

THE MULTI-COMPONENT AERODYNAMIC BALANCE CALIBRATION MATRIX OF DIMENSION ONE

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Abstract - This paper provides information on the calibration matrix of the aerodynamic six-component external balance of the TA-2 wind tunnel. The balance allows different sets of load cell arrangements.

Least squares method is used to fit the parameters of the calibration mathematical modelling. Data from eight campaigns were analysed considering the dimension of the matrix elements equal to one. Parameters and associated uncertainties which represent the behaviour of the balance over time were obtained.

Keywords: six-component external balance, wind tunnel tests, uncertainty in measurement

1. INTRODUCTION

Wind tunnels are aerodynamic facilities used to analyse the behaviour of structures submitted to aerodynamic flows. Multi-component balances are employed to measure aerodynamic forces and moments acting on the model being tested in the wind tunnel. The aerodynamic components are named drag, lateral and lift force and rolling, pitching and yawing moments. The calibration of the TA-2 wind tunnel external multi-component balance requires a system composed of a calibration cross, trays, cables, pulleys and approximately 100 weights of nominal mass equal to 10 kg (Fig. 1).

During calibration, known weights are applied to 14 trays of the calibration system. The applied forces and moments are transmitted throughout the balance and measured by the six load cells fixed to it (Fig. 2). There are two basic procedures: the alpha and the beta calibration. The former consists of 73 loadings and results in the estimation of 27 parameters while the latter consists of 219 loadings and results in the estimation of 39 parameters. A single calibration campaign lasts around one week.

The choice of the load cell arrangement is dependent on the testing requirements, *i. e.*, the expected aerodynamic forces and moments on the test article dictate the load capacity of the load cells to be employed in the tests.

For each set of load cells, it is necessary to calibrate the balance. The purpose of the calibration is to supply the calibration matrix, composed of polynomial parameters which relate aerodynamic forces and moments to load cell readings.

Prior to the balance calibration, the load cells are individually calibrated at the Low Force Laboratory of the Aerodynamics Division, using a Load Cell Calibration Machine (Fig. 3). A case is used to encapsulate the load cell to immobilise it during calibration and transportation to the external balance. At the Low Force Laboratory, first order polynomials are fitted to the data and the calibration certificate supplies linear and angular coefficients of the load cell calibration curves. Units for the angular and linear coefficients are $\text{mV} \times \text{kgf}^{-1}$ and mV , respectively. The values of the angular coefficients play an important role in the process of making the dimension of balance calibration matrix equal to one, as explained in section 2.7.

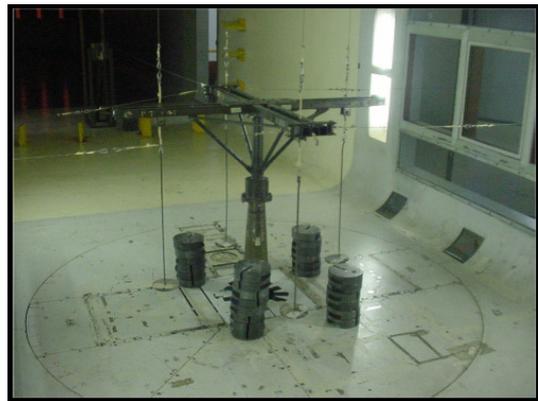


Fig. 1. The calibration system.



Fig. 2. The external six-component aerodynamic balance.

Studies have been conducted at TA-2 to develop a method to estimate the calibration parameters and associated

uncertainties using international standardisation([1],[2] and [3]).

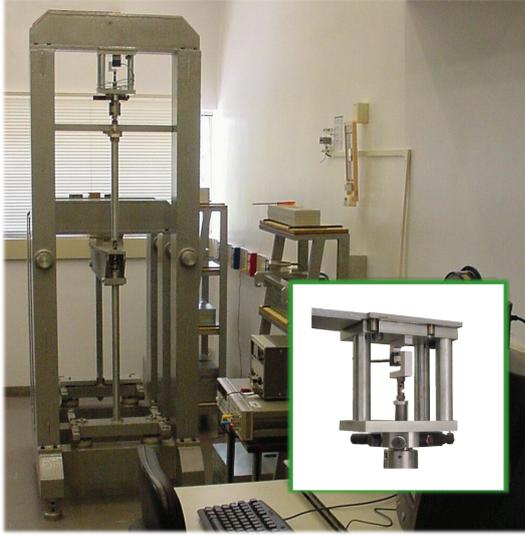


Fig. 3. The load cell calibration machine. In detail the load cell case.

