ANALYTICAL STRESS CALCULATION IN SPHERICAL TANK AND EXPERIMENTAL VERIFICATION

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Summary: *This paper presents analytical calculation of stresses in spherical tank which was used during designing of tank, and experimental verification of calculated stresses after the tank was produced. Derivation of equilibrium equations for the shell element is presented as well as procedure for determination of membrane state of stress. Experimental verification of tank is described. Experiments showed concentration of stress at the points of spherical tank support. Maximum equivalent stress exceeds yield stress for material out of which tank was built, but the area with exceeded stress is very small so it can be neglected. Therefore it is not necessary to reinforce the spherical tank at the points of support.*

Keywords: *spherical tank, experimental testing, analytical calculation*

1 Introduction

The spherical tank belongs to the group of stable elevated tanks designed for storing butane, propane or mixture of propane-butane with medium pressure [1]. It is loaded with the fluid pressure, hydrostatic pressure and forces that arise due to its own weight. The tank operates in moderate climate conditions, so temperature influence can be neglected. The expressions for stresses presented in this paper are practical for tank optimization used in the oil and gas industry, because in the initial phase of design we can get the basic dimensions of the spherical tank in a relatively short period of time [2].

2 Analytical Procedure

The stress state in elements of the shell in the form of surface of revolution can be determined using the equilibrium equations for the shell elements. Figure 1a shows the position of the shell element and Figure 1b shows the internal forces in that element [3]. According to [3], equilibrium equations for the shell element are given with:

$$\frac{\partial N_\theta}{\partial \theta} r_1 + N_{\theta\varphi} r_1 \cos\varphi + \frac{\partial (N_{\varphi\theta} r_0)}{\partial \varphi} + X r_0 r_1 = 0 \tag{1}$$

$$\frac{\partial N_{\theta\varphi}}{\partial \theta} r_1 - N_\theta r_1 \cos\varphi + \frac{\partial (N_{\varphi} r_0)}{\partial \varphi} + Y r_0 r_1 = 0 \tag{2}$$

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$$\frac{N_\theta}{r_2} + \frac{N_\varphi}{r_1} = -Z \tag{3}$$

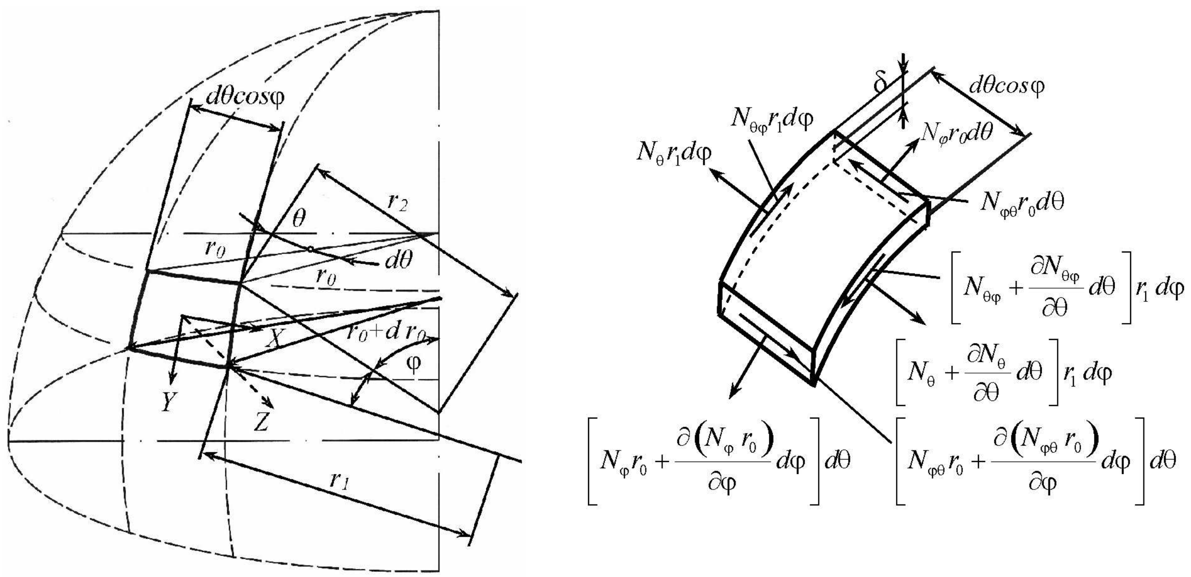

Figure 1: a) Position of the shell element in the form of surface of revolution, b) Internal forces in the element of the shell in the form of surface of revolution

In the membrane state of stress we can assume that the stresses acting parallel to the middle surface are uniformly distributed along the edges of the shell element $N_{\theta\varphi} = N_{\varphi\theta}$. For spherical tank $r_1 = r_2 = R$. Equilibrium equation for shell section is given by $2\pi r_0 N_\varphi \sin\varphi + Q = 0$. Since $r_0 = R\sin\varphi$ the previous equation becomes $2\pi R N_\varphi \sin^2\varphi + Q = 0$, where Q is resultant of the external loading.

