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SLIP-FLOW IN SONIC NOZZLES FOR FLOW RATE MEASUREMENTS

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Abstract **–** Sonic nozzles are wildly employed as a flow standard owing to their advantages in mechanical simplicity and long-term stability. The flow behaviours through the nozzles in normal conditions are well understood. However, in the applications where the size and the operating pressure are very small, the continuum assumption begins to break. Slip-flow, which is beyond the scope of ISO9300, becomes dominant. The scope of this study is to validate the method used to predict the effects in the nozzle.

Keywords: micro sonic nozzle, slip flow, rarefied gas

1. INTRODUCTION

The micro-scale size of a sonic nozzle as well as a very low operating pressure will reduce the large number of gas molecules and cause the gas to be rarefied. Consequently, due to the small number of gas molecules, the flow behaviours will be different from those of general gas, where the number of gas molecules is large enough to consider the gas as a continuum medium. The continuum medium assumption is not valid for the aforementioned cases if the flow behaves as slip, transition, or free molecular flow. The throat hydraulic diameter, D_H and throat pressure, p, are the two main factors of Knudsen number, Kn that characterize the flow regime through a sonic nozzle. The Knudsen number is defined as:

$$\mathrm{Kn} = \frac{\lambda}{D_H} \qquad (1)$$

where the mean free path, λ is a function of the pressure, p, the viscosity, μ and the most probable molecular velocity, $\tilde{v}$. Therefore, the molecular mean free path, which is estimated using Maxwellian theory is defined as:

$$\lambda = \frac{\sqrt{\pi}}{2}\frac{\mu\tilde{v}}{p} \qquad (2)$$

where the most probable molecular velocity is the function of the specific gas constant, R and the temperature, T as $\tilde{v} = \sqrt{2RT}$. From equation (1) and (2), Knudsen number could be written in term of the isentropic component, κ, the Mach number, M and the Reynolds number, Re as:

$$\mathrm{Kn} = \sqrt{\frac{\pi}{2}}\sqrt{\kappa}\,\frac{\mathrm{M}}{\mathrm{Re}}\,. \qquad (3)$$

With respect to the value of the Knudsen number, there are four distinct regimes as shown in Table 1.

Kn = 10 | *Kn = 0.1* | *Kn = 0.01* | *Kn → 0*

Free molecular regime	Transition regime	Slip-flow regime	Continuum regime	
			Viscous	Inviscid
Boltzman Equation without collisions	Boltzman Equation	Navier-Stokes + *slip BC.*	Navier-Stokes	Euler

Table 1. Classification of gas flow regime.

When Kn is very small, there are enough molecules for the gas to be considered as in a continuum regime. Slip-flow and other effects, such as temperature jump at a solid surface, start to appear at values of Kn greater than 0.001 and become dominant at around 0.01, where the slip-flow regime begins. As the gas becomes more and more rarefied, its flow is characterized as being in the transition and free molecular regimes, when Kn reaches 0.1 and 10, respectively. In order to predict the gas behaviour accurately, it is necessary to know its flow regime. Using incorrect assumptions can lead to large errors.

As well as the Knudsen number, the rarefaction parameter, δ is another quantity that is also used to describe the flow regime and is defined as:

$$\delta = \frac{\sqrt{\pi}}{2}\frac{1}{\mathrm{Kn}} = \frac{1}{\sqrt{2\kappa}}\frac{\mathrm{Re}}{\mathrm{M}}\,. \qquad (4)$$

From (3), Knudsen number of a flow of diatomic gas through a choked nozzle is plotted as a function of Reynolds number and shown in Fig. 1. In general, the Re of a flow in the slip flow regime is between 10^1 and 10^2.

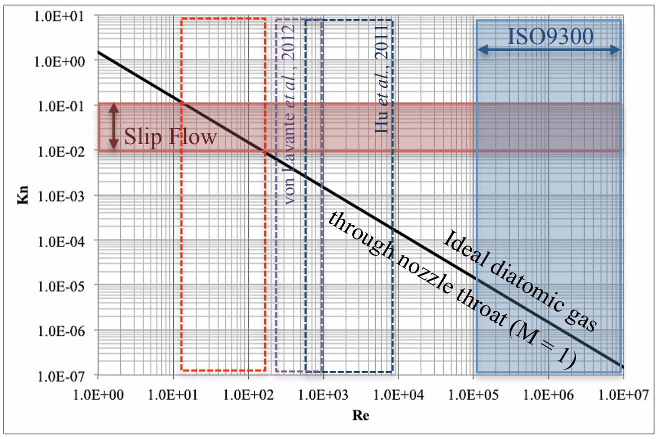

Fig. 1. Reynolds number versus Knudsen number of an ideal diatomic gas through a choked-flow nozzle throat.

