

## ACCURACY PARAMETERS OF CIRCUITS WITH RESISTIVE SENSORS

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**Abstract** – The novelty of this work is the analysis of accuracy measures and its visualization especially for two parameter measurements. The parameters of the resistance-to-voltage converters, i.e. different types of DC bridge-circuits with arbitrary variable resistive sensors (of relative resistance increments) and their accuracy measures are presented. Transfer coefficients, limited error and also standard uncertainty as the functions of relative increments of resistances are considered in a broad range for different types of supply and for different configurations of sensors.

**Keywords:** converter, resistance bridge, error, uncertainty

### 1. INTRODUCTION

Resistive sensors are the most common analog parametric sensors. They have to be connected in a circuit which transforms single or known combination of sensor resistance increments to a voltage or current signal. Then this signal can be converted into digital form for further processing [1].

In case of one variable measurements (1D), for producing a signal and for conditioning it, the transducers with resistive sensors are utilized. At the input of a measurement system the unbalanced resistive bridges are very often employed. These passive circuits are preferred in dynamic measurements as they carry a wide frequency band. Also, if the resistance change is large, there is no voltage saturation at circuit output as it happens in analogue converters with operational amplifiers. Analogue converters are discussed in detail in [2]. If the increments of resistances are conjugated in two or four arms of a bridge-circuit then the linear characteristic can be obtained.

The measurements of two variables (2D) with the use of one or two resistive sensors can be also implemented. It is possible in circuits where two different parameters are measured on their terminals (outputs) either sequentially or simultaneously. Some of 2D circuits are proposed in [3], [4] and described in detail in [5]. There are: the two-output cascade bridge-circuit [3-4] and a single closed loop (4R) supplied by two current sources (2J) or by one switchable single source (2x1J). Practical experiment with the design of 2J circuit for strain and temperature measurements is presented and discussed in [9].

Instantaneous errors, limited errors (the worst case), random errors and set of uncertainties are used to describe an accuracy of any measuring devices and to determine the accuracy of measurement result. It is done also for set of instruments connected in measurement systems. It is described in the ISO GUM Guide [11] how the standard

uncertainty type B and expanded uncertainty should be calculated. Considering the accuracy analysis of input sensor transducers it is required to estimate the uncertainty (type B) basing on limited errors of elements of their circuits. In this paper the limited errors are given as functions of the relative increments of resistances. They are accepted as the accuracy measures of circuit transfer coefficients.

Accuracy of the measuring systems with different types of sensors is usually considered in the literature only for one variable measurements (1D) and for small increments of sensor resistances [1]. In this paper a general description is presented for one (1D) and two (2D) simultaneously measured quantities. It is valid for the relative changes in resistance of one, two or four variable elements connected as single closed loop (4R) and supplied in different ways [6, 8]. The circuits are supplied by voltage or current source in different ways: classically - attached to the diagonal of the 4R network, and unconventionally – where two current sources are connected to the opposite arms of the 4R network (2J). Measurement accuracy for all types of circuits will be presented separately for both initial value and relative increment of a transfer coefficient [2].

The history of four-resistance bridge is now more than 170 years old. It started from S. Christie idea and it was applied after 10 years by Wheatstone. It is even beyond belief, but up to now, except our earlier works [4], [5], [9], authors did not find in literature the proper description of errors and uncertainties for arbitrary large relative increments of resistances in the bridge-circuit. The same is with the theory of 2D sensor circuits and their accuracy.

### 2. TRANSFER COEFFICIENTS AND ACCURACY PARAMETERS

#### 2.1. Unbalanced bridge circuit

Four resistances (4R) connected in the closed loop can work as the bridge shown in Fig. 1. It is the two-port circuit (type X) with two pair of terminals A-B and C-D. If any of its internal resistances  $R_i$  is variable then the output voltage  $U'_{DC}$  may change sign for some set of them. The ideal supply is preferred to use: by current  $I_{AB} \rightarrow J = \text{const.}$ ,  $R_G \rightarrow \infty$  or by voltage  $U_{AB} = \text{const.}$ ,  $R_G = 0$  and also the unloaded output, i.e.:  $R_L \rightarrow \infty$ ,  $U'_{DC} \rightarrow U_{DC}^{\infty}$ . For one variable measurements (1D) it is enough to know the changes of one terminal parameter and the open-circuit output voltage  $U'_{DC}$  is mostly used. Main terminal parameters of this circuit (bridge transfer coefficients and input resistance) for both supply cases (voltage and current) are given in Table 1. The bridge transfer impedance  $r_{21}$  and bridge transfer voltage  $k_{21}$

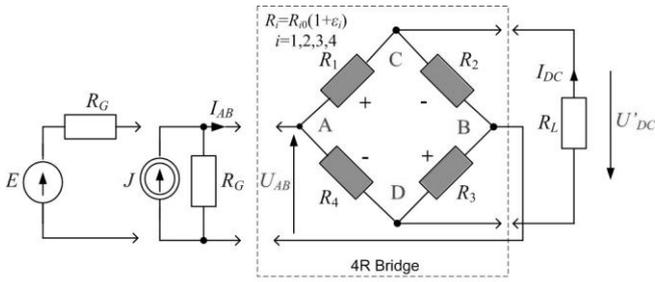


Fig 1. Unbalanced Wheatstone bridge supplied by current or voltage source.

