

HIGH PRECISION WIDE RANGE MEASUREMENT SYSTEMS WITH ADAPTIVE SENSOR ARRAYS

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Abstract – The paper presents a new approach to design of adaptive measurement systems (AMS) for top-precision wide range measurement of physical excitations (acceleration, pressure, etc.). AMS include the array of compensatory sensors adaptively adjusted using the analog-digital compensating circuits and digital data processing unit (DPU). The measurement of an input excitation is realized as iterative improvement of its estimates using data consequently delivered to DPU by the sensors with growing sensitivities. The principles of the system operation and optimal algorithm for the AMS control and estimation of the input excitation are described. The particularity of the AMS operation is elimination of saturation of the sensors at the confidence level provided by the proper choice of their sensitivities.

Keywords: adaptive sensor array, wide range measurement, optimization, accuracy limits.

1. INTRODUCTION

The trade-off between the accuracy of measurement and the dynamic range of measured values complicates substantially the design of systems for precise measurement of input excitations (measurands) varying in the wide ranges. This problem is being solved currently in different ways: by development of new classes of sensing elements, application of new materials and technologies, digital correction of nonlinearities, auto-calibration and so on. The perspective direction in this area is development of measurement systems employing the sensor arrays (banks). The latter one permits to increase the range and accuracy of current measurements of input excitation iteratively, by switching the more and more sensitive sensors depending on the results of previous measurements of the input excitation [1-2].

This paper is devoted to development of analytical backgrounds for design of the high precision analog-digital systems measuring wide range input excitations using the array of adaptive compensatory sensors (see Fig.1). The particularity of the considered adaptive measurement systems (AMS) is introduction of the data processing unit (DPU) which computes estimates of the measured input excitation using corresponding iterative estimation algorithm. Apart from the estimates computing, DPU synchronizes the work

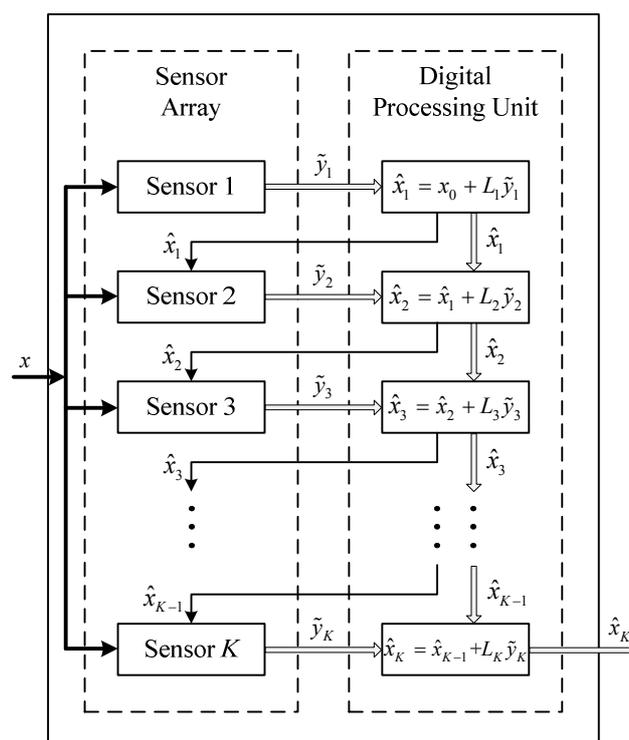


Fig. 1. General block diagram of AMS.

of the sensors and their switching, and adjusts the parameters of particular sensors in the way increasing the final accuracy of measurement. Another particularity of AMS is introduction of the analog-digital feedback forming the counteractions which compensate the influence of the input excitation on the sensing elements of the sensors.

The particularity of the presented approach is direct consideration of possible saturations of the compensatory sensors using the original methodology to optimization of the adaptive analog-digital estimation systems with feedback [3]. This methodology permits to design different classes of estimation systems, in particular measurement systems, which achieve the highest precision obtainable under the given level of noises, imperfections or errors of the parameters setting occurring in a considered system.