So far, the studies have focused on the dimensional matrix. In the present study, we are interested in the matrix whose elements have dimension one. More specifically, we want to know how the matrix elements and associated uncertainties respond to changes in the load cell arrangement. We seek to identify a behaviour pattern among the polynomial parameters throughout calibrations carried out over time. Based on this study, we want to answer the following question: is it possible to attribute a representative mean value and associated uncertainty to each element of the calibration matrix?

We analyse experimental data resulting from eight alpha calibrations in total: three carried out in 2010, one in 2009, two in 2006 and two in 2005. We consider that the calibrations are under reproducibility conditions because operators, instrumentation and environment change from one calibration to another [4].

2. METHODOLOGY

The adopted procedure to calibrate the TA-2 external balance is presented in section 2.1, the mathematical modelling of the calibration in section 2.2 and the data reduction steps are presented in sections 2.3, 2.4, 2.5 and 2.6. The procedure to reduce the calibration matrix to dimension one is presented in section 2.7. Section 2.8 shows how the elements of the calibration matrix and associated uncertainties of different calibration campaigns are analysed. We intend to obtain a calibration result which can represent the global behaviour of the aerodynamic balance.

2.1. Calibration procedure

The external balance calibration is a one-week activity at TA-2. A calibration cross is mounted in the place where

the model under test is to be positioned. Cables and pulleys are part of the system as shown in Fig. 4. Known weights of nominal values equal to 10 kg are applied to the trays to simulate the expected range of aerodynamic forces and moments in the tests. The loading procedure includes positive, negative and zero values of the 3 aerodynamic forces and 3 aerodynamic moments: drag, lateral and lift forces and rolling, pitching and yawing moments. The experiment combines null, single and double loadings. The single type experiment means that the weights are applied to the trays in such a way that only one of the six aerodynamic components is loaded. For the double type, a pair of aerodynamic components is loaded. In the null experiment, no weight is applied to the trays[1].

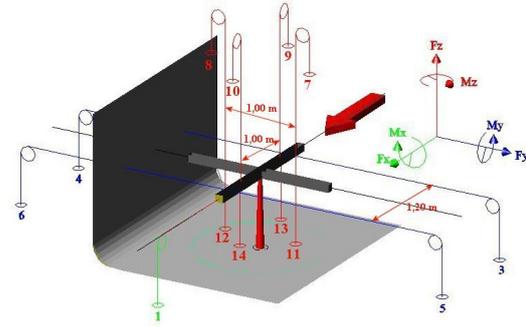


Fig. 4. Loading system. Trays 1 to 14

2.2. Mathematical modelling

The equation describing the mathematical modeling of the alpha calibration is:

$$F_{[73 \times 1]} = R_{[73 \times 27]} \cdot A_{[27 \times 1]} \quad (1)$$

The equation (1) is written in matrix formalism. Symbol F denotes the measurand (forces or moments), R the load cell readings and A the 27 unknowns. The number of equations equals the number of loadings performed in the calibration system, 73. There are six such equations, three for forces and three for moments.

Matrix R is called the design matrix and is a combination of readings of the load cells up to second order interactions.

As an example, for the drag force, the polynomial equation can be written as:

$$F_{drag} = a_1 R_1 + a_2 R_2 + a_3 R_3 + \dots + a_6 R_6 + a_7 R_1^2 + a_8 R_1 R_2 + a_9 R_1 R_3 + \dots + a_{26} R_5 R_6 + a_{27} R_6^2 \quad (2)$$

Besides the single terms of R , the second order terms RR are added to the equation because the balance's structure is not able to completely isolate interactions between the measured components.

In (2), we wrote the polynomial parameters for drag force as a_i , $i = 1, \dots, 27$. For the remaining five aerodynamic components, letters b , c , d , e and f can be

used to distinguish the parameters for lateral and lift forces, and rolling, pitching and yawing moments, respectively. Units for forces and moments are kilogram-force, kgf, and kilogram-force×meter, kgf×m, respectively. Readings R are in mili-volts, mV.