can be simplified to products of their initial sensitivities  $t_0'$ ,  $t_0''$  in the balance and normalized unbalance functions  $f'(\varepsilon_i)$ ,  $f''(\varepsilon_i)$ . Their formulas can be expressed by initial values  $R_{i0}$  and increments of all resistances, i.e.  $R_i = R_{i0}(1 + \varepsilon_i)$  and  $R_{i0}$  relating to one of the first arm, i.e.:  $R_{20} = mR_{10}$ ,  $R_{40} = nR_{10}$  and  $R_{30} = (m/n)R_{10}$  or  $R_{30} = (n/m)R_{10}$ . Dependences of transfer coefficients  $r_{21}$ ,  $k_{21}$  on  $\varepsilon$  (in simplified three cases of 4R bridge if  $R_{i0} = R_{10}$  and relative increments  $\pm\varepsilon$  of sensor resistances) are given in Fig. 2a,b [10]. If transfer impedance  $r_{21} = 0$  or  $k_{21} = 0$ , the bridge circuit is in a state of balance. In the two-variable measurement (2D) the output voltage  $U'_{DC}$  and the change of input  $R_{AB}^\infty$  or output  $R_{CD}^\infty$  resistance or a current of short-circuit output  $I_{DC}(R_L = 0)$  are measured.

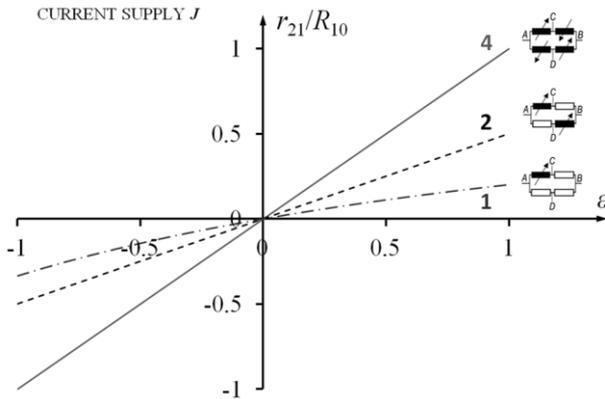


Fig 2a. Dependences of open-circuit transfer impedance  $r_{21}$  (divided by nominal resistance  $R_{10}$ ) on the relative resistance increment  $\varepsilon$  ( $m=1, n=1, |\varepsilon_i|=\varepsilon$ ) for three cases of  $4R_{10}$  bridge.

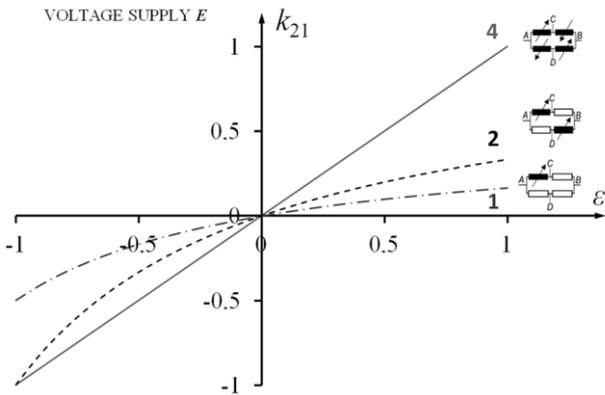


Fig 2b. Dependences of open-circuit transfer voltage  $k_{21}$  on the relative resistance increment  $\varepsilon$  ( $m=1, n=1, |\varepsilon_i|=\varepsilon$ ) for three cases of  $4R_{10}$  bridge.

Tab.1. Open-circuit transfer coefficients  $r_{21}$ ,  $k_{21}$  and input resistance (open-circuited terminals)  $R_{AB}^\infty$  of the 4R network supplied by current or voltage source in classic way.