The paper is organized as follows. General principles of AMS operation, the method of its optimization, as well as

AMS performance measures are described in Sect. 2. Results of selected exemplary simulation experiments with the computer model of AMS for measurement of acceleration are presented and discussed in Sect. 3. The results of investigations are summarised in Sect. 4.

2. PRINCIPLES OF AMS OPERATION

A general block diagram of the considered AMS is presented in Fig. 1. The sensor array in AMS consists of K sensors ($K \geq 3$). It is assumed that every sensor has the same output range $[-V_0, V_0]$, and the corresponding output voltage is converted into N_{ADC} -bit code by the analog-to-digital converter (ADC) integrated with the sensor. The input ranges $[-A_k, A_k]$ of particular sensors depend on their sensitivities C_k ($k = 1, \dots, K$) determined by the following relationship: $A_k = V_0/C_k$.

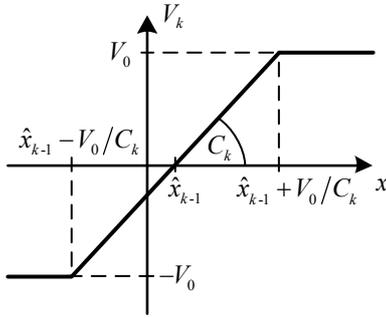


Fig. 2. Static transition function of compensatory sensor.

We assume that the excitation x_t acting at the AMS input is a stationary narrowband Gaussian process, its frequencies lay in the interval $[0, F]$, and particular measurements are carried out during a short time $\tau \ll T$, where $T = 1/2F$ determines the period of repeating the measurements. In these conditions, values of the input excitation measured by the system can be considered as the sequence $(x^{(1)}, x^{(2)}, \dots, x^{(M)})$ of normally distributed constant values $x^{(m)} = x(t - mT)$ measured independently and in the same manner. Therefore, the analysis of AMS operation can be reduced to consideration of a single measurement of the input excitation, which allows us to omit the superscripts (m) in notations of the input excitation ($x^{(m)} = x$) and other variables.

In practical measurements, to exclude saturation of the sensors, every user of a measurement system assesses, at least approximately, the mean value x_0 and variance σ_0^2 (or standard deviation σ_0) of the input excitation x . According to known result ([4], Sect. 7.4), under given x_0 and σ_0^2 , the normal distribution maximizes the entropy of the random values x that is a capability to say – which values x are more or less probable (“uncertainty” in terms of the information theory). In many cases, the normal distribution is a satisfactory good approximation of real probability distributions of the input excitations. For these reasons, below, the distribution of the values x is assumed to be normal with the parameters x_0, σ_0^2 .

Denoting by V the analog signal formed by the sensor, and by \tilde{y} its code formed by ADC, one can write the static transition functions of every (k -th) sensor in the following form (see Fig. 1):

$$\tilde{y}_k = \begin{cases} C_k(x - \hat{x}_{k-1}) + \xi_k & \text{if } C_k |x - \hat{x}_{k-1}| \leq V_0, \\ V_0 \operatorname{sgn}(x - \hat{x}_{k-1}) + \xi_k & \text{if } C_k |x - \hat{x}_{k-1}| > V_0, \end{cases} \quad (1)$$

where \hat{x}_{k-1} is a counteraction to the input excitation decreasing its influence on the sensing element. Variable ξ_k in (1) describes the quantization noise of ADC. If the array includes the sensors with asymmetrical input ranges $[A_{\min}, A_{\max}]$, values \hat{x}_{k-1} in (1) are to be replaced by $\hat{x}_{k-1} + (A_{\max} - A_{\min})/2$.

It is suggested that each sensor is protected against the physical destruction, if the input excitation exceeds its input range, as well as is calibrated, i.e. for the zero excitation, the output voltage has zero value.

2.1. Structure and general principles of AMS operation

The sensors in the AMS sensor array are ordered according to the growing sensitivities. The full dynamic range of the system $[-A_0, A_0]$ is determined by the first least sensitive sensor (Sensor 1 in Fig. 1): $[-A_0, A_0] = [-A_1, A_1] = [-V_0/C_1, V_0/C_1]$. Before each activation of AMS, all sensors stay in the nominal state, i.e. voltages at the inputs of ADCs have the values equal to 0. After the system activation, the input excitation x causes saturation of all sensors with the sensitivities $C_i > C_k$, where C_k is the sensitivity of the last in row of the non-saturated sensors beginning with Sensor 1. Further measurement of the excitation can be realized in different ways. Below, we describe only one of them assuming that activation of AMS under given excitation x causes saturation of all sensors except of Sensor 1.