The first six polynomial parameters a_1 to a_6 have similar units: kgf×mV⁻¹ for forces and kgf×m×mV⁻¹ for moments. The remaining polynomial parameter units are kgf×mV⁻² and kgf×m×mV⁻², for forces and moments, respectively.

In an ideal balance, only the first term $a_1 R_1$ of the right hand side of (2) would be included in the mathematical modelling of the drag force. By similar reasoning, only the second term $a_2 R_2$ would appear for the lateral force, and so on.

2.3. Curve fit

Least-squares fitting is applied to experimental data resulting in the estimation of the 27 polynomial parameters of the mathematical modelling (2). The data reduction consists of the steps presented in the following equations:

$$\begin{aligned} R_{[27 \times 73]}^t \cdot F_{[73 \times 1]} &= R_{[27 \times 73]}^t \cdot R_{[73 \times 27]} \cdot A_{[27 \times 1]} \\ R_{[27 \times 73]}^t \cdot F_{[73 \times 1]} &= (R^t R)_{[27 \times 27]} \cdot A_{[27 \times 1]} \\ \underbrace{(R^t R)_{[27 \times 27]}^{-1}}_{\text{ErrorMatrix}} \cdot R_{[27 \times 73]}^t \cdot F_{[73 \times 1]} &= \underbrace{(R^t R)^{-1} \cdot (R^t R)}_{\text{IdentityMatrix}_{[27 \times 27]}} \cdot A_{[27 \times 1]} \end{aligned} \quad (3)$$

which leads to the evaluation of the 27 polynomial parameters:

$$A_{[27 \times 1]} = (R^t R)_{[27 \times 27]}^{-1} \cdot R_{[27 \times 73]}^t \cdot F_{[73 \times 1]} \quad (4)$$

In equation (4), R^t is the transpose matrix of R . The inverse matrix $(R^t R)^{-1}$ is called the error matrix because its diagonal elements are the variances of the fitted parameters. The off-diagonal elements are the co-variances between the fitted parameters. Each aerodynamic force and moment has its own set of estimated parameters.

We can generalise and extend the vector $A_{[27 \times 1]}$ to a calibration matrix $A_{[27 \times 6]}$ to represent the 27 polynomial parameters of each of the six aerodynamic components. The first column belongs to drag force, second to lateral force, etc.

The quality of the fit for each of the six aerodynamic components is quantified by the chi-squared quantity [5]:

$$\chi^2 = \sum_{j=1}^n \left(\frac{F_{j\text{applied}} - F_{j\text{fitted}}}{u_{F_j}} \right)^2 \quad (5)$$

where n is equal to 73, F_{applied} values are weights applied to the trays of the calibration system, F_{fitted} are estimated values supplied by the least squares method and u_{F_j} are the uncertainties assigned to F_{applied} . A number corresponding to a good fit is equal to the number of degrees of freedom. As the calibration data provides a set of 73 equations and 27 unknowns, the number of degrees of freedom is 46. If χ^2 is

divided by the number of degrees of freedom, the reduced chi-squared, χ_{red}^2 , is obtained and should result in a number close to 1. A departure from unity means that either the mathematical modelling of the fitted curve or the uncertainty assigned to data points is not adequate.

In this paper, no uncertainty is assigned to the independent variable R . The uncertainty in the aerodynamic forces and moments, u_{F_j} , is considered in three different approaches: i) equal to 1, ii) equal to the standard deviation of the curve fit and iii) equal to the standard deviation of experimental measurement. Differences between i, ii and iii will be explained in sections 2.4, 2.5 and 2.6.

2.4. Uncertainty in data points equal to 1

In this case, we consider that no information is available on the uncertainty u_F in the measurand F and we assign a value equal to 1 to the associated uncertainty. The elements of the 73 lines and 27 columns of the design matrix R are divided by $u_F = 1$ (6). The unit of u_F is kilogram-force for forces and kilogram-force×meter for moments. In sequence, we apply the least-squares fitting to the calibration data to obtain the calibration matrix. This process is a basis for the other two presented in sections 2.5 and 2.6.