Current supply $J$ , ( $R_G \rightarrow \infty$ )	
$U'_{DC} \rightarrow U_{DC}^\infty = I_{AB} r_{21}$	
$r_{21} \equiv \frac{U_{DC}^\infty}{I_{AB}} = \frac{R_1 R_3 - R_2 R_4}{\sum R_i} \equiv t_0 f(\varepsilon_i)$	
$t_0 \equiv \frac{m n R_{10}}{(1+m)(1+n)}$	$f(\varepsilon_i) = \frac{\Delta L(\varepsilon_i)}{1 + \varepsilon_{\Sigma R}} \quad \varepsilon_{\Sigma R} = \frac{\varepsilon_1 + m \varepsilon_2 + n(\varepsilon_4 + m \varepsilon_3)}{(1+m)(1+n)} \quad m, n > 0$
Voltage supply $E$ , ( $R_G = 0$ )	
$U'_{DC} \rightarrow U_{DC}^\infty = U_{AB} k_{21}$	
$k_{21} \equiv \frac{U_{DC}^\infty}{U_{AB}} = \frac{R_1 R_3 - R_2 R_4}{(R_1 + R_2)(R_3 + R_4)} \equiv k_0 f_E(\varepsilon_i)$	
$k_0 \equiv \frac{m}{(1+m)^2}$	$f_E(\varepsilon_i) \equiv \frac{\Delta L(\varepsilon_i)}{(1 + \varepsilon_{12})(1 + \varepsilon_{34})} \quad \varepsilon_{12} \equiv \frac{\varepsilon_1 + m \varepsilon_2}{1+m} \quad \varepsilon_{43} \equiv \frac{\varepsilon_4 + m \varepsilon_3}{1+m} \quad m > 0$
Balance condition $R_{10} R_{30} = R_{20} R_{40}$	
where: $R_i = R_{i0}(1 + \varepsilon_i)$ , $R_{20} = mR_{10}$ , $R_{40} = nR_{10}$ , $R_{30} = mnR_{10}$	
$\varepsilon_i = [\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4]^T \quad \Delta L(\varepsilon_i) = \varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4 + \varepsilon_1 \varepsilon_3 - \varepsilon_2 \varepsilon_4$	
Input resistance (open-circuited terminals) $R_{AB}^\infty$	
Its initial value $R_{AB0}$ and relative increment $\varepsilon_{AB}$ :	
$R_{AB}^\infty = \frac{(R_1 + R_2)(R_4 + R_3)}{\sum R_i} = R_{AB0}(1 + \varepsilon_{AB})$	
$R_{AB} = R_{10} \frac{(1+m)n}{1+n} \quad \varepsilon_{AB} = \frac{1}{1 + \varepsilon_{\Sigma R}(\varepsilon_i)} \left( \frac{n \varepsilon_{12} + \varepsilon_{43}}{1+n} + \varepsilon_{12} \varepsilon_{43} \right)$	

## 2.2. Unconventional double current circuit

Measurements 1D and 2D can be also realized by the unconventionally supplied bridge as it is presented in Fig. 3.

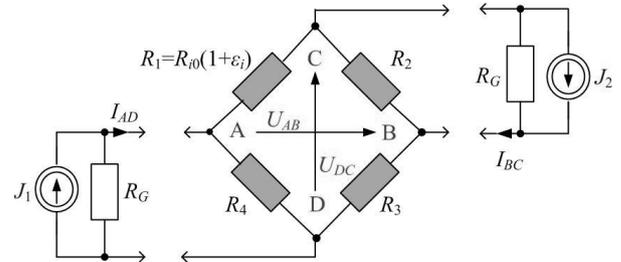


Fig 3. Idea of two output resistance converter named as double current bridge (2J) - resistor combination (4R) is unconventionally supplied by two current sources.

It is the concept of 4R network given by one of authors [3], [4]. The circuit is unconventionally powered by two current sources  $J_1$  and  $J_2$  connected to opposite arms of the single closed loop (4R). For this type of circuit symbol 2J was proposed [3], [4]. The circuit has two voltage outputs  $U_{AB}$ ,  $U_{CD}$ . In general case their balance conditions, presented in Tab. 2, are different for each bridge diagonal. Only if all the initial resistances  $R_{i0}$  are equal, conditions are the same as for the classically supplied bridge and for two diagonals of the 2J circuit if it is powered by two equal current sources  $J_1 = J_2 = J$ . Two transfer impedances ( $r_{AB}$ ,  $r_{CD}$ ) depend on resistance increments of the 4R network in different ways.

Tab. 2. Open-circuit transfer impedances  $r_{AB}$ ,  $r_{CD}$  of the unconventionally supplied 4R circuit (if  $J_1=J_2=J$ ,  $R_G \rightarrow \infty$ ,  $R_L \rightarrow \infty$ ).

Unconventional double current supply $2J$ , ( $R_G \rightarrow \infty$ )		
Balance conditions		
a) not concurrently		
$r_{AB0} = 0$	$r_{CD0} = 0$	
$R_{10} \cdot R_{40} = R_{20} \cdot R_{30}'$	or $R_{10} \cdot R_{20} = R_{30}'' \cdot R_{40}$	
for $R_{20}=mR_{10}$ , $R_{40}=nR_{10}$		
$R_{30}'=(m/n)R_{10}$	$R_{30}''=(n/m)R_{10}$	
b) concurrently: for $n=1/m$		
$R_{20}=m R_{10}$ , $R_{40}=(1/m)R_{10}$ , $R_{30}=R_{10}$		
$r_{AB} \equiv \frac{U_{AB}}{J} = \frac{R_1 R_4 - R_2 R_3}{\sum R_i} \equiv t_0' f'(\varepsilon_i)$		
$t_0' \equiv \frac{m n R_{10}}{(m+n)(1+m)}$	$f'(\varepsilon_i) = \frac{\Delta L'(\varepsilon_i)}{1 + \varepsilon_{\Sigma R}}$	$\varepsilon'_{\Sigma R} = \frac{m(m\varepsilon_2 + \varepsilon_1) + n(m\varepsilon_4 + \varepsilon_3)}{(m+n)(1+m)}$
$\Delta L'(\varepsilon_i) = \varepsilon_1 - \varepsilon_2 - \varepsilon_3 + \varepsilon_4 + \varepsilon_1 \varepsilon_4 - \varepsilon_2 \varepsilon_3$ , $m, n > 0$		
$r_{CD} \equiv \frac{U_{CD}}{J} = \frac{R_1 R_2 - R_3 R_4}{\sum R_i} \equiv t_0'' f''(\varepsilon_i)$		
$t_0'' \equiv \frac{m n R_{10}}{(m+n)(1+n)}$	$f''(\varepsilon_i) = \frac{\Delta L''(\varepsilon_i)}{1 + \varepsilon'_{\Sigma R}}$	$\varepsilon'_{\Sigma R} = \frac{m(n\varepsilon_2 + \varepsilon_3) + n(n\varepsilon_4 + \varepsilon_1)}{(m+n)(1+n)}$
$\Delta L''(\varepsilon_i) = \varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4 + \varepsilon_1 \varepsilon_2 - \varepsilon_3 \varepsilon_4$ , $m, n > 0$		

The 2J circuits were verified experimentally [8], [9].