Sensor 1 delivers the code \tilde{y}_1 to DPU which computes the first estimate \hat{x}_1 of the input excitation x according to the equation:

$$\hat{x}_1 = x_0 + L_1 \tilde{y}_1, \quad (2)$$

where the coefficient L_1 has the value dependent on the sensitivity C_1 of Sensor 1 (see below, Sect. 2.2). The computed estimate \hat{x}_1 is saved in DPU. Simultaneously, DPU routes the estimate \hat{x}_1 to the input of the compensating unit of Sensor 2 using a digital-to-analog converter (DAC) (not shown in Fig. 1).

Appearance of the signal \hat{x}_1 activates the compensating unit of Sensor 2 which forms the counteraction \hat{x}_1 applied to the sensing element (to clarify the main effects, in the current research, influence of possible errors in the counteraction setting, and possible external and internal analog noises are assumed to be negligibly small). The compensated excitation $e_1 = x - \hat{x}_1$ creates, at the Sensor 2 output, the voltage $V_2 = C_2(x - \hat{x}_1)$ converted by ADC into the code \tilde{y}_2 routed to DPU, and the next stage of the excitation x measurement begins.

In the next stage, operations realized at the previous one are repeated: DPU computes the second estimate \hat{x}_2 according to the equation $\hat{x}_2 = \hat{x}_1 + L_2 \tilde{y}_2$, replaces the estimate \hat{x}_1

by \hat{x}_2 and routs \hat{x}_2 to the compensating unit of Sensor 3. Sensor 3 forms the next code \tilde{y}_3 and further stages of the excitation measurement repeat the operations described above. The final estimate \hat{x}_K is routed to addressee.

2.2. Particularities of AMS realization

Implementation of the proposed measurement scheme requires the advanced analytical support. The principle problem is exclusion of saturation of every sensor – each saturation causes unacceptably large distortions of the final estimates and radically worsens the accuracy of measurements.

To solve this problem, we employ the method proposed in [3] for elimination of saturation with the given probability $\mu \ll 1$. Being applied to AMS, this method determines the input range $[-A_k, A_k] = [-V_0 / C_k, V_0 / C_k]$ of every sensor in the array ($k = 1, \dots, K$) which guarantee that its saturation may appear with a probability not greater than μ . This condition can be expressed by the relationship:

$$\Pr \{C_k |x - \hat{x}_{k-1}| < V_0\} \geq 1 - \mu. \quad (3)$$

For the normally distributed excitations with the zero mean and known variance σ_k^2 , formula (3) gives the following result. The corresponding boundaries A_k of the k -th sensor input range should not be smaller than $A_k = \alpha \sigma_k$, where σ_k is the standard deviation of the compensated excitation acting at the sensing element of the k -th sensor, and α (common for all sensors saturation factor [5]) satisfies the equation [3,6]:

$$\frac{1}{\sigma_k \sqrt{2\pi}} \int_0^{\alpha} \exp\left(-\frac{x^2}{2\sigma_k^2}\right) dx = \frac{1-\mu}{2}. \quad (4)$$

Taking into account the dependence $A_k = V_0 / C_k$, one may find the sensitivities of the “statistically fitted” sensors:

$$C_k = \frac{V_0}{A_k} = \frac{V_0}{\alpha \sigma_k}, \quad (5)$$

which practically exclude their saturation. Thus, if we wish to exclude saturation of AMS as a whole at the confidence level μ , and input excitations have the variance σ_0^2 , the least sensitive Sensor 1 should have the input range $[-\alpha \sigma_0, \alpha \sigma_0]$, and its sensitivity should have the value $C_1 = C_0 = V_0 / \alpha \sigma_0$.

Remark 1: Elimination of the sensors’ saturation at the level μ (e.g. for $\mu < 10^{-4}$) permits to consider them de facto as linear units. This enables application of the methods of optimal adaptive estimation [3,6] for improvement of the quality of estimates formed by AMS.