$$R = \begin{bmatrix} \frac{R_{1,1}}{u_F} & \frac{R_{2,1}}{u_F} & \dots & \frac{R_{5,1} R_{6,1}}{u_F} & \frac{R_{6,1}^2}{u_F} \\ \frac{R_{1,2}}{u_F} & \frac{R_{2,2}}{u_F} & \dots & \frac{R_{5,2} R_{6,2}}{u_F} & \frac{R_{6,2}^2}{u_F} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{R_{1,72}}{u_F} & \frac{R_{2,72}}{u_F} & \dots & \frac{R_{5,72} R_{6,72}}{u_F} & \frac{R_{6,72}^2}{u_F} \\ \frac{R_{1,73}}{u_F} & \frac{R_{2,73}}{u_F} & \dots & \frac{R_{5,73} R_{6,73}}{u_F} & \frac{R_{6,73}^2}{u_F} \end{bmatrix} \quad (6)$$

2.5. Uncertainty in data points equal to the standard deviation of the curve fit

The second approach begins with the estimation of the standard deviation of the fit which is provided using results of the curve fitting obtained using $u_F = 1$ in (6). The standard deviation of the fit, S_{fit} , is the positive square root of the equation:

$$S_{\text{fit}} = \sqrt{\frac{\sum_{j=1}^n (F_{j\text{applied}} - F_{j\text{fitted}})^2}{n - 27}} \quad (7)$$

where $j = 1, \dots, n$, for $n = 73$, identifies the number of the loading process. This value is considered as the associated uncertainty of the measurand F , and is included in the design matrix: $u_F = S_{\text{fit}}$ in (6). Thus, as we do not know the uncertainty in the 73 experimental data points, we assign a mathematically quantified uncertainty value to the data.

There are six estimated standard deviations, S_{fit} , one for each aerodynamic component and they are included in the design matrix. In this case, the design matrix contains information about the uncertainty $u_F = S_{\text{fit}}$ associated to the dependent variable F , besides the measured values of the independent variable R [6]. The difference between

the design matrices arranged in the first approach described in section 2.4 and in the second approach described in the present section is that in the former the functions of R are divided by one ($u_F = 1$) and the curve fitting is based on a design matrix similar to all aerodynamic components. In the latter, the functions of R are divided by standard uncertainties u_F equal to the standard deviation values S_{fit} supplied by (7) and the curve fit is accomplished by using six different design matrices. All the elements of the vector F also have to be divided by the associated uncertainty $u_F = S_{fit}$.

2.6. Uncertainty in data points equal to the standard deviation of experimental data

The last approach discussed in this paper involves a type A evaluation of measurement uncertainty [4].

The TA-2 data acquisition system is set to read the six load cell measurements 500 times. Therefore, each one of the individual 73 loading stages of the calibration process provides 500 readings for R_1 , 500 readings for R_2 , etc. In order to evaluate the uncertainty in data points according to the method proposed in this third approach, the mean and standard deviation values of the raw data are estimated as follows.

The load cell readings are multiplied by the polynomial parameters, as established by the mathematical modelling (1), to evaluate the corresponding aerodynamic component F . This calculation is performed 500 times and uses the polynomial parameters obtained in the curve fitting described in section 2.4, which considers uncertainty in data points equal to 1. The result is a sample of 500 values of the measurand F , whose mean and standard deviation are evaluated. The mean value will give the elements of the vector $F_{[73 \times 1]}$. The experimental standard deviation of the 500 measurements, which we named S_{500} , will be included in the design matrix as a standard uncertainty. Each line of (6) will be divided by the corresponding data point standard uncertainty, $u_F = S_{500}$.

We also carried out the calculation considering the experimental standard deviation of the mean, $S_{500} = S_{500} \times (\sqrt{500})^{-1}$, as the uncertainty u_F .

2.7. The calibration matrix of dimension one

Extending (4) and arranging the calibration parameters in a single matrix, we obtain $A_{[27 \times 6]}$, the calibration matrix.