The flow characteristics through a small-size sonic nozzle were previously investigated for low flow rate measurements by Hu *et al.* and E. von Lavante *et al.* [1][2]. The Reynolds number ranges of both researches, which are smaller than the scope of ISO9300 (10^5 to 10^7), are shown in Fig. 1. The corresponding Knudsen numbers for both ranges are below those of the slip-flow range in which continuum assumption is still applicable. However, it is clear that if the Reynolds number is smaller than those of von Lavante *et al.* the flow will enter the slip-flow regime, which is the investigation scope of the present work.

2. SLIP-FLOW

The slip-flow regime is a slightly rarefied one, which could occur either in a gas flow through a small passage or through a very low-pressure condition. It is typically corresponds to a Knudsen number raging between 0.01 and 0.1 (δ between 8.9 and 89), easily reached for flow either through a micrometer scale nozzle throat or in nozzle that operate in a rough vacuum condition. The Knudsen layer plays a fundamental role in the slip-flow regime. This thin layer, one or two molecular mean free paths in thickness, is a region of local non-equilibrium, observed in any gas flow near a surface. In non-rarefied flow, the Knudsen layer is too thin to have any significant influence but, in the slip-flow regime, it needs to be considered [3].

Although the Navier-Stokes equations are not valid in the Knudsen layer, due to non-linear stress/strain-rate behaviour within it [4], their use with appropriate boundary velocity slip and temperature jump conditions can provide an accurate prediction of mass flow rate [5]. The slip flow velocity at the passage wall was originally proposed by Maxwell as:

$$u_{slip} = \frac{2-\alpha}{\alpha}\lambda\left[\frac{\partial u}{\partial n} - \frac{3}{2}\frac{\mu}{\rho T}\frac{\partial^2 T}{\partial s\partial n}\right]_{wall} + \frac{3}{4}\left[\frac{\mu}{\rho T}\frac{\partial T}{\partial s}\right]_{wall} \quad (5)$$

where s and n denote the tangential and the normal directions to the wall. The tangential momentum accommodation coefficient, α is unity for perfect diffuse reflection and is zero for purely specular reflection of molecules with the wall. If isothermal flow is assumed, the previous equation, (5), could be reduced to:

$$u_{slip}\Big|_{wall} = u_{gas} - u_{wall} = \frac{2-\alpha}{\alpha}\lambda\frac{\partial u}{\partial n}\Big|_{wall} \quad (6)$$

where u_{gas} is the gas velocity and u_{wall} is the wall velocity.

The validity of Maxwell slip-flow model was verified by the first author in his previous literature [5]. Fig. 2 presents the comparison of Maxwell slip flow model with kinetic model and the experiments in the form of reduced flow rate (G*) through a rectangular microchannel versus rarefaction parameter (δ_0). The kinetic model bases on the linearized BGK method. The reduced flow rate is numerically solved by the discrete velocity method (DVM). The experimental data are obtained by Pitakarnnop *et al.* with constant-volume flow rate measurement method [5] and by Lalonde with constant-pressure flow rate measurement method [6].

From Fig. 2, Maxwell model underestimates the results of slip flow in the entire regime. However, in the beginning of the slip-flow regime, the prediction of Maxwell model is still within the uncertainty of the measurements.

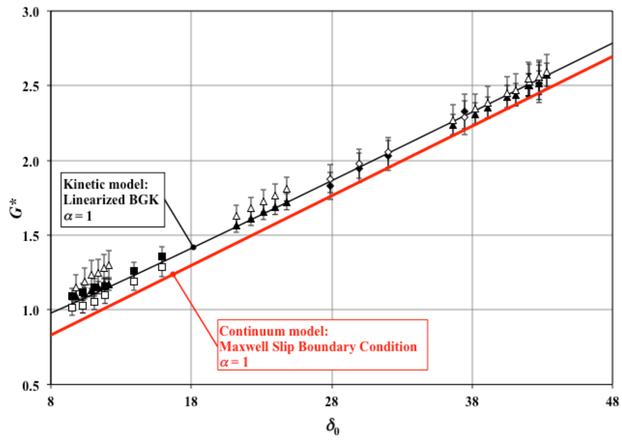

Fig. 2. Comparison of Maxwell slip flow model with the kinetic model and the experiments in the form of reduced flow rate versus rarefaction factor. Experimental data obtained by Pitakarnnop *et al.* with the constant volume measurement (square and diamond) [5] and Lalonde (triangle) with the constant pressure measurement [6].