2.3. Optimization of AMS parameters

Like it was done in [3,6], one can determine, in the main order, the values of the parameters C_k, L_k of particular sensors which minimize, for each $k = 1, \dots, K$, the mean square error (MSE) P_k of the measured (estimated) values of the input excitation:

$$P_k = E[x - \hat{x}_k(\tilde{y}_1^k)]^2, \quad (6)$$

under the guaranteed probability of the sensors’ saturation not greater than μ (as it was said above, possible analog noises and errors of counteractions forming are not considered). Variable \tilde{y}_1^k in (6) denotes the set of subsequent codes at the outputs of ADCs $\tilde{y}_1^k = [\tilde{y}_1, \tilde{y}_2, \dots, \tilde{y}_k]$ in the first particular sensors. The optimal values of parameters C_k and L_k are determined by the following relationships [3,6]:

$$C_k = \frac{V_0}{\alpha \sqrt{\sigma_v^2 + P_{k-1}}}, \quad L_k = \frac{C_k P_k}{\sigma_\xi^2 + C_k^2 \sigma_v^2}. \quad (7)$$

The relationship for C_k follows from fitting condition (3), and $\sigma_k^2 = \sigma_v^2 + P_{k-1}$, ($k = 2, \dots, K$, $\sigma_1^2 = P_0 = \sigma_0^2$). Variables σ_ξ^2 and σ_v^2 in formula (7) are the variances of quantization noises related to the resolution of ADC and the resolution of the compensating excitation determined by DAC. Their values can be assessed using the known Widrow’s formula [7]:

$$\sigma_\xi^2 = \Delta_{ADC}^2 / 12 = V_0^2 \cdot 2^{-2N_{ADC}} / 3, \quad (8)$$

$$\sigma_v^2 = \Delta_{DAC}^2 / 12 = A_0^2 \cdot 2^{-2N_{DAC}} / 3, \quad (9)$$

where V_0, N_{ADC} and A_0, N_{DAC} are the input ranges and resolutions (in number of bits) of ADC and the compensating units, respectively. In turn, values P_k in (7) are the values of minimal MSE (MMSE) of estimates formed by AMS at the k -th stage of the excitation measuring under coefficients (7), and determined by the recursive equation of the Riccati type [3]:

$$P_k = \frac{(\sigma_\xi^2 + C_k^2 \sigma_v^2) P_{k-1}}{\sigma_\xi^2 + C_k^2 (\sigma_v^2 + P_{k-1})}, \quad (10)$$

where C_k is calculated according to (7), and $P_0 = \sigma_0^2$. MSE values determine the root mean square error (RMS) and standard deviation of measurement: $RMS_k = \sqrt{MSE_k} = \sqrt{P_k}$.

2.4. AMS performance measures

The AMS as a whole can be considered as the converter of the values of physical excitations into the binary codes. This allows to describe the accuracy (“uncertainty” [8]) of measurements by the resolution N_{AMS} of the system, i.e. by the number of significant bits in the codes \hat{x}_K at the AMS output (K is the number of the last sensor). To introduce this measure correctly, one should separate the input range $[-\alpha \sigma_0, \alpha \sigma_0]$ of AMS into $2^{N_{AMS}}$ adjacent intervals Δ_{AMS} referring to the corresponding binary code, and every code word \hat{x}_K determines the analog value \hat{x}_K^a placed in the middle of the interval Δ_{AMS} . To exclude possible rough errors, intervals Δ_{AMS} should have the lengths guaranteeing that all the values x of excitation satisfying the inequality $|x - \hat{x}_K^a| \leq \Delta_{AMS} / 2$ will be referred to the code \hat{x}_K with a probability not smaller than μ , i.e. at the same confidence level as saturation errors. Accurate evaluations [9] of the length of the intervals satisfying the latter requirement give the following relationship:

$$\Delta_{AMS} = 2\alpha \sqrt{P_K}. \quad (11)$$

According to the said above, the (binary) resolution of AMS has the value:

$$N_{AMS} = \log_2 \left(\frac{2A_0}{\Delta_{AMS}} \right) = \frac{1}{2} \log_2 \left(\frac{\sigma_0^2}{P_K} \right) \text{ [bit]}. \quad (12)$$

Remark 2: One should emphasize that formula (11) determines the confidence interval of measurement errors under the confidence level μ that corresponds to the definition of the measurement uncertainty B given in GUM [8]. In practical applications, quality of digital measurement systems can be assessed by the more clear measure (12) completed by the values A_0 and μ .