So far, at TA-2 we have dealt with the dimensional calibration matrix. To reduce the calibration matrix to dimension one, the elements of the design matrix R in section 2.4 are divided by the value of the angular coefficients declared in the load cell calibration certificates. The linear coefficient is discharged because the aerodynamic balance is submitted to a zero adjustment during calibration. As an example, R_1 is divided by the angular coefficient of the load cell selected to measure drag force; the crossed terms such as $R_1 R_2$, are divided by the product of the angular coefficients of drag and lateral forces, and so on... As the unit of the angular coefficients resulting from

the calibration of the load cell is $\text{kgf} \times \text{mV}^{-1}$, the modified elements of the new design matrix have kgf or $\text{kgf} \times \text{kgf}$ units, depending on whether the terms in the design matrix are a single or a second order function of R . The new design matrix is now normalized and the effect of having load cell arrangements of different load capacities is minimized.

Applying the steps presented in (3) with this new design matrix, will lead to a dimension one calibration matrix, which we name $A_{one[27 \times 6]}$. The elements of A_{one} have associated uncertainties also obtained by using (3). The aim of this procedure is to analyse balance calibration data originating from different calibration campaigns.

2.8. Analysis of different calibration data

We intend to identify a calibration matrix and associated error matrix which are representative of the aerodynamic balance, despite the load cell arrangement. The understanding of the behaviour of the balance over time will help extend the schedule interval between successive calibration campaigns.

After performing the calculation described in section 2.7, we obtain eight calibration matrices of dimension one, $A_{one[27 \times 6]}$, and associated uncertainties. Each column of the eight matrices A_{one} and associated uncertainties, which correspond to the polynomial coefficients of a particular aerodynamic component are grouped together in a sample and analysed. The analysis consists of evaluating the mean and standard uncertainty values of the sample. The estimation is accomplished by employing least squares fit, which in this case can be considered as a weighted mean value estimation.

Applying the fitting process to all columns of A_{one} will provide a single calibration matrix composed of elements which are mean values of the elements of the eight previous calibration matrices. Error matrices corresponding to the associated uncertainties of the mean values will also be supplied. We hope in this process that both matrices can be used to represent the behaviour of the TA-2 aerodynamic balance.

3. RESULTS

Codes using MatLab® and LabVIEW® were written for this study. Although data reduction encompassed all the aerodynamic components, only part of the results is presented in this paper due to the large amount of data. The relevant results will be considered and discussed.

We start by considering the calibration campaign named 2010.4, carried out in 2010. The elements, a_i , and associated uncertainties, u_{a_i} , of the drag force component of the dimensional calibration matrix are shown in Table 1. They were obtained by using the calculation expressed in (4). The first column in Table 1 is related to coefficients estimated as explained in section 2.4 ($u_F = 1$), the second as in section 2.5 ($u_F = S_{fit}$) and the third and fourth columns as in section 2.6 ($u_F = S_{500}$ and $u_F = S_{500}$).

The reason for carrying out studies using different values for u_F is to find the one which results in a better quality fit, *i. e.*, a value must be assigned to the uncertainty in data points and we are looking for which one would result in $\chi_{red}^2 = 1$.

In Table 1, units for a_1 to a_6 are $\text{kgf} \times \text{mV}^{-1}$ for forces and $\text{kgf} \times \text{m} \times \text{mV}^{-1}$ for moments. Units for a_7 to a_{27} are $\text{kgf} \times \text{mV}^{-2}$ and $\text{kgf} \times \text{m} \times \text{mV}^{-2}$, for forces and moments, respectively.

As previously mentioned in section 2.2, the ideal aerodynamic balance would be represented by $F = a_1 R_1$ for the drag force. As expected, the contribution of a_1 is the largest among the 27 coefficients, as shown in Table 1, while the contributions of coefficients a_2 to a_{27} are less important to the mathematical modelling (2). Although not shown in Table 1, an important interaction between R_2 and R_6 was observed for lateral force, and besides b_2 , the value of b_6 was also significant.

A criterion which can be adopted to identify relevant contributions is the ratio between the parameter value and its associated uncertainty. Polynomial coefficients whose values are of the same magnitude, or lower, than the associated uncertainties, can be considered negligible.

Table 1. Polynomial coefficients and associated uncertainties $a_i(u_{a_i})$, for drag force. Calibration 2010.4.