3. PROBLEM STATEMENT

Two different axisymmetric micro-nozzles are considered. The configurations in geometry and flow conditions follow E. von Lavante *et al.* [2] and Alexeenko et *al.* [7] for the studies in continuum flow regime and slip-flow regime respectively. The flows through both micro-nozzles are investigated using ANSYS FLUENT, which is well-known commercial CFD software. FLUENT is capable to determine the flow from laminar to turbulent as well as viscous and inviscid flows. In addition, it also gives an additional option for the "Low Pressure Boundary Slip (LPBS)" condition. Since the flow of the investigated nozzle is in the low Reynolds number range, it is clear that the flow is laminar with high viscous effects. LPBS option, which is based on Maxwell's model, is enable to determine the slip-flow effect at the nozzle walls.

LPBS uses different model to calculate the value of mean free path, which slightly results the deviations in the Kn, the velocity profile and the flow rate comparing to those previously proposed. To avoid the confusion, the following mean free path model is used for all calculations:

$$\lambda = \frac{k_B T}{\sqrt{2}\pi\,\sigma_{LJ}^2\,p} \quad (11)$$

where k_B and σ_{LJ} are the Boltzmann constant and the Lennard-Jones characteristic length respectively.

LPBS was found to be inaccurate for predicting slip near the sharp and the wide corners of triangular and trapezoidal cross sections [3]. However, the anomaly due to discontinuity of the section at the corner is not found in the axisymmetric flow simulation as shown in Fig. 3.

The Maxwell slip-flow model is accurate enough to use in the low Kn (early slip-flow regime). Along with the LPBS in FLUENT, they are the excellent tools to simulate the slip-flow through a complex axisymmetric geometry. Therefore, they are selected to be employed in the present work to predict the flow through a critical nozzle.

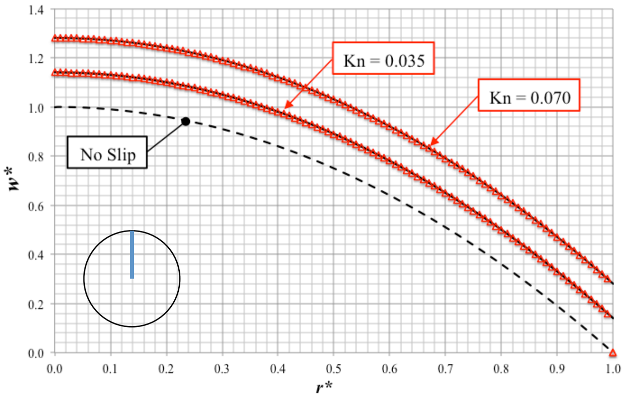

Fig. 3. Dimensionless velocity along the radius of a tube; comparison between no-slip result (--), analytical result for slip-flow (–) and LPBS (Δ) for Kn = 0.035 and 0.070.

4.1. Continuum flow through micro-nozzle

The punched shape micro-nozzle with a throat diameter of 25 µm and an expansion angle of 17° is taken from E. von Lavante *et al.* [2]. The geometry of the nozzle is shown in Fig. 4. The flow domain including the inlet reservoir has 10900 quadrilateral mesh cells.

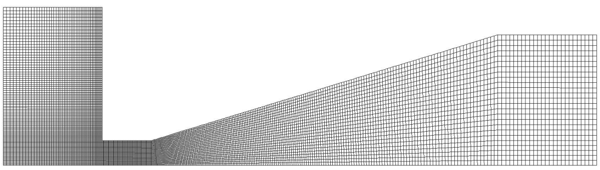

Fig. 4. Geometry and mesh of the investigated micro nozzle in continuum flow regime [2].

Only the flow in forward direction (from left to right) is investigated. The inflow is air at atmospheric conditions. The flow is determined for two outlet-to-inlet pressure ratio: 0.4 and 0.3. The regime of flow through the nozzle is remained within the continuum flow regime. As shown in Fig. 5, the Knudsen number at the throat and the outlet are less than 0.005.

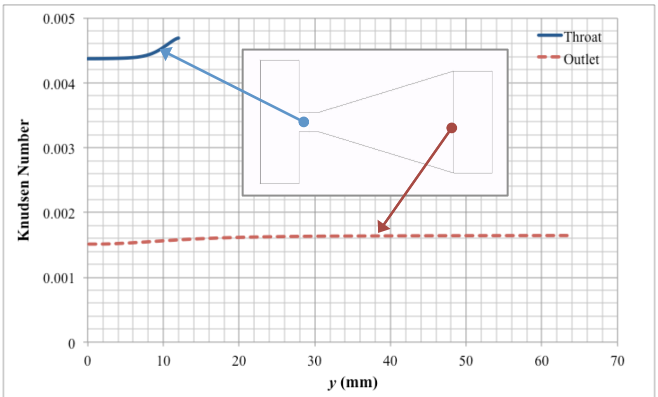

Fig. 5. Knudsen number plots at the throat (–), and the outlet (--) of the punched shape nozzle for the outlet-to-inlet pressure ratio equal to 0.3.