Remark 3: Considering AMS as the “communication channel” delivering the information about the values of input excitation to addressee (further units processing the estimates formed by AMS), one may assess the instant and mean information efficiency (bit rate [bit/s]) of the system:

$$R_{AMS}^{inst} = \frac{1}{2\tau} \log_2 \left(\frac{\sigma_0^2}{P_K} \right) \text{ [bit/s]}, \quad (13)$$

$$R_{AMS}^{mean} = \frac{1}{2T} \log_2 \left(\frac{\sigma_0^2}{P_K} \right) \text{ [bit/s]}. \quad (14)$$

The practical assessments of the presented performance measures can be realized using the empirical MSE calculated according to the following formula:

$$\hat{P}_k = \frac{1}{M} \sum_{m=1}^M [x^{(m)} - \hat{x}_k^{(m)}]^2, \quad (15)$$

where M is the number of independent random values of input excitations measured in the experiment.

3. SIMULATION EXPERIMENTS

As the illustration of the proposed approach, the results of analysis of the computer model of AMS designed for accurate and wide range measurement of acceleration are presented below. The goal of the performed simulations was analysis of the accuracy of acceleration estimates provided by AMS. It was assumed that the considered system measures the acceleration in the range from -500 [g] to 500 [g] ($A_0 = 500$ [g]). The input range of ADC used at the output of the particular accelerometers was from -5 [V] to 5 [V] ($V_0 = 5$ [V]). The other parameters used in the experiment were as follows: $N_{DAC} = 12$, and $\alpha = 4$ that corresponds to the probability of the sensors saturation not greater than $\mu \sim 10^{-4}$.

In the first series of simulation experiments, the performance measures of AMS for different numbers of internal accelerometers (sensors) and for different resolutions of internal ADC were studied. Testing (reference) accelerations were modeled as the sequence of $M = 10^5$ random values $x^{(m)}$ normally distributed with the parameters $x_0 = 0$, $\sigma_0 = A_0/\alpha = 125$. Each value of acceleration was processed by the model of the system realized according to the scheme presented in Fig. 1 and the relationships presented above.

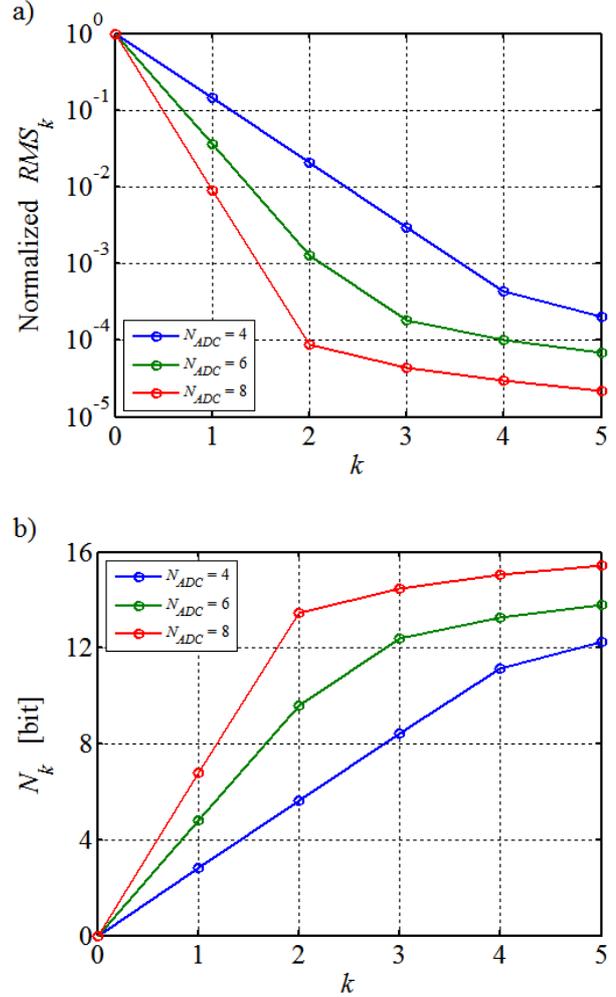


Fig. 3. Results of simulation experiments: (a) normalized RMS after subsequent iterations k (or for AMS with different number of sensors) for different resolutions of ADC, (b) AMS resolution (in bit) depending on the number of sensors used in AMS.

