FATIGUE PREDICTION BY MULTIAXIAL CRITERIA IN THE REGION OF LIMITED LIFETIME

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Summary: *The paper describes validation of selected multiaxial fatigue criteria in the region of limited lifetime. Used test data and methodology of validation are described. Results of validation are presented.*

Keywords: *fatigue of metals, multiaxial methods, finite life, fatigue strength*

1 Introduction

Many stress-based multiaxial criteria were proposed for high-cycle fatigue (HCF) prediction. When investigating fatigue behavior of materials it is often difficult to say, which criteria should be used for which fatigue life domain. The concept of S-N curves based on the assumption of the elastic response is often used even in low-cycle fatigue (LCF) domain, where it is not possible to guarantee that no plastic strain occur. FKM analysis guide [5] thus accepts the prediction by S-N curves from the HCF domain till 10.000 cycles, i.e. at least touching the LCF domain. Our team has not found any proof that the multiaxial prediction based on the S-N curve provides adequate prediction results. This paper therefore describes results of predictions realized at substantially lower lifetimes than the typical HCF domain is. Not only the relation between the number of cycles and quality of the results of multiaxial criteria was studied but also the influence of the type of material and of the loading type was analyzed.

2 Test data

Validation was realized on experimental data obtained from tests on smooth specimens to suppress the notch effect. All specimens were loaded by a combination of axial (biaxial in some cases) and torsion channels with sinusoidal or constant signals. The phase shift on involved load channels can vary. A high number of experimental results are required for an adequate validation. It was necessary to process whole fatigue curves in order to get fatigue test data also for lower lifetimes than the usual fatigue limits processed e.g. in [2] or [3]. The final test batch contains 105 fatigue curves for multiaxial or uniaxial loading with or without mean stress value. 88 curves relate to ferrous materials, while 17 curves concern aluminum alloys. In total, 495 data points (combinations of local stresses at an evaluated number of cycles) were gathered in the test batch. Detailed description of particular fatigue curves can be found in [6]. To realize the evaluation of the prediction quality, it was necessary to know not only the complex load combinations, but also so-called material data, which relate to stress values at examined number of cycles for fully reversed and fully repeated axial and torsion loadings. If the examined experiments were not affected by mean stress, we do not need the data for repeated loadings. It is very rare to find fatigue data for repeated torsion. The only criteria, which need experimental data in this load case are Papuga's PCr and PQ criteria [3]. Papuga suggests to replace this experimental input (if absent) by calculating its estimated value according to formula proposed by Liu et al in [8]. This proposal was accepted here.

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3 Type of evaluation

The evaluation process commonly used in the HCF domain was used also for fatigue data for medium-cycle fatigue (MCF) or low-cycle fatigue (LCF). All criteria can be formulated in the form, where the left hand side represents the damage parameter (a composition of weighted local stresses), while a material parameter (the fatigue strength in fully reversed axial loading at evaluated number of cycles) is located on the right side.

$$a \cdot f(C) + b \cdot g(N) \leq f_{-1} \; (LHS \leq RHS) \tag{1}$$

The material parameters a and b express, how the particular criterion balances the damage effect caused by shear stress C and normal stress N. If the left side is lower than the right side of the criteria, the component would withstand the applied load. Because we deal with experimentally set load combinations for the examined numbers of cycles, the right side should be equal to the left side of the particular criteria. Such situation happens quite rarely because neither the experimental data nor the criteria are ideal. Therefore, the fatigue index error is defined to be

$$\Delta FI = \left(\frac{LHS(load) - RHS(material)}{RHS(material)} \right) \cdot 100\,\% \tag{2}$$

Values of the fatigue index error below zero mean, that the criterion does not predict failure, although the examined specimen actually broke, which is non-conservative behaviour. Values higher than zero correspond to conservative prediction. Results of ΔFI for all investigated methods are described by three basic statistic values- mean value, standard deviation (S.D.) and range of results (i.e. max-min).

4 Processing of input data

First, the raw test data were retrieved from literature experiment by experiment and then fatigue curves were prepared using the Kohout-Věchet regression [1]. For particular numbers of cycles, the stress values were deduced from fatigue curves as local loads (for LHS of Eq. (1)) or material property (its RHS). Fatigue index errors were calculated for several multiaxial methods using PragTic freeware fatigue solver [4]. As described in [2], results for experiments loaded with negative mean stress are over-conservative, so these load cases were excluded from the evaluation in this paper.

It was found during the evaluation of results of particular methods, that one group of experimental data for aluminum alloys gives results very different from other results [6]. There are only few experimental sources available for aluminum alloys. It was not thus possible to decide, if there is a problem related to this specific experimental data set, or it is a typical behavior of aluminum alloys. Experimental data for ferrous alloys therefore are treated here only.

5 Methods and test results

Altogether six experimental methods were examined in the area of limited lifetime. These are Dang Van criterion (D), Findley criterion (F), McDiarmid maximum shear stress range criterion (M), Crossland criterion (C) (see all in [2]), and two Papuga's criteria PCr and PQ [3]. Results of fatigue index error of particular methods were sorted according to the load combination acting during fatigue experiments, according to the material of specimens and the number of cycles to failure. Because there is not enough space to present all results here, only main characteristics of behavior of particular criteria will be described.