4.1. Continuum flow through micro-nozzle

The axisymmetric de Laval nozzle which has a throat diameter of 300 µm, a converging angle of 35°, a diverging angle of 15°, and an exit to throat area ratio of 100 is taken from Alexeenko et *al.* [7]. The geometry of the nozzle is shown in Fig. 6. The flow domain including the outlet reservoir has 6900 quadrilateral mesh cells.

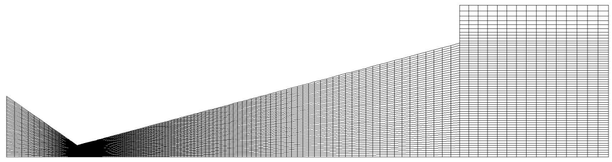

Fig. 6. Geometry and mesh of the investigated micro nozzle in slip flow regime [7].

The inflow nitrogen has a stagnation pressure and stagnation temperature of 10 kPa and 300 K respectively. Fig. 7 shows the plot of Knudsen number along the throat and the outlet of the nozzle. As in the previous case, the regime of flow at the throat is continuum as Kn is less than 1. However, due to rapid expansion of flow to vacuum at the outlet, the flow is in the slip-flow regime as Kn is between 0.01 and 0.1.

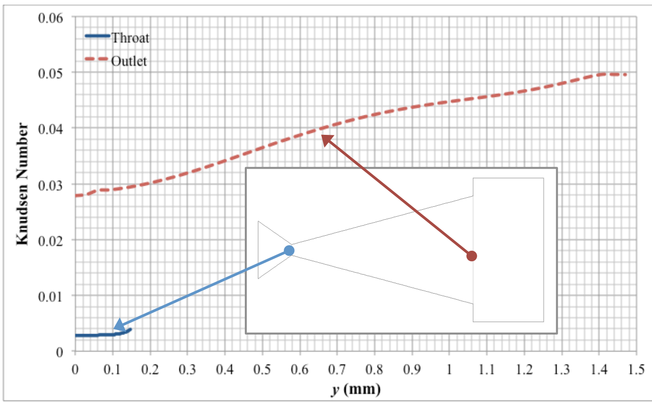

Fig. 7. Knudsen number plots at the throat (–), and the outlet (--) of the de Laval nozzle.

4. RESULTS AND DISCUSSION

To investigate gas flow through micro-nozzle, the numerical results from Ansys FLUENT are compared with those of the experiment from von Lavante et al. [2] and of the validated DSMC from Alexeenko et al. [7] in the continuum flow regime and slip-flow regime respectively.

4.1. Continuum flow through micro-nozzle

The simulation results of the laminar and the turbulent flow through a 25 micrometer diameter punched-shape sonic nozzle are compared with the experimental results from von Lavante et al. [2] and presented in table 2 and table 3 for the conditions where the outlet to the inlet pressure ratio are 0.4 and 0.3 respectively. The flow results are presented in term of the discharge coefficient, C_D.

Table 2. Discharge coefficient (C_D) when p_{out}/p_{in} = 0.4.

p_{out}/p_{in} = 0.4	$\dot{m}_{ideal}$ (kg/s)	$\dot{m}_{CFD}$ (kg/s)	C_D
FW - CFD - Laminar	117.5	83.466	0.710
FW - CFD - Slip	117.5	85.074	0.724
FW - CFD - Turbulent - SA	117.5	78.679	0.670
FW - CFD - Turbulent - k-ε	117.5	78.139	0.665
von Lavante *et al.*, 2012 - CFD	117.4	81.28	0.692
von Lavante *et al.*, 2012 - Experiment			0.662

Table 3. Discharge coefficient (C_D) when p_{out}/p_{in} = 0.3.

p_{out}/p_{in} = 0.3	$\dot{m}_{ideal}$ (kg/s)	$\dot{m}_{CFD}$ (kg/s)	C_D
FW - CFD - Laminar	117.5	83.611	0.712
FW - CFD - Slip	117.5	85.247	0.726
FW - CFD - Turbulent - SA	117.5	78.825	0.671
FW - CFD - Turbulent - k-ε	117.5	78.212	0.666
von Lavante *et al.*, 2012 - CFD	117.4	83.53	0.711
von Lavante *et al.*, 2012 - Experiment			0.664

From above tables, for the same nozzle geometry and flow conditions, the results of the method using in the investigation are close to those of von Lavante *et al.* The results from Laminar flow are close to the CFD simulations. The results from Spalart-Allmaras and k-epsilon turbulent flow models are very close to those from the experiments. However, the viscous effect is dominant for a gas in the slip-flow regime. Therefore, the validated laminar method is used to determine all the slip-flow through a nozzle in the present work.