The considered version of AMS included $K = 5$ consequently activated accelerometers. The normalized to $P_0 = \sigma_0^2$ RMS for each stage of the measurement $k = 1, \dots, 5$ were calculated according to the formula:

$$RMS_k = \sqrt{\frac{\hat{P}_k}{\sigma_0^2}} = \frac{1}{\sigma_0} \sqrt{\frac{1}{M} \sum_{m=1}^M [\hat{x}_k^{(m)} - x^{(m)}]^2}. \quad (16)$$

The results of simulations are presented in Fig. 3. Fig. 3a illustrates the rate of decrease of the normalized RMS of estimates in subsequent iterations of measurement, which is equivalent to the measurement by AMS with different number of sensors ($k = 1, \dots, K$). The presented plots are obtained for AMS employing ADC with different resolutions $N_{ADC} = 4, 6, 8$ bits.

The plots in Fig. 3b show the changes of the binary resolution of AMS N_k in subsequent iterations obtained for the same parameters. The AMS resolutions were calculated according to (12) and using the empirical values of RMS computed according to (16).

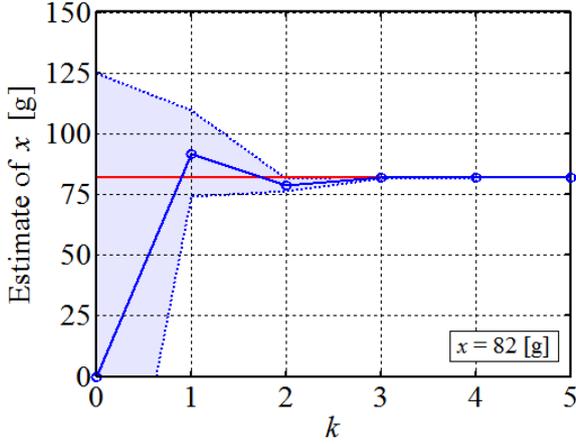


Fig. 4. Exemplary estimates of acceleration measured by AMS after subsequent iterations k of measurement for $x = 82$ [g].

Fig. 4 shows the changes of estimates (presented as small circles) obtained in subsequent iterations of measurement of the fixed value of acceleration ($x = 82$ [g]). The highlighted grey area in Fig. 4 corresponds to the set of realizations of estimates which lay inside the field confined by their RMS. In this experiment, the following resolution of ADC was assumed: $N_{ADC} = 4$.

There was also carried out a group of experiments concerning the important for practice question: how discrepancies between the nominal (7) and actually set values of the coefficients C_k and L_k influence on MSE of measurements. These differences can be caused by the technological dispersion of sensors parameters, non-idealities of characteristics of A/D and D/A converters, setting errors, and so on.

The experiments were carried out for two cases: the coefficients C_k were set accurately but L_k were set with the errors varying in the range $L_k = L_k^{nom}(1 + \delta_L)$, and the reverse situation – constant accurate L_k and varying C_k . The model and nominal parameters had the same values as in previous simulations. Both groups of experiments showed qualitatively similar effect illustrated in Fig. 5, where the results corresponding to the ADC resolution $N_{ADC} = 6$ are presented. Namely: there exists a quite wide range of deviations of the parameters from their nominal values, for which RMS is practically non-sensitive to the appearing discrepancies of the C_k and L_k values. This field is wider for AMS employing ADC with smaller resolution, and narrower for ADC with greater resolution. In all cases, RMS attains, after 5 stages of measurements, the values of 10^{-4} order and not significantly increases for ADC with lesser resolution ($N_{ADC} = 4$).

