (23)

$$N_{x\max} = k \frac{R_4}{R_1 \cdot R_3} \cdot R_{x\max} = R_{x\max}, \quad (23)$$

where  $R_{x\max}$  - the maximum value of  $R_x(t)$ .

The summation (15,16), performed in the active balanced bridge, averages the measurement result  $N_x$  over the time period  $T$ .

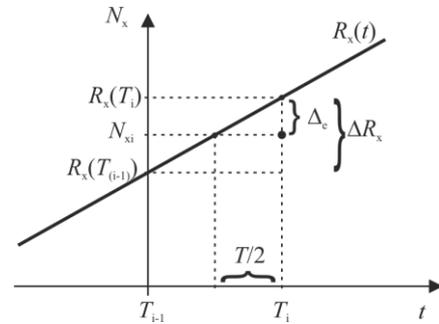


Fig. 7. Dynamic error  $\Delta_e$  of the switchable AMZ structure

For a linear change (Fig. 7) of the measured resistance  $R_x(t)$  during the time interval  $T$ , by  $\Delta R_x$  equal (24)

$$\Delta R_x = R_x(T_i) - R_x(T_{i-1}) = \frac{dR_x(t)}{dt} \cdot T, \quad (24)$$

at the instance of readout  $T_i$  the measured value  $N_{xi}$  equals (25)

$$N_{xi} = R_x(T_{i-1}) + \frac{\Delta R_x}{2}. \quad (25)$$

The dynamic error  $\Delta_e$  (26) assigned to the instant of readout  $T_i$  is 50% of the measured quantity change  $\Delta R_x$  during the averaging time interval  $T$ :

$$\Delta_e = R_x(T_i) - N_{xi} = \frac{\Delta R_x}{2} = \frac{T}{2} \cdot \frac{dR_x(t)}{dt}. \quad (26)$$

The magnitude of dynamic error defined that way is directly proportional to the rate of change of the measured resistance  $dR_x(t)/dt$  and to the averaging time  $T$ .

In order to maintain the dynamic error value  $\Delta_e$  below the quantization error the expression (27) shall be satisfied

$$\Delta_e < \frac{N_{x\max}}{k}. \quad (27)$$

Hence, taking into consideration (22), (23) and (26), we obtain the condition for limiting the measured signal rate of rise (28)

$$\frac{dN_x(t)}{dt} < \frac{2N_{x\max}}{k \cdot T}. \quad (28)$$

When the analysed active balanced bridge is applied to temperature measuring by means of a thermoresistor described by the first-order inertial model and time constant  $\tau$ , the allowable rate of change of the measured signal is determined by relation (29)

$$\frac{dN_x(t)}{dt} \leq \frac{N_{x\max}}{\tau}. \quad (29)$$

For the purposes of further analysis has been assumed the maximum rate of change of the thermoresistor resistance (29). Then, the relation (28) implies the dependence (30) which defines relationships between the AMZ parameters: the averaging period  $T$ , resolution  $k$  and the thermoresistor time constant  $\tau$ :

$$T < \frac{2\tau}{k}. \quad (30)$$

The reduction of the averaging time  $T$  is limited by the minimum synchronization time  $t_g$ , which must be longer than the switching time of switch P.

The maximum resolution  $k_{\max}$  for the minimum synchronization time  $t_{g\min}$  is determined from relation (30), taking into account (18)

$$k_{\max} < \sqrt{\frac{2\tau}{t_{g\min}}}. \quad (31)$$

The minimum synchronization time determines maximum readout frequency  $f_{\max}$  (32):

$$f_{\max} = \frac{1}{k \cdot t_{g\min}}. \quad (32)$$

Resistance temperature sensors have a relatively large inertia. Their time constant is normally not smaller than several seconds, seldom it is  $< 1$  s.

Assuming  $\tau = 1$  s and  $t_{g\min} = 20$  ns, the maximum sensitivity value calculated from (31) is  $k_{\max} = 10000$ , and maximum readout frequency is  $f_{\max} = 5$  kHz.

The presented analysis applies to the dynamic error defined in such a way that the measurement result is assigned to the instant of the averaging termination.

When the measurement result is assigned to another instant within the averaging time, e.g. the midpoint of the averaging interval (33), the analysis of dynamic error  $\Delta_m$  brings quite different outcomes.

$$\Delta_m = R_x \left( T_i - \frac{T}{2} \right) - N_{xi} \quad (33)$$

For a linear change (Fig. 7) of the measured resistance  $R_x(t)$  value during the time interval  $T$ , the dynamic error magnitude at the midpoint of the averaging interval is (34)

$$\Delta_m = 0. \quad (34)$$

The analysis shows that the dynamic error  $\Delta_e$  (26) can be corrected taking into account a time shift resulting from the instant to which the measurement result is assigned. This time shift is smaller than the averaging time  $T$ , and for linearly changing quantities it equals  $T/2$ . This correction is possible if the measurement result is not needed instantly, i.e. before time  $T$  elapses. In many applications (e.g. a visual readout) a time shift resulting from the indications readout frequency greater or equal to several Hz is imperceptible thus the correction is not necessary.

In the case of temperature measurements, the measured resistance value does not change linearly but according to the exponential function. For short averaging times the assumption about linearity may not introduce significant errors. Otherwise such errors should be evaluated or corrected, e.g. by computing the instances within averaging periods, to which the measurement results should be assigned in order to avoid dynamic error.

## CONCLUSION AND SUMMARY

The paper presents an original active balanced bridge configuration. It has been demonstrated that this bridge is also balanced if the values of resistances of resistors connected in the bridge arms do not satisfy the balance condition for Wheatstone bridge.

It has been shown that it is possible to measure the current flowing in the bridge arm in such a manner that the input impedance of a measuring system is neither distorting the current nor loading the bridge. That feature allows obtaining information, contained in the measuring-accessible current signal, about the value of the resistance being tuned in the process of balancing the bridge. It also enables construction of miniature measuring transducers utilizing a balanced bridge.

The paper describes a basic configuration of the switchable active balanced bridge employing a capacitor for measuring the current in the bridge arm, and introduces the balance equation for the basic switchable AMZ bridge. In the special case this equation reduces to the Wheatstone bridge balance equation. The analysis of this equation has shown many new and advantageous metrological properties which enable to utilize the balanced bridge in contemporary, particularly digital, measurement techniques.

The paper provides results of calibration of a low resistances digital meter developed on the basis of the analysed switchable AMZ structure.

The AMZ bridge can also co-operate with parametric sensors of non-electrical quantities. The range of applicability of that solution has been defined using an example of a temperature measurement by means of a thermistor. The allowable rate of change of a measured quantity has been determined. It has also been shown that the correction of dynamic error at the expense of an insignificant delay of the result readout is feasible. In such a case high-resolution AMZ circuits can be used in most applications of resistive temperature sensors.

## REFERENCES

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