4.2. Slip flow through micro-nozzle

The slip-flow through an axisymmetric conical nozzle is studied. The results are compared with those of the Direct Simulation Monte Carlo (DSMC) Method. The DSMC results are obtained from Alexeenko *et al.*. The flow conditions and the nozzle geometry are available in the literature [7]. The adiabatic condition along with the tangential momentum accommodation coefficient equals to 0.8 is set at the nozzle wall for the slip-flow simulation. The comparison of velocity profiles along the nozzle axis presented in Fig. 8 shows only a small difference between the solutions of slip-flow model (Δ) and DSMC (–). The results are in an excellent agreement from x = 0.5 until the exit of the nozzle. Near the nozzle throat, the continuum approach without the slip-flow boundary condition (o) gives almost identical velocity along the axis to those obtained from the slip-flow model. However, without slip-flow model, the results are underestimated at the nozzle exit where the gas is rarefied.

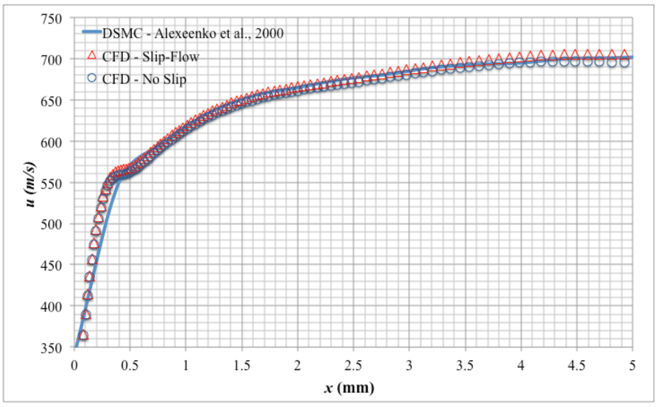

Fig. 8. Velocity along the symmetry axis (y = 0); comparison between DSMC (–) [7], Slip-Flow (Δ) and No Slip (o).

Fig. 9 shows the comparison of a velocity profile at the exit plane of the nozzle. The comparison of slip-flow model and DSMC confirms an excellent agreement of velocity profiles near the axis. However, Maxwell's first order slip boundary condition underestimates the velocity near the lip, as the continuum approach is inaccurate inside the Knudsen layer.

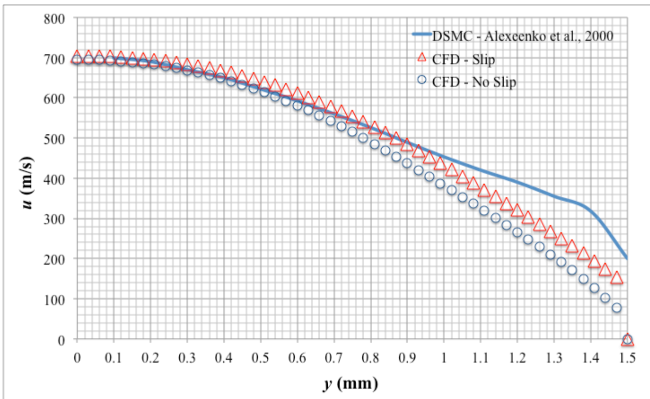

Fig. 9. Velocity profile at the nozzle exit (x = 5.038 mm); comparison between DSMC (–) [7], Slip-Flow (Δ) and No Slip (o).

6. CONCLUSIONS AND PERSPECTIVES

The investigation shows that the continuum assumption with an additional boundary condition is suitable to calculate high Knudsen low Reynolds number of a critical flow though an axisymmetric nozzle in the slip-flow regime. The first order Maxwell boundary condition is employed in the present work. However, for a flow through a passage with sharp or wide corner cross section, the boundary condition must be used with care in CFD. Due to discontinuity of flow at the corner, some deviation could be found. In addition, the first order slip boundary condition underestimates the slip velocity near nozzle wall. Even though, the issue causes only a small deviation in an overall flow but, in some applications, it is needed to be considered carefully.

The investigated method could be further used in the design of the efficient micro-nozzle and/or nozzle in low operating pressure.

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