### 2.3. Cascade bridge circuit

Another circuit for 2D measurements is given in Fig. 4. It contains two four-arm resistance bridges (4R) connected in cascade [4], [6], [7]. Input resistance of bridge 1 is connected as one arm of bridge 2. This circuit is supplied by single DC current source  $J$ . It has two voltage outputs  $U_{DC}$ ,  $U_{AB}'$ . It is balanced for initial values  $R_{i0}$  of all resistances of the tested bridge 1. If the bridge is unbalanced then the first equation in Tab. 3 is obtained from output voltage  $U_{DC}$  formula. It depends on the increments of arm resistances. The second one results from output voltage  $U_{AB}'$  of the bridge 2, which depends on increment  $\varepsilon_{AB}$  of input terminal resistance  $R_{AB}^\infty$  of the bridge 1.

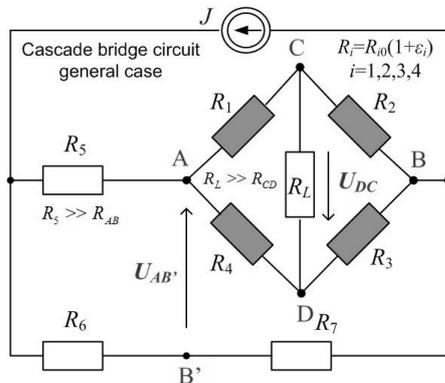


Fig. 4. Idea of two output resistance converter - cascade bridge circuit (double-bridged structure).

Tab. 3. Open-circuit transfer impedances  $r'_{AB}$ ,  $r'_{CD}$  and input resistance  $R_{AB}^\infty$  of the cascade bridge circuit.

Cascade bridge circuit ( $R_L \gg R_{CD}$ )
$r'_{CD} \equiv \frac{U_{DC}}{J} = \frac{R_1 R_3 - R_2 R_4}{\sum R_i} \equiv t_0''' f'''(\varepsilon_i)$
$r'_{AB} \equiv \frac{U_{AB}'}{J} = \frac{R_5 R_7 - R_{AB}^\infty R_6}{R_5 + R_6 + R_7 + R_{AB}^\infty} \equiv t_0'''' f''''(\varepsilon_i)$
$R_{AB}^\infty = \frac{(R_1 + R_2)(R_4 + R_3)}{\sum R_i} \equiv R_{AB0}(1 + \varepsilon_{AB}(\varepsilon_i))$

### 3. ACCURACY PARAMETERS OF UNBALANCED BRIDGE WITH EQUAL INITIAL RESISTANCES

The relative instantaneous values of errors  $\delta_{Ri}$  of sensor resistances  $R_i = R_{i0}(1 + \varepsilon_i)$  (where  $i=1,2,3,4$ ) may be presented as the initial error  $\delta_{i0}$  (for nominal value of  $R_{i0}$ ) and the relative error  $\delta_{\varepsilon i}$  of resistance increment  $\varepsilon_i$

$$\delta_{Ri} \equiv \frac{\Delta_i}{R_i} = \delta_{i0} + \frac{\Delta_{\varepsilon i}}{1 + \varepsilon_i} = \delta_{i0} + \frac{\varepsilon_i}{1 + \varepsilon_i} \delta_{\varepsilon i} \quad (1)$$

Bridge accuracy can be represented by the instantaneous (actual) values of measurement errors. They can be created for two transfer coefficients -  $r_{21}$  and  $k_{21}$ . They result from the total differential of analytical equations and they are known as the error propagation functions. After ordering all components of  $\delta_{Ri}$ , the absolute error of transfer impedance  $r_{21}$  is [5], [10]

$$\Delta_{r_{21}} = R_1 \frac{R_3 - r_{21}}{\sum R_i} \delta_{R1} - R_2 \frac{R_4 + r_{21}}{\sum R_i} \delta_{R2} + R_3 \frac{R_1 - r_{21}}{\sum R_i} \delta_{R3} - R_4 \frac{R_2 + r_{21}}{\sum R_i} \delta_{R4} \equiv \sum_{i=1}^4 w_{Ri} \delta_{Ri} \quad (2)$$

Equation (2) can be transformed to the following (generalized) form

$$\Delta_{r_{21}} = \sum_{i=1}^4 w_{Ri} (\delta_{i0} + \frac{\varepsilon_i}{1 + \varepsilon_i} \delta_{\varepsilon i}), \quad (3)$$

where a weight coefficient equals

$$w_{Ri} = \frac{t_0}{1 + \varepsilon_{\Sigma R}} [(-1)^{i+1} (1 + \varepsilon_j) - \frac{R_{i0}}{R_{j0}} \Delta L(\varepsilon_i)] (1 + \varepsilon_i), \quad (4)$$

where: symbols:  $i=1, 2, 3, 4$  and  $j=3, 4, 1, 2$  (in order), multiplier  $(-1)^{i+1} = +1$  (positive) if  $i$  is 1, 3 or  $-1$  (negative) if  $i$  is 2, 4.