2.1 Own weight loading

The load on the spherical tank due to its own weight is shown in Figure 2. Internal forces due to tanks own weight above the supports are:

$$N_\varphi = -\frac{gR}{1+\cos\varphi} \text{ and } N_\theta = -gR\left(\cos\varphi - \frac{1}{1+\cos\varphi}\right) \tag{4}$$

Internal forces due to tanks own weight below the supports are:

$$N_\varphi = -\frac{gR}{1-\cos\varphi} \text{ and } N_\theta = -gR\left(\cos\varphi - \frac{1}{1-\cos\varphi}\right) \tag{5}$$

2.2 Internal pressure loading

The spherical tank supported along a parallel circle B-B (Fig. 2) and filled with liquid which has the specific weight γ is loaded with pressure $p = -Z = \gamma R(1-\cos\varphi) + p_g$, where p_g represents the uniformed pressure superposed on hydrostatic pressure [3]. Internal forces due to hydrostatic pressure and internal gas pressure above the supports are:

$$N_\varphi = \frac{\gamma R^2}{6}\left[1 - \frac{2\cos^2\varphi}{1+\cos\varphi}\right] + \frac{Rp_g}{2} \text{ and } N_\theta = \frac{\gamma R^2}{6}\left[5 - 6\cos\varphi + \frac{2\cos^2\varphi}{1+\cos\varphi}\right] + \frac{Rp_g}{2} \tag{6}$$

Internal forces due to hydrostatic pressure and internal gas pressure below the supports are:

$$N_\varphi = \frac{\gamma R^2}{6}\left[5 + \frac{2\cos^2\varphi}{1-\cos\varphi}\right] + \frac{Rp_g}{2} \text{ and } N_\theta = \frac{\gamma R^2}{6}\left[1 - 6\cos\varphi + \frac{2\cos^2\varphi}{1-\cos\varphi}\right] + \frac{Rp_g}{2} \tag{7}$$

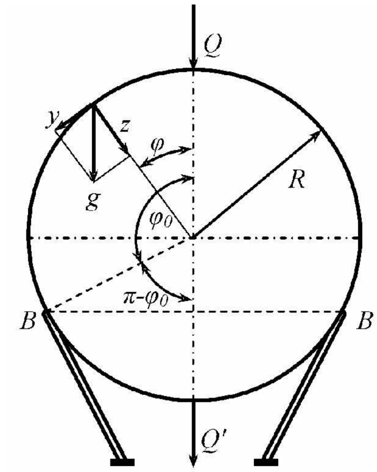

Figure 2: Own weight loading of the spherical tank

3 Experimental verification

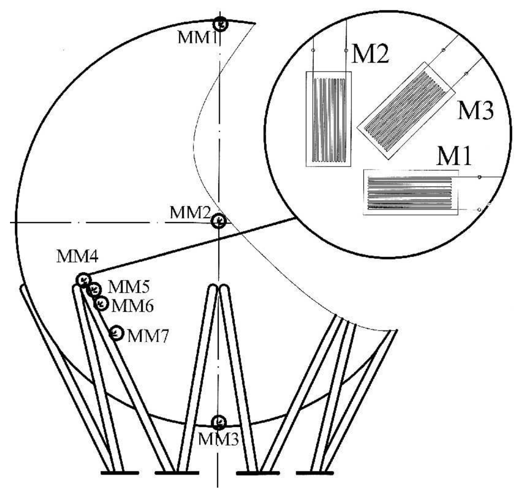

Figure 3: Layout of the measuring pointsk

Verification of theoretical analysis was performed by measuring stresses at 7 measuring points with 21 strain gauges. The layout of the measuring points is shown in Figure 3. This type of strain gauges setup makes measuring easier and does not require knowledge of the direction of the principal stresses propagation.

4 Results

The comparative values of stresses, for the operating pressure of $1.67\,\mathrm{MPa}$ and for the test pressure of $2.5\,\mathrm{MPa}$, obtained analytically at the characteristic points, and experimentally, are presented in Table 1.

5 Conclusion

The carried out research showed the high level of correspondence of the results obtained analytically with the results of experimental testing of spherical tank. This correspondence of the results allows analytical expressions to be used for dimensioning spherical tanks. In the case of analyzed tank for the pressure of

Table 1: Obtained stress for $1.67\,\mathrm{MPa}$ and $2.5\,\mathrm{MPa}$ pressure

Measuring site	$\sigma_e\,(\mathrm{N/mm^2})$ Analytical	$\sigma_e\,(\mathrm{N/mm^2})$ Experiment	$\sigma_e\,(\mathrm{N/mm^2})$ Analytical	$\sigma_e\,(\mathrm{N/mm^2})$ Experiment
MM1	185.3	169.6	277.4	256.9
MM2	192.4	175.6	284.4	262.9
MM3	199.1	182.3	291.2	272.9
MM4	-	343.1	-	483.4
MM5	194.6	201	287.8	282.3
MM6	195.7	180.1	289.6	257.3
MM7	196.6	182.3	290.8	261.6

$2.5\,\mathrm{MPa}$ the equivalent stress exceeded yield stress at the points of tank connection with the supports. Since this high stress is concentrated in the small region of the connection area we draw a conclusion that this value of stress is not critical and that construction of spherical tank can proceed without need to reinforce the spherical tank at the points of its connection with the supports.

6 Acknowledgements

The part of this research is supported by Ministry of Education and Science, Republic of Serbia, Grant TR 32036.

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