The established effect weakens the requirements on the precision of technological processes used in AMS manufacturing. Moreover, it shows that design of more complex AMS with high resolution ADCs, apart from growing costs of the systems would meet the increased requirements on the technology. In turn, AMS with low cost and low bit ADC (3-6 bits) provide sufficiently high quality of measurements, simultaneously being substantially more convenient for manufacturing.

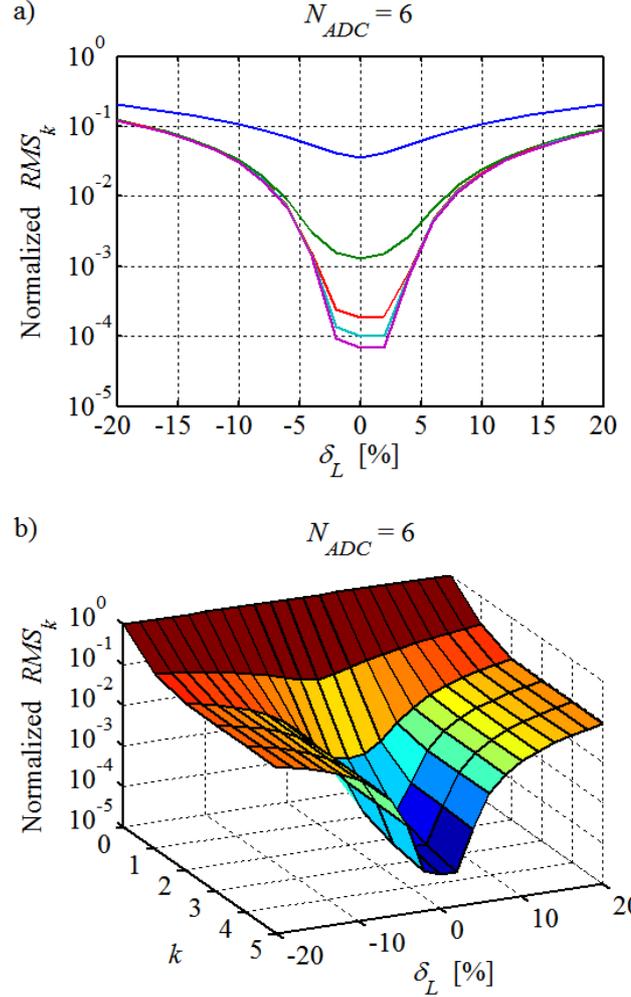


Fig. 5. Normalized RMS of measurements for AMS with discrepancies between the values of parameters C_k and L_k , (a) 2D plots for different errors δ_L (lines from top to bottom correspond to subsequent iterations: $k = 1, 2, \dots, 5$), (b) 3D presentation.

4. CONCLUSIONS

The results of research confirm a capability of AMS with adaptively adjusted sensor array to extend the range of measurements and, simultaneously, to improve their accuracy. This is the result of optimal adaptive compensation of the input excitations and growing sensitivity of subsequently activated sensors. The values of the sensors sensitivities determined by the proposed algorithm together with the adaptive compensation of the input excitation provide the highest accuracy of measurement possible to achieve in the given conditions and for the assumed probability of the sensors saturation.

The practical versions of AMS: the adaptive wide-range high precision compensatory voltmeters and accelerometers with directly controlled sensitivity (not in array structure) were studied in [10] (on the hardware prototype of the voltmeter) and in [11] (analytically and in simulations), respectively. The obtained results confirmed the efficiency of adaptive adjusting of the sensing elements of these systems.

Preliminary analysis of difficulties in realization of AMS for the measurement of non-electrical quantities shows that the array architecture of the systems is more convenient for practical realization. It also ensures more reliable measurements than AMS realized as the sensors with special unit directly adjusting their sensitivities. The established small sensitivity of the considered AMS to technological errors seems to be especially interesting for their potential realization, especially in nanotechnologies.

The current and preliminary researches ([10-12] and others) show also that optimal utilization of the feedback and the presented method of adaptive analog-digital measurement systems optimization and design opens wide possibilities for the further development of the theory as well as for elaboration of new efficient measurement systems.

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