The transfer impedance  $r_{21}$  of 4R network (Tab. 1) has the absolute instantaneous error

$$\Delta_{r_{21}} = \Delta_{r_{210}} + \Delta_{r_{21r}} = t_0 \delta_{210} + f(\varepsilon_i) \delta_{r_{21r}}(\varepsilon_i), \quad (5)$$

where:  $\delta_{210} \equiv \frac{\Delta_{r_{210}}}{t_0} = \delta_{10} + \delta_{20} + \delta_{30} + \delta_{40}$  - relative zero error,

$r_{210}$  - initial transfer impedance,  $\delta_{r_{21r}}(\varepsilon_i)$  - the relative error of difference  $r_{21} - r_{210}$ .

Error  $\delta_{r_{21r}}(\varepsilon_i)$  has the following form

$$\delta_{r_{21r}}(\varepsilon_i) \equiv \frac{\Delta_{r_{21}} - \Delta_{r_{210}}}{r_{21}} = \frac{\delta_{r_{21}} - \delta_{r_{210}}}{f(\varepsilon_i)} = \sum_{i=1}^4 w'_{r_{i0}} \delta_{i0} + \sum_{i=1}^4 w'_{r_{ei}} \delta_{ei}, \quad (6)$$

where: 
$$w'_{r_{i0}} = (-1)^{i-1} \frac{\varepsilon_i + \varepsilon_j + \varepsilon_i \varepsilon_j - \varepsilon_{\Sigma R}}{\Delta L(\varepsilon_i)} - t_0 \frac{1 + \varepsilon_i}{R_{j0}(1 + \varepsilon_{\Sigma R})}, \quad (6a)$$

$$w'_{r_{ei}} = \left[ \frac{(-1)^{i-1} (1 + \varepsilon_j)}{\Delta L(\varepsilon_i)} - \frac{t_0}{R_{j0}(1 + \varepsilon_{\Sigma R})} \right] \varepsilon_i. \quad (6b)$$

Weight coefficients (6a,b) are finite for any value of  $r_{21}$ , including  $r_{21} = 0$ , because if all  $\varepsilon_i \rightarrow 0$  also  $\Delta L \rightarrow 0$ . Error  $\delta_{r_{21r}}$  represents adequately as the error  $\delta_{ei}$  (of increment  $\varepsilon_i$ ) in formula (1).

The absolute limited error of transfer impedance  $r_{21}$  is defined by inequality

$$|\Delta_{r_{21}}| \leq |\Delta_{r_{210}}| + |\Delta_{r_{21r}}| \leq |t_0| |\delta_{210}| + |f(\varepsilon_i)| |\delta_{r_{21r}}(\varepsilon_i)|, \quad (7)$$

After dividing both sides of (7) by maximum value of transfer impedance  $|r_{21\max}|$  one receives

$$\frac{|\Delta_{r_{21}}|}{|r_{21\max}|} \leq \frac{|\delta_{210}|}{|f_{\max}|} + \frac{|f(\varepsilon_i)|}{|f_{\max}|} |\delta_{r_{21r}}(\varepsilon_i)|, \quad (8)$$

where:  $f_{\max}$  - is a maximum value of  $f(\varepsilon_i)$ .

Random (standard) measure  $\bar{\delta}_{r_{21r}}$  is calculated from equation

$$\bar{\delta}_{r_{21r}} = \sqrt{\sum_{i=1}^4 w'_{r_{i0}}{}^2 \delta_{i0}^2 + \sum_{i=1}^4 w'_{r_{ei}}{}^2 \delta_{ei}^2}. \quad (9)$$

Limited errors  $|\delta_{r_{21r}}|$  and random (standard) measures (of random errors or type B uncertainties)  $\bar{\delta}_{r_{21r}}$  in two particular cases, used with Pt100 sensors, '1' – only  $R_1$  variable or '2' – variable resistances  $R_1$  and  $R_3$  and their relative increments equal to  $\varepsilon$ , are presented in Tab. 4.

### 3.1. Example

- Determination of limited error of  $r_{21}$  (related to  $r_{21\max}$ )

According to IEC751 norm, the limited error of nominal resistance Class B Pt100 sensor equals  $|\delta_{10}| = 0.12\%$  and the limited error of increment  $|\delta_{e1}| = 0.32\%$ . It is assumed that the limited errors of nominal resistances  $R_2, R_3, R_4$  are equal  $|\delta_{20}| = |\delta_{30}| = |\delta_{40}| \equiv |\delta_0| = |\delta_{10}| = 0.12\%$ . The limited relative zero error  $|\delta_{210}|$  of the bridge comes to 0.48%. The initial sensitivity  $t_0$  equals 25  $\Omega$  and therefore the limited absolute zero error  $|\Delta_{r_{210}}| = 0.12 \Omega$ . For the full range of the converter  $\varepsilon_{1\max}$  is equal 2.296 (at  $T = 650^\circ\text{C}$ ). On the basis of equation in Tab. 4 'Case 1', the limited relative error  $|\delta_{r_{21r}}|$  (related to  $t_0 = 0.25 R_{10}$ ) of the one-sensor double-symmetric bridge ( $R_{i0} = R_{10}$ ) equals

$$|\delta_{r_{21r}}| = \frac{\frac{1}{2} (1 - \frac{1}{8} \varepsilon_{1\max}) |\delta_{10}| + \frac{1}{2} (1 - \frac{5}{8} \varepsilon_{1\max}) |\delta_0| + |\delta_{e1}|}{1 + 0.25 \varepsilon_{1\max}} = 0.22\%. \quad (10)$$