$u_{F_{drag}}=1$	$u_{F_{drag}}=S_{fit}$	$u_{F_{drag}}=S_{500}$	$u_{F_{drag}}=S_{500}$
3.5 1(6)	3.50 6(3)	3.50 6(8)	3.505 9(3)
0.0 1(7)	0.00 9(4)	0.01 0(9)	0.009 7(4)
-0.0 1(8)	-0.00 5(4)	-0.00 5(10)	-0.005 3(4)
0.0 0(3)	0.00 17(19)	0.00 2(5)	0.002 29(22)
0.0 04(27)	0.00 38(16)	0.00 3(4)	0.003 44(19)
0.0 03(25)	0.00 33(15)	0.00 3(4)	0.003 45(16)
0.00 10(27)	0.001 03(15)	0.001 1(4)	0.0010 61(17)
-0.00 0(9)	-0.000 2(5)	-0.00 05(13)	-0.0005 3(6)
-0.00 1(10)	-0.001 1(6)	-0.00 12(11)	-0.0012 4(5)
0.00 0(4)	0.000 00(25)	0.000 1(6)	0.0001 20(29)
0.00 0(4)	0.000 02(21)	-0.000 0(7)	-0.0000 0(3)
-0.00 0(3)	-0.000 01(19)	-0.000 1(4)	-0.0000 93(17)
-0.0 01(19)	-0.00 09(11)	-0.00 17(24)	-0.001 72(11)
-0.0 0(5)	-0.00 01(27)	-0.00 0(6)	-0.000 18(26)
0.0 01(21)	0.00 05(12)	0.00 05(29)	0.000 46(13)
0.0 00(17)	0.000 3(10)	0.00 05(24)	0.000 51(11)
0.0 00(15)	0.000 2(9)	-0.00 02(22)	-0.0001 6(10)
-0.0 01(24)	-0.00 09(14)	-0.00 07(30)	-0.000 68(13)
-0.0 01(23)	-0.00 06(13)	-0.00 13(27)	-0.001 25(12)
-0.0 01(19)	-0.00 06(11)	-0.00 02(25)	-0.000 21(11)
-0.0 00(18)	-0.00 02(10)	-0.00 03(23)	-0.000 31(10)
-0.00 1(5)	-0.000 56(27)	-0.000 4(6)	-0.0004 06(29)
-0.00 0(8)	-0.000 1(5)	-0.00 03(13)	-0.0002 8(6)
0.00 1(8)	0.000 5(5)	0.00 07(10)	0.0006 9(5)
-0.00 0(3)	-0.000 14(18)	-0.000 2(5)	-0.0002 06(21)
0.00 0(6)	0.000 1(4)	0.00 02(10)	0.0002 5(4)
0.00 0(3)	0.000 36(18)	0.000 3(4)	0.0003 39(19)

Notice that uncertainty in data points equal to one (first column) resulted in the largest uncertainty in a_1 . The uncertainty in data points equal to the experimental standard deviation of the mean provided the lowest value (fourth column). It could be inferred from Table 1 that this latter condition, $u_F = S_{500}$, should be applied to data points. But the complete data reduction has yet to be analysed. The final goal is to apply the curve fit to the coefficients obtained in the eight balance calibrations carried out from 2005 to 2010 and to obtain the reduced chi-squared quantity χ_{red}^2 close to unity. For this purpose, we still have to obtain the eight calibration matrices of dimension one, A_{one} , to arrange the polynomial coefficients in samples and to estimate their mean and uncertainty values. An example of such a sample is shown in Fig. 5. The picture shows the display of the LabVIEW® code.

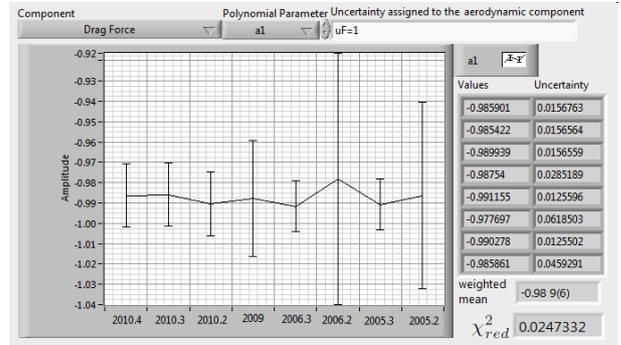


Fig. 5. The output of the LabVIEW® code.