6 Test results and their analysis

Results of experimental data were first classified according to the number of cycles to failure into three groups (Table 1). It is evident that in general, ranges of ΔFI for MCF are not dramatically different compared to HCF region, which is the most important conclusion for practical use of investigated methods. Approaching the LCF domain, we see that the values of ranges and standard deviations in this domain are for the best three evaluated methods (D, PQ, PCr) lower than the ranges and standard deviations of Findley, McDiarmid and Crossland methods in the region of HCF. Therefore, only results of three best criteria will be described further.

Table 1: Fatigue results for ferrous materials

	HCF (128 experimental data)						MCF (129 experimental data)						LCF (120 experimental data)					
Cycles	200,000 ≤ nr. of cycles. ≤ 10,000,000						50,000 ≤ nr. of cycles ≤ 200,000						< 50,000					
ΔFI [%]	D	F	M	C	PCr	PQ	D	F	M	C	PCr	PQ	D	F	M	C	PCr	PQ
Mean	-0.2	6.8	-6.2	-6.6	1.8	-0.4	-4.0	3.8	-7.4	-6.9	2.9	0.9	-3.6	1.2	-1.9	-4.0	4.4	3.0
S.D.	8.5	14.3	13.2	11.6	3.7	5.4	10.2	13.7	13.8	13.2	4.6	6.6	9.7	9.5	13.6	14.8	5.1	8.9
Range	48	71	56	53	25	36	48	70	55	54	28	38	40	45	52	57	21	39

Table 2: Fatigue results for ferrous materials - mean values of ΔFI for D, PCr and PQ criterion

	HCF				MCF				LCF			
Loading groups	N. of data	D	PCr	PQ	N. of data	D	PCr	PQ	N. of data	D	PCr	PQ
nMS, P	70	1.9	2.2	0.2	37	3.5	3.0	0.7	18	5.9	4.6	2.2
MS, NP	34	-2.3	1.2	-2.3	53	-6.5	3.8	0.8	82	-3.9	5.6	4.2
MS, uniax	15	-3.2	-0.2	-0.9	23	-8.7	-0.1	-0.6	20	-10.9	-0.6	-1.2
FA, ductile	94	-1.1	1.8	-1.1	106	-5.1	2.9	0.4	85	-4.6	4.4	2.3

nMS, loading without mean stress; MS, loading with mean stress; P, proportional loading; NP, non-proportional loading; uniax, uniaxial loading; FA, ductile, group of results for ferrous ductile materials

Table 3: Fatigue results for ferrous materials - values of standard deviations of ΔFI for D, PCr and PQ criterion

	HCF				MCF				LCF			
Loading groups	N. of data	D	PCr	PQ	N. of data	D	PCr	PQ	N. of data	D	PCr	PQ
nMS, P	70	3.6	2.8	3.4	37	3.4	2.6	3.1	18	4.1	3.6	3.6
MS, NP	34	11.9	4.7	7.5	53	10.4	4.9	8.1	82	9.9	5.3	10.3
S, uniax	15	9.8	1.6	4.0	23	8.2	1.2	2.9	20	3.8	0.8	1.8
FA, ductile	94	9.1	3.9	5.6	106	10.6	4.5	6.5	85	9.5	5.3	9.3

nMS, loading without mean stress; MS, loading with mean stress; P, proportional loading; NP, non-proportional loading; uniax, uniaxial loading; FA, ductile, group of results for ferrous ductile materials

Mean values of fatigue index error closest to optimum zero were obtained for Dang Van, PCr and PQ methods. Dang Van criterion is implemented in several commercial fatigue solvers, but better results provide criteria suggested by Papuga [3], the PCr and PQ criterion, because the standard deviations and ranges are significantly greater for the Dang Van method than for the PCr and PQ method. Mean values of results of the Dang Van, PCr and PQ criterion for particular groups of loading are presented in Table 2. Some trends were observed by these criteria:

- The average of ΔFI for the Dang Van criterion for proportional experiments without mean stress is conservative in all lifetime domains and increases from HCF to LCF lifetime domain.

- The mean value of ΔFI for the PCr criterion for proportional experiments without mean stress and for experiments loaded non-proportionally with mean stress is conservative. We can see a rise of conservativeness of the mean value of ΔFI from HCF to LCF lifetime domain.

- The mean value of ΔFI for the PQ criterion for proportional experiments without mean stress is also conservative in all lifetime domains, with the same rise of conservativeness as stated by PCr. The mean values of ΔFI for the PQ criterion are for this type of loading closer to optimum (zero value) than the mean value of ΔFI for the PCr criterion.

- Results for loading combinations with non-proportional loading without mean stress and for loading combinations with proportional loading with mean stress were not evaluated here because of small number of gathered experimental data for these type of loading.

- Results for PCr and PQ criteria are well balanced for all tested groups of loading.

7 Conclusion

The assumption, that it is possible to use multiaxial stress-based criteria also in the area of limited lifetime was confirmed. Considering mean values, standard deviations and ranges, the best results for all lifetime domains were obtained by using the PCr method. Further work should focus on gathering more experimental data, mainly for aluminum alloys, brittle and extra-ductile materials.

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