For full-range value of  $\varepsilon_1$  the unbalance function of the bridge equals

$$f_{\max} = \frac{\varepsilon_{1\max}}{1 + 0.25 \varepsilon_{1\max}} = 1.46 \quad (11)$$

and full-range transfer impedance is equal to

Tab. 4. Accuracy measures of the 4R unbalanced (Wheatstone) bridge ( $R_L = \infty$ ) supplied by current source with equal initial resistances ( $4R_{10}$ ).

Relative accuracy measures of difference $r_{21} - r_{210}$ (related to $r_{21}$ )
<p><b>Case '1'</b> variable <math>R_1 = R_{10}(1 + \varepsilon_1), R_2 = R_3 = R_4 = R_{10}, \varepsilon_1 \geq -1</math></p> $r_{21} = \frac{R_{10}}{4} \frac{\varepsilon_1}{1 + 0.25 \varepsilon_1} \quad r_0 = \frac{R_{10}}{4}$
$\delta_{r_{21r}} = \frac{(\frac{1}{2} - \frac{1}{16} \varepsilon_1) \delta_{10} + (\frac{1}{2} - \frac{3}{16} \varepsilon_1) \delta_{30} + \frac{1}{16} \varepsilon_1 (\delta_{20} + \delta_{40}) + \delta_{e1}}{1 + 0.25 \varepsilon_1}$
$ \delta_{r_{21r}}  = \frac{\frac{1}{2} (1 - \frac{1}{8} \varepsilon_1)  \delta_{10}  + \frac{1}{2} (1 - \frac{5}{8} \varepsilon_1)  \delta_0  +  \delta_{e1} }{1 + 0.25 \varepsilon_1}$
$\bar{\delta}_{r_{21r}} = \frac{\bar{\delta}_{10}, \bar{\delta}_{e1}, \bar{\delta}_{20} = \bar{\delta}_{30} = \bar{\delta}_{40} \equiv \bar{\delta}_0, \sqrt{(\frac{1}{8} \varepsilon_1)^2 \bar{\delta}_{10}^2 + (1 - \frac{3}{4} \varepsilon_1 + \frac{11}{64} \varepsilon_1^2) \bar{\delta}_0^2 + 4 \bar{\delta}_{e1}^2}}{2(1 + 0.25 \varepsilon_1)} \quad \text{for } corr_{ij} = 0$
<p><b>Case '2'</b> variable <math>R_1 = R_3 = R_{10}(1 + \varepsilon), R_2 = R_4 = R_{10}, \varepsilon \geq -1</math></p> $r_{21} = \frac{R_{10}}{4} 2\varepsilon$
$\delta_{r_{21r}} = 0.5 (\delta_{10} + \delta_{30} + \delta_{e1} + \delta_{e3})$ $ \delta_{r_{21r}}  = 0.5 ( \delta_{10}  +  \delta_{30}  +  \delta_{e1}  +  \delta_{e3} )$ $\bar{\delta}_{r_{21r}} = \frac{1}{\sqrt{2}} \sqrt{\bar{\delta}_{10}^2 + \bar{\delta}_{30}^2 + \bar{\delta}_{e1}^2 + \bar{\delta}_{e3}^2} \quad \text{for } corr_{ij} = 0$
$ \delta_{10}  =  \delta_{30} ,  \delta_{20}  =  \delta_{40}  \equiv  \delta_0 ,  \delta_{e1}  =  \delta_{e3}  \equiv  \delta_e $ $ \delta_{r_{21r}}  =  \delta_{10}  +  \delta_e $
$\bar{\delta}_{10} = \bar{\delta}_{30}, \bar{\delta}_{e1} = \bar{\delta}_{e3} = \bar{\delta}_e, \bar{\delta}_{20} = \bar{\delta}_{40} \equiv \bar{\delta}_0$ $\bar{\delta}_{r_{21r}} = \sqrt{\bar{\delta}_{10}^2 + \bar{\delta}_0^2 + 2\bar{\delta}_e^2} \quad \text{for } corr_{ij} = 0$

$$r_{21\max} = \frac{R_{10}}{4} f_{\max} = 36.5 \Omega. \quad (12)$$

For  $f(\varepsilon_i) = f_{\max}$ , according to (8), the limited error of transfer impedance  $r_{21}$  (related to  $r_{21\max}$ ) is equal to

$$\frac{|\Delta_{r_{21}}|}{|r_{21\max}|} \leq \frac{|\delta_{210}|}{|f_{\max}|} + |\delta_{r_{21r}}(\varepsilon_i)| \leq \frac{0.48\%}{1.46} + 0.22\% \leq 0.55\%. \quad (13)$$

- Determination of the standard uncertainty of  $r_{21}$  (related to  $r_{21\max}$ )

Let us now determine the expanded uncertainty when the parameters of particular sensor as well as parameters of the bridge-circuit have unknown values. However, these parameters have uniform distributions of the same range as above mentioned limited errors.