The sample corresponds to the polynomial coefficient a_1 , for the initial condition $u_F = 1$. The x -axis corresponds to the identification of the calibration campaign. The numerical values of a_1 in the y -axis have dimension one. Vectors a_1 and u_{a_1} are shown on the right hand side of the picture. Applying the least squares fitting to this sample will result in the estimation of a single weighted mean value, \bar{a}_1 , whose uncertainty is supplied by the error matrix. This time, the design matrix is a vector $[8 \times 1]$ and the error matrix is $[1 \times 1]$. These values, \bar{a}_1 and the uncertainty, are also presented in Fig. 5.

We see at the bottom right of the picture that the quality of fit for this sample is less than unity, as quantified by the reduced chi-squared, χ_{red}^2 . An extension of the results already shown in Fig. 5 is presented in Table 2. The uncertainty in data points u_F is modified to S_{fit} , S_{500} and S_{500} .

Examining Fig. 5 and Table 2, we conclude that among the four u_F values used, the best condition for χ_{red}^2 was achieved for the experimental standard deviation S_{500} .

Tables 3 and 4 summarize some results of the calibration data analysis. Table 3 presents the quality of the weighted fit for the main parameters a_1 , b_2 , c_3 , d_4 , e_5 and f_6 . One observes that although there is not an ideal value for the reduced chi-squared values, $u_F = S_{500}$ provides the best quality of fit, excluding c_3 . This indicates that the

Table 2. Reduced qui-squared quantity values for different u_F .

year	$a_1(u_{a_1})$		
	$u_{Fdrag} = S_{fit}$	$u_{Fdrag} = S_{500}$	$u_{Fdrag} = S_{500}$
2010.4	-0.985 9(9)	-0.98 58(22)	-0.985 77(10)
2010.3	-0.98 54(11)	-0.98 51(22)	-0.985 06(10)
2010.2	-0.989 9(8)	-0.99 1(9)	-0.991 23(27)
2009	-0.987 5(6)	-0.98 8(7)	-0.988 22(21)
2006.3	-0.991 2(4)	-0.9 92(21)	-0.991 9(7)
2006.2	-0.977 7(6)	-0.97 8(9)	-0.977 84(28)
2005.3	-0.990 3(9)	-0.9 91(21)	-0.991 1(7)
2005.2	-0.985 9(4)	—	—
$\bar{a}_1(u_{\bar{a}_1})$	-0.986 69(22)	-0.98 56(15)	-0.9856 9(6)
χ_{red}^2	57.399	0.266	260.721

Table 3. Quality of fit for a_1, b_2, c_3, d_4, e_5 and f_6 .

$u_F =$	χ_{red}^2			
	1	S_{fit}	S_{500}	S_{500}
\bar{a}_1	0.0247	57.3993	0.2665	260.7212
\bar{b}_2	0.1367	147.3498	0.7150	691.1815
\bar{c}_3	4.5595	777.7519	326.1298	277562.9425
\bar{d}_4	0.0053	18.3514	0.2995	219.7286
\bar{e}_5	0.0061	6.0302	0.4424	292.1783
\bar{f}_6	0.0535	34.3593	4.7907	3866.5414

uncertainty assigned to data points must be reviewed for the lift force component.

The mean values $\bar{a}_1, \bar{b}_2, \bar{c}_3, \bar{d}_4, \bar{e}_5$ and \bar{f}_6 and associated uncertainties are presented in Table 4, for the chosen uncertainty in data points $u_F = S_{500}$. Only part of the 27×6 polynomial coefficients is shown. The criterion for presentation is the ratio parameter/uncertainty greater than 3. We assume that these values will be representative of the external multi-component aerodynamic balance of the subsonic wind tunnel TA-2.

4. CONCLUSIONS

Calibration and error matrices of eight calibrations of the TA-2 wind tunnel external balance were analysed. Data reduction considered different uncertainty values in experimental data points. Mean values and associated uncertainties of the main elements of the dimensionless calibration matrix were supplied. It was shown that these elements presented stability over time, which was revealed by the chi-squared quantity. So far, we have analysed the diagonal elements of the error matrices, which are the variances of the polynomial parameters. Future work to consider the analysis of the covariances, *i. e.*, the off-diagonal elements of the error matrices, is planned.