Then type B standard uncertainties of sensor  $R_1$  and other resistances  $R_2, R_3, R_4$  in the bridge are as follows

$$\bar{\delta}_{10} = \frac{|\delta_{10}|}{2\sqrt{3}}, \bar{\delta}_{e1} = \frac{|\delta_{e1}|}{2\sqrt{3}}, \bar{\delta}_{20} = \bar{\delta}_{30} = \bar{\delta}_{40} \equiv \bar{\delta}_0. \quad (14)$$

They are components of the combined standard uncertainty of bridge transfer impedance  $r_{21}$ . According to Table 4, this is expressed as

$$\bar{\delta}_{r_{21r}} = \frac{\sqrt{\left(1 - \frac{1}{8}\varepsilon_{1\max}\right)^2 \bar{\delta}_{10}^2 + \left(1 - \frac{3}{4}\varepsilon_{1\max} + \frac{11}{64}\varepsilon_{1\max}^2\right) \bar{\delta}_0^2 + 4\bar{\delta}_{\varepsilon_1}^2}}{2(1 + 0.25\varepsilon_{1\max})} = 0.20\%. \quad (15)$$

The standard uncertainty of zero  $\bar{\delta}_{210}$  equals 0.07%.

The standard uncertainty of transfer impedance  $r_{21}$  (related to  $|r_{21\max}|$ ) is equal to

$$\frac{\bar{\Delta}_{r_{21}}}{r_{21\max}} = \sqrt{\frac{\bar{\delta}_{210}^2}{f_{\max}^2} + \bar{\delta}_{r_{21r}}^2} = 0.21\% \quad (16)$$

In this way the expanded uncertainty  $U_{r_{21}} = 0.42\%$  is determined (coverage factor  $k=2$  based on Gauss distribution, level of confidence  $p=0.95$ ). It is assumed that all correlation coefficients are equal to 0.

#### 4. ACCURACY PARAMETERS OF DOUBLE-CURRENT CIRCUIT WITH EQUAL INITIAL RESISTANCES

The parameters  $r_{AB}$ ,  $r_{CD}$  of another circuit (2J, Tab. 2) and the differences  $r_{AB} - r_{AB0}$ ,  $r_{CD} - r_{CD0}$  are calculated in a similar way to  $r_{21}$ . One defines the instantaneous errors related to values  $r_{AB}$  i  $r_{CD}$  and the absolute instantaneous errors :

$$\delta_{r_{AB}r} \equiv \frac{\Delta_{r_{AB}} - \Delta_{r_{AB0}}}{r_{AB}} = \frac{t_0' \delta_{r_{AB}} - t_0'(\delta_{10} - \delta_{20} - \delta_{30} + \delta_{40})}{t_0' f'(\varepsilon_i)} = \frac{\delta_{r_{AB}} - (\delta_{10} - \delta_{20} - \delta_{30} + \delta_{40})}{f'(\varepsilon_i)},$$

$$\Delta_{r_{AB}} = t_0' \delta_{210}' + f'(\varepsilon_i) \delta_{r_{AB}r}, \quad (17a,b)$$

$$\delta_{r_{CD}r} \equiv \frac{\Delta_{r_{CD}} - \Delta_{r_{CD0}}}{r_{CD}} = \frac{\delta_{r_{CD}} - (\delta_{10} + \delta_{20} - \delta_{30} - \delta_{40})}{f''(\varepsilon_i)},$$

$$\Delta_{r_{CD}} = t_0'' \delta_{210}'' + f''(\varepsilon_i) \delta_{r_{CD}r}. \quad (18a,b)$$

The limited errors of 4R network are presented in Fig. 5-8. They are made for the case if the initial resistances  $R_{i0}$  ( $i=1,2,3,4$ ) are equal and the increments of resistances  $R_1$  and  $R_2$  are of opposite sign ( $\varepsilon_2 = -\varepsilon_1$ ,  $\varepsilon_3 = 0$ ,  $\varepsilon_4 = 0$ ).

It is possible to present the limited errors and the standard uncertainties (type B) in graphical form. It has been made for two types of supply for Wheatstone bridge and for 2J bridge circuit. If there are two measured variables  $X_1$ ,  $X_2$  and if  $\varepsilon_1 = \varepsilon_A(X_1) + \varepsilon_B(X_2)$  and  $\varepsilon_2 = -\varepsilon_A(X_1) + \varepsilon_B(X_2)$ .

The plots of the limited errors  $|\delta_{r_{21}}|$ ,  $|\delta_{r_{21r}}|$ ,  $|\delta_{r_{AB}}|$  and  $|\delta_{r_{CD}}|$  in function of the relative increment of resistance  $\varepsilon_1$  are presented in Fig. 5-8.

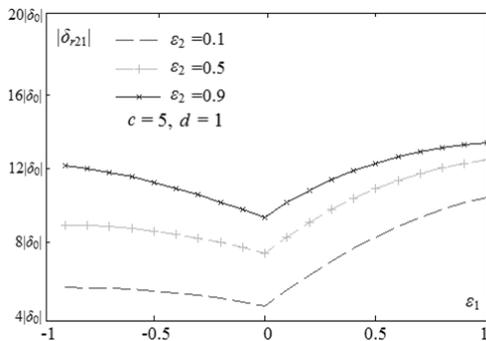


Fig. 5. Limited error  $|\delta_{r_{21}}|$  of the bridge with equal initial arm resistances  $4R_{10}$  ( $|\delta_{10}|=|\delta_{20}|=|\delta_{30}|=|\delta_{40}|=|\delta_0|$ , current supply  $J$ ).

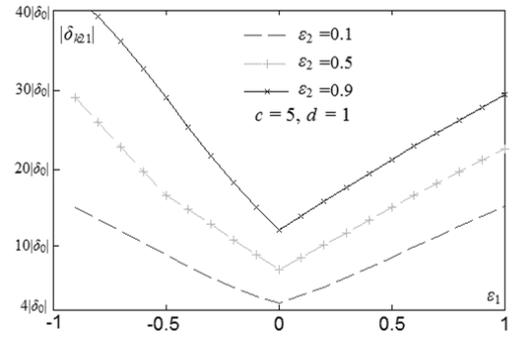


Fig. 6. Limited error  $|\delta_{k21}|$  of the bridge with equal initial arm resistances  $4R_{10}$  ( $|\delta_{10}|=|\delta_{20}|=|\delta_{30}|=|\delta_{40}|=|\delta_0|$ , voltage supply  $E$ ).

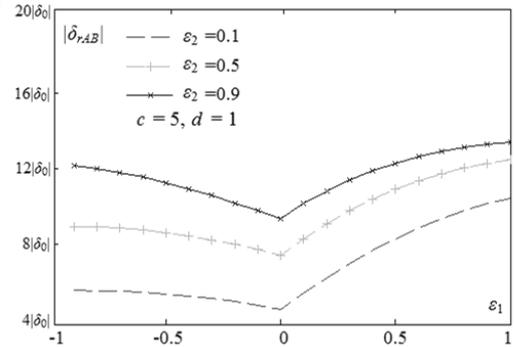


Fig. 7. Limited error  $|\delta_{r_{AB}}|$  of the bridge with equal initial arm resistances  $4R_{10}$  ( $|\delta_{10}|=|\delta_{20}|=|\delta_{30}|=|\delta_{40}|=|\delta_0|$ , double current supply  $J_1=J_2=J$ ).

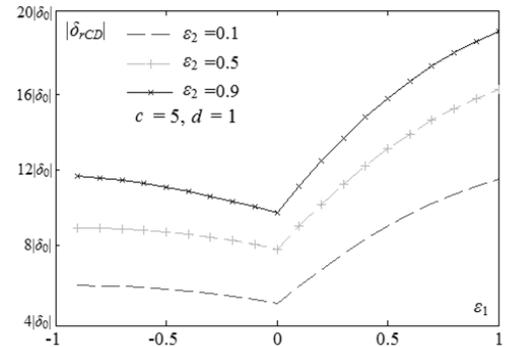


Fig. 8. Limited error  $|\delta_{r_{CD}}|$  of the bridge with equal initial arm resistances  $4R_{10}$  ( $|\delta_{10}|=|\delta_{20}|=|\delta_{30}|=|\delta_{40}|=|\delta_0|$ , double current supply  $J_1=J_2=J$ ).

The relative increment of resistance  $\varepsilon_2$  is a parameter. Additionally, one assumed that

$$c = \frac{|\delta_{\varepsilon_1}|}{|\delta_0|} = 5, \quad d = \frac{|\delta_{\varepsilon_2}|}{|\delta_{\varepsilon_1}|} = 1. \quad (19a,b)$$

In a similar way it is possible to find functions for type B standard uncertainties.

#### 4. CONCLUSIONS

The four resistors (4R) connected in single closed loop was supplied in different ways. The equations for three circuits with open-circuited terminals were presented. In each case a transfer coefficient was differently dependent on

the relative increments of resistances  $\varepsilon_i$ . Each of this circuit can work as a resistance-to-voltage converter. It can convert a single resistance (one variable measurement 1D) or a combination of resistances (two variable measurement 2D). In case of Wheatstone bridge the output voltage and input resistance are measured. If double current bridge or the cascade bridge is utilized then two voltages on the diagonals of circuit are measured.

From the comparison of Fig. 5 and Fig. 6 it follows that the limited errors of transfer voltage  $k_{21}$  (voltage supply) are of higher values than the limited errors of transfer impedance  $r_{21}$  (current supply).

The plots of the limited errors of  $r_{AB}$  and  $r_{CD}$  are different (Fig. 7 and Fig. 8) in function of increments  $\varepsilon_1$  and  $\varepsilon_2$ . It confirms an asymmetry of both output of the double current bridge circuit.

The uncertainties (type B) of transfer coefficients of all types of circuits can be obtained from the instantaneous and limited errors equations. The analysis must assume a uniform distribution of the errors of individual resistances  $R_i$ . In the two variable measurement where two signals are further processed it should be taken into account that their limiting errors are interrelated and type B uncertainties correlated. Detailed exploration of this issue will be the subject of another publication.

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