Table 4. Relevant weighted mean values for force and moment parameters. Unit of dimension one. ($u_F = S_{500}$).

forces	$u_f = S_{500}$	moments	$u_f = S_{500}$
$\bar{a}_1(u_{\bar{a}_1})$	-0.98 56(15)	$\bar{d}_2(u_{\bar{d}_2})$	-0.008 82(27)
$\bar{a}_7(u_{\bar{a}_7})$	0.0000 77(20)	$\bar{d}_3(u_{\bar{d}_3})$	-0.27 31(20)
$\bar{b}_1(u_{\bar{b}_1})$	-0.005 9(9)	$\bar{d}_4(u_{\bar{d}_4})$	6.09 21(17)
$\bar{b}_2(u_{\bar{b}_2})$	2.127 8(4)	$\bar{d}_5(u_{\bar{d}_5})$	-0.005 4(8)
$\bar{b}_5(u_{\bar{b}_5})$	-0.00 40(11)	$\bar{d}_6(u_{\bar{d}_6})$	0.011 7(7)
$\bar{b}_6(u_{\bar{b}_6})$	2.15 07(10)	$\bar{d}_{12}(u_{\bar{d}_{12}})$	-0.0004 01(23)
$\bar{b}_{27}(u_{\bar{b}_{27}})$	-0.0002 6(8)	$\bar{d}_{14}(u_{\bar{d}_{14}})$	0.000 86(12)
$\bar{c}_1(u_{\bar{c}_1})$	-0.008 2(7)	$\bar{d}_{19}(u_{\bar{d}_{19}})$	0.003 7(7)
$\bar{c}_2(u_{\bar{c}_2})$	-0.014 1(3)	$\bar{d}_{21}(u_{\bar{d}_{21}})$	0.000 79(26)
$\bar{c}_3(u_{\bar{c}_3})$	26.36 20(25)	$\bar{e}_1(u_{\bar{e}_1})$	0.003 5(6)
$\bar{c}_4(u_{\bar{c}_4})$	-0.01 07(22)	$\bar{e}_2(u_{\bar{e}_2})$	-0.027 58(24)
$\bar{c}_5(u_{\bar{c}_5})$	-0.00 40(10)	$\bar{e}_3(u_{\bar{e}_3})$	-0.04 77(18)
$\bar{c}_6(u_{\bar{c}_6})$	-0.02 06(10)	$\bar{e}_4(u_{\bar{e}_4})$	-0.00 98(15)
$\bar{c}_{13}(u_{\bar{c}_{13}})$	0.00010 1(8)	$\bar{e}_5(u_{\bar{e}_5})$	-3.063 4(7)
$\bar{c}_{17}(u_{\bar{c}_{17}})$	0.0002 0(5)	$\bar{e}_6(u_{\bar{e}_6})$	-0.008 1(7)
$\bar{c}_{22}(u_{\bar{c}_{22}})$	0.002 8(3)	$\bar{e}_9(u_{\bar{e}_9})$	-0.0003 8(7)
$\bar{c}_{25}(u_{\bar{c}_{25}})$	0.0003 0(6)	$\bar{e}_{11}(u_{\bar{e}_{11}})$	0.0000 91(23)
$\bar{c}_{27}(u_{\bar{c}_{27}})$	0.0003 1(7)	$\bar{e}_{17}(u_{\bar{e}_{17}})$	-0.0007 0(4)
—	—	$\bar{e}_{20}(u_{\bar{e}_{20}})$	-0.002 15(23)
—	—	$\bar{e}_{24}(u_{\bar{e}_{24}})$	0.001 30(20)
—	—	$\bar{e}_{27}(u_{\bar{e}_{27}})$	-0.0006 9(5)
—	—	$\bar{f}_1(u_{\bar{f}_1})$	0.003 0(7)
—	—	$\bar{f}_2(u_{\bar{f}_2})$	-0.019 2(3)
—	—	$\bar{f}_3(u_{\bar{f}_3})$	0.01 42(24)
—	—	$\bar{f}_6(u_{\bar{f}_6})$	-3.053 5(8)
—	—	$\bar{f}_{10}(u_{\bar{f}_{10}})$	0.0002 1(6)
—	—	$\bar{f}_{12}(u_{\bar{f}_{12}})$	0.0002 45(24)
—	—	$\bar{f}_{13}(u_{\bar{f}_{13}})$	0.00004 3(7)
—	—	$\bar{f}_{16}(u_{\bar{f}_{16}})$	-0.0001 8(4)

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